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Invisible Decays of Z-boson to Two Dark Matters in Gauge-Invariant-Simplified Model of Dark Matter-Neutrino Interaction Scenario.

Sittha Jeamburaset,[a] Udom Robkob,[a] Patipan Uttayarat[b]

[a] *Department of Physics, Mahidol University, 272 Rama VI Road, Ratchathewi, Bangkok 10400, THAILAND*

[b] *Department of Physics, Srinakharinwirot University, 114 Sukhumvit 23rd Rd., Wattana, Bangkok 10110, THAILAND*

Abstract: We calculate the invisible decay width of Z-boson decays to two dark matters final state in a gauge-invariant-simplified model of dark matter-neutrino interaction framework. We find that in the case of a scalar dark matter, the amplitudes for this process involves with three-point scalars functions which have different massive internal lines. We compute the scalar functions in the analytic expressions based on 't Hooft and Veltman work.

Contents

1 Introduction

Evidence from astrophysics and cosmology points to the fact that the Universe is filled with a large quantity of non-relativistic matter that is weakly interacting. Determining the nature of the dark matter (DM) remains one of the primary open questions in understanding of particle physics. A wide variety of experiments are currently on search of DM interactions in the lab and sky, and evolve the new data to constrain the theoretical model.

Experiment at the Large Hadron Collider (LHC) has played an important role in observing the interaction between dark matter (DM) and the Standard Model (SM) particles. Because DM is invisible to the LHC detectors, its LHC signature involves a visible object that depends on the detail of SM particles interacting against the invisible DM. For understanding the nature of dark matter, we consider the gauge-invariant-simplified DM-neutrino interaction Model which enriches collider signatures of the dark sector. This model allows us to calculate an invisible decay width of Z-boson to two DMs through fermion one-loops diagrams. For most approaches to the calculation of one-loop amplitudes, the knowledge of scalar one-loop integral is sufficient. The methods used in the literature are based on the work of Passarino and Veltman and 't Hooft and Veltman which are the techniques for reducing the tensor integrals to scalar integrals.This work therefore concentrates on the integrals with different massive internal lines and these integrals can be evaluated analytically. We can therefore enhance the collider signatures by using the results of the invisible decay width to constrain our model scenario.

Our work is organized as follows. In section 2, we review the techniques to calculate one-loop integrals in the general way. In sections 3, we present the Lagrangain and couplings of the dark matter and mediator to each other and with the SM. In sections 4, we compute the amplitude for the Z-boson decay into 2DMs and in sections 5 derive the decay width of this process. We conclude and discuss in section 6.

2 One-Loop Calculations

In this section, we review traditional techniques for the one-loop calculations using in the sequel. We derive an analytic expression for scalar-one loop integrals (one, two and three-point functions) based on 't Hooft and Veltman's work and also investigate tensor integrals that can be reduced to linear combinations of scalar integrals times an external kinematic parameters (external momenta) known as Passarino-Veltman's reduction.

2.1 Passarino-Veltman Reduction

Most of the computations of one-loop integrals associates with tensor integrals. The Lorentz covariance structure of the integrals allows us to decompose the tensor integrals into tensors constructed from the external kinematic parameters (external momenta p_i), and the metric tensor $g_{\mu\nu}$ with totally symmetric coefficient functions (scalar integrals). This techniques is purposed by Passarino-Veltman in 1979. We will try to explain them and therefore summarize all of the integrals using in this work. Here, we use the same conventions presented in [6].

2.1.1 Reduction of Two and Three-Point Functions

Using the Lorentz decomposition of the tensor integrals the invariant functions can be reduced to scalar integrals. For tensors rank 1 and rank 2 three-point functions, it can be expressed as

$$C^{\mu} = p_1^{\mu} C_1 + p_2^{\mu} C_2 \tag{2.1}$$

$$C^{\mu\nu} = g^{\mu\nu} C_{00} + \sum_{i,j=1}^{2} p_i^{\mu} p_j^{\nu} C_{ij}, \qquad \text{where } C_{21} = C_{12} \tag{2.2}$$

We contract Eq.(2.1) with p_1 and p_2, we obtain the numerators expressed in term of the denominators.

$$\begin{aligned} l \cdot p_1 &= \frac{1}{2}\left(f_1 + d_2 - d_1\right), \quad f_1 = m_2^2 - m_1^2 - p_1^2\,, \\ l \cdot p_2 &= \frac{1}{2}\left(f_2 + d_3 - d_2\right), \quad f_2 = m_3^2 - m_2^2 - p_2^2 - 2p_1 \cdot p_2\,. \end{aligned} \tag{2.3}$$

Where $d_i = \left(l + \sum_{k=1}^{i-1} p_k\right)^2 - m_i^2, i = 1, 2, 3, ...$ For C^{μ}, we obtain the following results.

$$R_1^{[c]} = p_{1\mu} C^{\mu} = p_1^2 C_1 + (p_1 \cdot p_2) C_2 = \frac{1}{2}\left(f_1 C_0(1,2,3) + B_0(1,3) - B_0(2,3)\right) \tag{2.4}$$

$$R_2^{[c]} = p_{2\mu} C^{\mu} = (p_1 \cdot p_2) C_1 + p_2^2 C_2 = \frac{1}{2}\left(f_2 C_0(1,2,3) + B_0(1,2) - B_0(1,3)\right) \tag{2.5}$$

We find a system of equations,

$$\begin{pmatrix} R_1^{[c]} \\ R_2^{[c]} \end{pmatrix} = \begin{pmatrix} p_1 \cdot p_1 & p_1 \cdot p_2 \\ p_1 \cdot p_2 & p_2 \cdot p_2 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

We obtain,

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{p_1^2 p_2^2 - (p_1 \cdot p_2)^2} \begin{pmatrix} p_1 \cdot p_1 & -p_1 \cdot p_2 \\ -p_1 \cdot p_2 & p_2 \cdot p_2 \end{pmatrix} \begin{pmatrix} R_1^{[c]} \\ R_2^{[c]} \end{pmatrix} \tag{2.6}$$

For rank-two tensor, contract Eq.(2.2) with p_1 and p_2 we obtain

$$\begin{aligned} R_1^{[c]\nu} &= p_{1\mu} C^{\mu\nu} = p_1^{\nu}\left(p_1 \cdot p_1 C_{11} + p_1 \cdot p_2 C_{12} + C_{00}\right) + p_2^{\nu}\left(p_1 \cdot p_1 C_{12} + p_1 \cdot p_2 C_{22}\right) \qquad (2.7) \\ &= \frac{1}{2}\left(f_1 C^{\nu}(1,2,3) + B^{\nu}(1,3) - B^{\nu}(2,3)\right) \\ R_2^{[c]\nu} &= p_{2\mu} C^{\mu\nu} = p_1^{\nu}\left(p_1 \cdot p_2 C_{11} + p_2 \cdot p_2 C_{12}\right) + p_2^{\nu}\left(p_1 \cdot p_2 C_{12} + p_2 \cdot p_2 C_{22} + C_{00}\right) \qquad (2.8) \\ &= \frac{1}{2}\left(f_2 C^{\nu}(1,2,3) + B^{\nu}(1,2) - B^{\nu}(1,3)\right) \end{aligned}$$

Inserting Lorentz decomposition of $R_1^{[c]\nu}$ and $R_2^{[c]\nu}$,

$$R_1^{[c]\nu} = R_1^{[c1]} p_1^{\nu} + R_1^{[c2]} p_2^{\nu}$$

$$R_2^{[c]\nu} = R_2^{[c1]} p_1^{\nu} + R_2^{[c2]} p_2^{\nu}$$

This gives

$$R_1^{[c1]} p_1^\nu = \frac{1}{2}\left(f_1 C_1(1,2,3) + B_1(1,3) + B_0(2,3)\right) p_1^\nu \tag{2.9}$$

$$R_1^{[c2]} p_2^\nu = \frac{1}{2}\left(f_1 C_2(1,2,3) + B_1(1,3) - B_1(2,3)\right) p_2^\nu \tag{2.10}$$

When $B^\nu(2,3)$ is decomposed as follows

$$\begin{aligned}
B^\nu(2,3) &= \frac{1}{i\pi^2}\int d^d l \frac{l^\nu}{((l+p_1)^2 - m_2^2)((l+p_1+p_2)^2 - m_3^2)} \\
&= \frac{1}{i\pi^2}\int d^d l \frac{l^\nu - p_1^\nu}{(l^2 - m_2^2)((l+p_2)^2 - m_3^2)} \\
&= \frac{1}{i\pi^2}\int d^d l \frac{l^\nu}{(l^2 - m_2^2)((l+p_2)^2 - m_3^2)} - \frac{1}{i\pi^2}\int d^d l \frac{p_1^\nu}{((l+p_1)^2 - m_2^2)((l+p_1+p_2)^2 - m_3^2)} \\
B^\nu(2,3) &= p_2^\nu B_1(2,3) - p_1^\nu B_0(2,3)
\end{aligned}$$

and also

$$R_2^{[c1]} p_1^\nu = \frac{1}{2}\left(f_2 C_1(1,2,3) + B_1(1,2) - B_1(1,3)\right) p_1^\nu \tag{2.11}$$

$$R_2^{[c2]} p_2^\nu = \frac{1}{2}\left(f_2 C_2(1,2,3) - B_1(1,3)\right) p_2^\nu \tag{2.12}$$

Thus,

$$R_1^{[c1]} = p_1\cdot p_1 C_{11} + p_1\cdot p_2 C_{12} + C_{00} = \frac{1}{2}\left(f_1 C_1(1,2,3) + B_1(1,3) + B_0(2,3)\right) \tag{2.13}$$

$$R_2^{[c1]} = p_1\cdot p_2 C_{11} + p_2\cdot p_2 C_{12} = \frac{1}{2}\left(f_2 C_1(1,2,3) + B_1(1,2) - B_1(1,3)\right) \tag{2.14}$$

$$R_1^{[c2]} = p_1\cdot p_1 C_{12} + p_1\cdot p_2 C_{22} = \frac{1}{2}\left(f_1 C_2(1,2,3) + B_1(1,3) - B_1(2,3)\right) \tag{2.15}$$

$$R_2^{[c2]} = p_1\cdot p_2 C_{12} + p_2\cdot p_2 C_{22} + C_{00} = \frac{1}{2}\left(f_2 C_2(1,2,3) - B_1(1,3)\right) \tag{2.16}$$

We obtain a system of equations,

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \frac{1}{p_1^2 p_2^2 - (p_1\cdot p_2)^2}\begin{pmatrix} p_1\cdot p_1 & -p_1\cdot p_2 \\ -p_1\cdot p_2 & p_2\cdot p_2 \end{pmatrix}\begin{pmatrix} T_1^{[c1]} \\ T_2^{[c1]} \end{pmatrix} \tag{2.17}$$

$$\begin{pmatrix} C_{12} \\ C_{22} \end{pmatrix} = \frac{1}{p_1^2 p_2^2 - (p_1\cdot p_2)^2}\begin{pmatrix} p_1\cdot p_1 & -p_1\cdot p_2 \\ -p_1\cdot p_2 & p_2\cdot p_2 \end{pmatrix}\begin{pmatrix} T_1^{[c2]} \\ T_2^{[c2]} \end{pmatrix} \tag{2.18}$$

Where $T_1^{[c1]} = R_1^{[c1]} - C_{00}$, $T_2^{[c1]} = R_2^{[c1]}$, $T_1^{[c]} = R_1^{[c2]}$ and $T_2^{[c2]} = R_2^{[c2]} - C_{00}$
To get C_{00} contract Eq. (2.2) with $g_{\mu\nu}$, we have

$$\begin{aligned}
g_{\mu\nu}C^{\mu\nu} &= \frac{1}{i\pi^2}\int d^d l \frac{l^2 - m_1^2}{d_1 d_2 d_3} + m_1^2 C_0 = D C_{00} + T_1^{[c1]} + T_2^{[c2]} \\
C_{00}(1,2,3) &= \frac{1}{2(D-2)}\left(2m_1^2 C_0(1,2,3) - f_2 C_2(1,2,3) - f_1 C_1(1,2,3) + B_0(2,3)\right)
\end{aligned} \tag{2.19}$$

Following from the above process, we have an expression for two-point functions summarized in the below. For tensors rank 1 two-point functions, it can be expressed as

$$B^{\mu} = p^{\mu} B_1 \tag{2.20}$$

Where p is the external momentum. By contracting through with p the form factor can be expressed entirely in terms of scalar integrals. The results are,

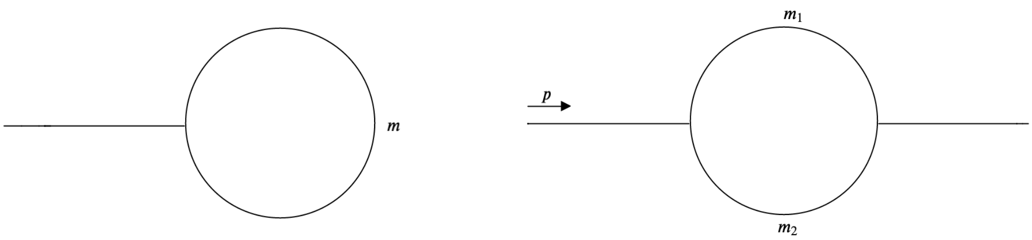
$$B_1(1,2) = \frac{1}{2p_1^2}\left[f_1 B_0(1,2) + A_0(1) - A_0(2)\right] \tag{2.21}$$

$$B_1(1,3) = \frac{1}{2(p_1+p_2)^2}\left[(f_1+f_2) B_0(1,3) + A_0(1) - A_0(3)\right] \tag{2.22}$$

$$B_1(2,3) = \frac{1}{2p_2^2}\left[(f_2 + 2p_1 \cdot p_2) B_0(2,3) + A_0(2) - A_0(3)\right] \tag{2.23}$$

2.2 Scalar Loop Integrals

With techniques using in the last section all one-loop integrals can be reduced to scalar-loop integrals. Here we derive the general analytic results for one-point (A_0), two-point (B_0) and three-point (C_0) scalar integrals based on 't Hooft and Veltman's result presented in [2].



(a) One-point scalar loop

(b) Two-point scalar loop

Figure 1: The one-point and two-point Green functions, the corresponding expression will be discussed below

2.2.1 One-Point Function

$$\begin{aligned}
A_0(m^2) &= \frac{\mu^{4-d}}{i\pi^2}\int d^d l \frac{1}{l^2 - m^2} \\
&= (2\pi)^{d-2}(-1)\frac{\Gamma(1-d/2)}{2^d \pi^{d/2}\Gamma(1)}(m^2)^{d/2-1}(\mu^2)^{2-d/2} \\
&= -m^2 \pi^{d/2-2}\left(\frac{m^2}{\mu^2}\right)^{d/2-2}\Gamma(1-d/2)
\end{aligned}$$

When $d = 4-\epsilon$, we have $\pi^{d/2-2} = 1 - \frac{\epsilon}{2}\ln\pi$, $\left(\frac{m^2}{\mu^2}\right)^{d/2-2} = \left(1 - \frac{\epsilon}{2}\ln\frac{m^2}{\mu^2}\right)$
and $\Gamma(1-d/2) = -\left(\frac{2}{\epsilon} - \gamma_\epsilon + 1\right)$

$$\begin{aligned}
A_0(m^2) &= m^2\left(1 - \frac{\epsilon}{2}\ln\pi\right)\left(1 - \frac{\epsilon}{2}\ln\frac{m^2}{\mu^2}\right)\left(\frac{2}{\epsilon} - \gamma_\epsilon + 1\right) \\
&= m^2\left(\frac{2}{\epsilon} - \gamma_\epsilon + 1 - \ln\frac{m^2}{\mu^2} - \ln\pi\right) \\
A_0(m^2) &= m^2\left(\frac{2}{\epsilon} + 1 - \ln\frac{m^2}{\widetilde{\mu}^2}\right) \qquad \text{where } \; e^{\gamma_\epsilon}\pi\widetilde{\mu}^2 = \mu^2
\end{aligned} \tag{2.24}$$

2.2.2 Two-Point Function

$$B_0(p^2; m_1^2, m_2^2) = \frac{\mu^{4-d}}{i\pi^2}\int d^dl \frac{1}{(l^2 - m_1^2)((l+p)^2 - m_2^2)}$$

Introducing Feynman parameters in denominator and shifting the momentum. We get,

$$\begin{aligned}
D &= (1-x)(l^2 - m_1^2) + x((l+p)^2 - m_2^2) \\
&= l^2 + x(1-x)p^2 - (1-x)m_1^2 - xm_2^2 \\
D &= l^2 - \Delta \quad , \Delta = x(x-1)p^2 + (1-x)m_1^2 + xm_2^2
\end{aligned}$$

We have,

$$\begin{aligned}
B_0(p^2; m_1^2, m_2^2) &= \frac{\mu^{4-d}}{i\pi^2}\int_0^1 dx \int d^dl \frac{1}{(l^2-\Delta)^2} \\
&= \pi^{d/2-2}\Gamma(2-d/2)\int_0^1 dx \left(\frac{\Delta}{\mu^2}\right)^{d/2-2} \\
&= \left(\frac{2}{\epsilon} - \gamma_\epsilon - \ln\pi\right) - \int_0^1 dx \ln\frac{(x^2p^2 - (p^2 + m_1^2 - m_2^2)x + m_1^2)}{\mu^2}
\end{aligned}$$

Where $\Delta = p^2(x-x_1)(x-x_2) = x^2p^2 - (p^2 + m_1^2 - m_2^2)x + m_1^2$
and $x_1, x_2 = \frac{1}{2p^2}\left((p^2 + m_1^2 - m_2^2) \pm \lambda^{1/2}(p^2, m_1^2, m_2^2)\right) = \eta \pm \Delta\eta$
$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$
Consider,

$$\begin{aligned}
\int_0^1 dx \ln\left((x-x_1)(x-x_2)\right) &= -2 + \ln\left(1 - x_1 - x_2 - x_1x_2\right) - \eta\ln\left(\frac{(x-x_1)(x-x_2)}{x_1x_2}\right) \\
&\quad + \Delta\eta\ln\left(\frac{(x_1-1)x_2}{(x_2-1)x_1}\right) \\
&= -2 - \ln k^2 + \ln(m_1m_2) + \frac{m_2^2 - m_1^2}{p^2}\ln\left(\frac{m_2}{m_1}\right) \\
&\quad - \frac{\lambda^{1/2}(p^2, m_1^2, m_2^2)}{p^2}\ln\left(\frac{m_1^2 + m_2^2 - p^2 + \lambda^{1/2}(p^2, m_1^2, m_2^2)}{2m_1m_2}\right)
\end{aligned}$$

Finally,

$$
\begin{aligned}
B_0(p^2;m_1^2,m_2^2) &= \left(\frac{2}{\epsilon} - \gamma_\epsilon - \ln\pi\right) + 2 - \ln\frac{(m_1 m_2)}{\mu^2} + \frac{m_1^2 - m_2^2}{p^2}\ln\left(\frac{m_2}{m_1}\right) \\
&+ \frac{\lambda^{1/2}(p^2,m_1^2,m_2^2)}{p^2}\ln\left(\frac{m_1^2+m_2^2-p^2+\lambda^{1/2}(p^2,m_1^2,m_2^2)}{2m_1m_2}\right) \\
B_0(p^2;m_1^2,m_2^2) &= \frac{2}{\epsilon} + 2 - \ln\left(\frac{m_1m_2}{\widetilde{\mu}^2}\right) + \frac{m_1^2 - m_2^2}{p^2}\ln\left(\frac{m_2}{m_1}\right) \\
&+ \frac{\lambda^{1/2}(p^2,m_1^2,m_2^2)}{p^2}\ln\left(\frac{m_1^2+m_2^2-p^2+\lambda^{1/2}(p^2,m_1^2,m_2^2)}{2m_1m_2}\right)
\end{aligned}
\tag{2.25}
$$

2.2.3 Three-Point Function

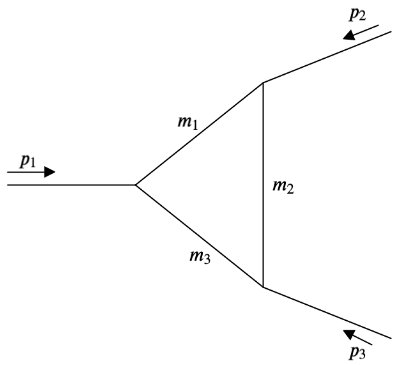


Figure 2: Three-point scalar loop

$$
C_0(p_1^2,p_2^2;m_1^2,m_2^2,m_3^2) = \frac{1}{i\pi^2}\int d^dl \frac{1}{(l^2-m_1^2)((l+p_1)^2-m_2^2)((l+p_1+p_2)^2-m_3^2)}
$$

Introducing Feynman parameters in denominator and shifting the momentum $(l \to l - p_1(1-y) - p_2(1-x))$, we get

$$
\begin{aligned}
D &= y(l^2-m_1^2) + (x-y)((l+p_1)^2-m_2^2) + (1-x)((l+p_1+p_2)^2-m_3^2) \\
&= l^2 - p_1^2y^2 - p_2^2x^2 - 2p_1\cdot p_2xy + (m_3^2-m_2^2+p_2^2)x + (m_2^2-m_1^2+p_1^2+2p_1\cdot p_2)y - m_3^2 \\
D &= l^2 + ax^2 + by^2 + cxy + dx + ey + f = l^2 - \Delta \quad, \Delta = -(ax^2+by^2+cxy+dx+ey+f)
\end{aligned}
$$

Where

$$
\begin{aligned}
a &= -p_2^2 \\
b &= -p_1^2 \\
c &= -2p_1\cdot p_2 = -(p_1+p_2)^2 + p_1^2 + p_2^2 \\
d &= \left(m_3^2 - m_2^2 + p_2^2\right) \\
e &= \left(m_2^2 - m_1^2 + p_1^2 + 2p_1\cdot p_2\right) \\
f &= -m_3^2
\end{aligned}
\tag{2.26}
$$

Then

$$\begin{aligned}C_0(p_1^2,p_2^2;m_1^2,m_2^2,m_3^2) &= \frac{i\Gamma(3)(-1)^3}{i\pi^2}\int_0^1 dx\int_0^x dy\int d\Omega_{d-1}\int_0^\infty dq_E\frac{q_E^{d-1}}{\left(q_E^2+\Delta\right)^3}\\ &= \frac{-i\Gamma(3)}{i\pi^2}\int_0^1 dx\int_0^x dy\frac{\pi^{d/2}}{\Gamma(d/2)}\frac{(\Delta)^{d/2-3}\Gamma(3-d/2)\Gamma(d/2)}{\Gamma(3)}\\ &= \frac{-1}{\pi^2}\int_0^1 dx\int_0^x dy\pi^{d/2}(\Delta)^{d/2-3}\Gamma(3-d/2)\end{aligned}$$

Hence around $n=4$ we have

$$C_0(p_1^2,p_2^2;m_1^2,m_2^2,m_3^2) = \int_0^1 dx\int_0^x dy\frac{1}{ax^2+by^2+cxy+dx+ey+f}$$

The problem of above integral is the appearance of the two quadratic contributions x^2 and y^2, we make the change of the variables, namely

$$y = y' + \alpha x$$

We get

$$\begin{aligned}D' &= ax^2+b(y'+\alpha x)^2+cx(y'+\alpha x)+dx+e(y'+\alpha x)+f\\ &= \left(b\alpha^2+c\alpha+a\right)x^2+by'^2+xy'(c+2\alpha b)+x(d+e\alpha)+ey'+f\end{aligned}$$

Where α parameter is one of the roots of the equation

$$b\alpha^2+c\alpha+a=0 \tag{2.27}$$

Without loss of generality, we may select one of α, for example

$$\alpha = \frac{p_1\cdot p_2+\sqrt{\Delta_3}}{-p_1^2}$$

Where $\Delta_3 = (p_1\cdot p_2)^2 - p_1^2 p_2^2$ is the corresponding Gram determinant. then the denominator reads

$$D' = by'^2+xy'(c+2\alpha b)+x(d+e\alpha)+ey'+f$$

Which becomes linear in x. Now we are going to perform the series of the change of the variables

$$\int_0^1 dx\int_0^x dy = \int_0^1 dx\int_{-\alpha x}^{(1-\alpha)x} dy' = \int_{-\alpha}^0 dy'\int_{-\frac{y'}{\alpha}}^1 dx + \int_0^{1-\alpha} dy'\int_{\frac{y'}{1-\alpha}}^1 dx$$

This gives

$$\begin{aligned}C_0 = &\int_{-\alpha}^0 dy'\frac{1}{y'(c+2\alpha b)+d+e\alpha}\ln\left(by'^2+xy'(c+2\alpha b)+x(d+e\alpha)+ey'+f\right)\Big|_{-\frac{y'}{\alpha}}^1\\ &+\int_0^{1-\alpha} dy'\frac{1}{y'(c+2\alpha b)+d+e\alpha}\ln\left(by'^2+xy'(c+2\alpha b)+x(d+e\alpha)+ey'+f\right)\Big|_{\frac{y'}{1-\alpha}}^1\\ = &\int_{-\alpha}^0 dy'\frac{1}{y'(c+2\alpha b)+d+e\alpha}\ln\frac{by'^2+y'(c+2\alpha b+e)+d+e\alpha+f}{by'^2-y'^2(c+2\alpha b)/\alpha-y'(d+e\alpha)/\alpha+ey'+f}\\ &+\int_0^{1-\alpha} dy'\frac{1}{y'(c+2\alpha b)+d+e\alpha}\ln\frac{by'^2+y'(c+2\alpha b)+d+e\alpha+ey'+f}{by'^2+y'^2(c+2\alpha b)/(1-\alpha)+y'(d+e\alpha)/(1-\alpha)+ey'+f}\end{aligned}$$

Let us define $N(y') = y'(c+2\alpha b) + d + \alpha e$, we have

$$
\begin{aligned}
C_0 = & \int_{-\alpha}^{1-\alpha} dy' \frac{1}{N(y')} \ln\left(by'^2 + ey' + f + N(y')\right) \\
& - \int_{-\alpha}^{0} dy' \frac{1}{N(y')} \ln\left(by'^2 + \mathrm{ey}' + f - \frac{y'}{\alpha} N(y')\right) \\
& - \int_{0}^{1-\alpha} dy' \frac{1}{N(y')} \ln\left(by'^2 + ey' + f + \frac{y'}{1-\alpha} N(y')\right)
\end{aligned}
$$

Notice that expression $1/N(y')$ has singularity at $y_0 = -(d+\alpha e)/(c+2\alpha b)$. In order to have residuum equal to zero at y_0 we add to every integral the expression $-\ln\left(by_0^2 + \mathrm{ey}_0 + f\right)$

$$
\begin{aligned}
C_0 = & \int_{-\alpha}^{1-\alpha} dy' \frac{1}{N(y')} \left\{\ln\left(by'^2 + ey' + f + N(y')\right) - \ln\left(by_0^2 + ey_0 + f\right)\right\} \\
& - \int_{-\alpha}^{0} dy' \frac{1}{N(y')} \left\{\ln\left(by'^2 + ey' + f - \frac{y'}{\alpha} N(y')\right) - \ln\left(by_0^2 + ey_0 + f\right)\right\} \\
& - \int_{0}^{1-\alpha} dy' \frac{1}{N(y')} \left\{\ln\left(by'^2 + ey' + f + \frac{y'}{1-\alpha} N(y')\right) - \ln\left(by_0^2 + ey_0 + f\right)\right\}
\end{aligned}
$$

This additional term allows studying the integrals with complex α. Now we make the substitution $y' = y - \alpha,\ y = -y'/\alpha$ and $y = y'/(1-\alpha)$ respectively.
And also $y_{01} = y_0 + \alpha, y_{02} = -y_0/\ \alpha$ and $y_{03} = y_0/(1-\alpha)$ when $N(y_0) = 0$.

We have

$$
\begin{aligned}
C_0 = & \frac{1}{(c+2\alpha b)} \int_0^1 \frac{dy}{y - y_{01}} \left\{\ln\left(by^2 + y(e+c) + a + d + f\right) - \ln\left(by_{01}^2 + (c+e)y_{01} + a + d + f\right)\right\} \\
& + \frac{1}{(c+2\alpha b)} \int_0^1 \frac{dy}{y - y_{02}} \left\{\ln\left(ay^2 + dy + f\right) - \ln\left(ay_{02}^2 + dy_{02} + f\right)\right\} \\
& - \frac{1}{(c+2\alpha b)} \int_0^1 \frac{dy}{y - y_{03}} \left\{\ln(y^2(a+b+c) + y(d+e) + f) - \ln((a+b+c)y_{03}^2 + (e+d)y_{03} + f)\right\} \\
= & \frac{1}{(c+2\alpha b)} \left[S_3(y_{01}, y_{11}, y_{21}) + S_3(y_{02}, y_{12}, y_{22}) - S_3(y_{03}, y_{13}, y_{23})\right]
\end{aligned}
$$

$$
C_0 = \frac{1}{2\sqrt{\Delta_3}} \left[S_3(y_{01}, y_{11}, y_{21}) + S_3(y_{02}, y_{12}, y_{22}) - S_3(y_{03}, y_{13}, y_{23})\right] \tag{2.28}
$$

Where

$$
\begin{aligned}
y_{01} &= y_0 + \alpha = -\frac{\alpha c + 2a + d + \alpha e}{c + 2\alpha b} \\
y_{02} &= -\frac{y_0}{\alpha} = -\frac{d + \alpha e}{\alpha c + 2a} \\
y_{03} &= \frac{y_0}{1-\alpha} = -\frac{d + \alpha c}{c + 2\alpha b + \alpha c + 2a}
\end{aligned}
$$

y_{11} and y_{21} are the roots of $by^2 + y(e+c) + a + d + f = 0$, y_{12} and y_{22} are the roots of $ay^2 + dy + f = 0$ and y_{13} and y_{23} are the roots of $y^2(a+b+c) + y(d+e) + f = 0$

Consider the integral

$$S_3\left(y_0,y_1,y_2\right)=\int_0^1 dy\frac{1}{y-y_0}\left[\ln\left(ay^2+by+c\right)-\ln\left(ay_0^2+by_0+c\right)\right]$$

where a is real, while b, c and y_0. may be complex, with the restriction that the imaginary part of the argument of the first logarithm has always the same sign in the y range $[0,1]$
Let

$$ay^2+by+c=a\left(y-y_1\right)\left(y-y_2\right),\quad y_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Notice that

$$a\left(y-y_1\right)\left(y-y_2\right)=ay^2-ay\left(y_1+y_2\right)+ay_1y_2,\quad -\left(y_1+y_2\right)=\frac{b}{a},\quad y_1y_2=\frac{c}{a}$$

For arbitrary y imaginary parts read $-a\,\mathrm{Im}\left(y_1+y_2\right)+a\,\mathrm{Im}\left(y_1y_2\right)$ while for $y=0$ reads $a\,\mathrm{Im}\left(y_1y_2\right)$. Thus, the sign of $-a\,\mathrm{Im}\left(y_1+y_2\right)+a\,\mathrm{Im}\left(y_1y_2\right)$ must be the same sign as $a\,\mathrm{Im}\left(y_1y_2\right)$.
Let ϵ and δ be infinitesimal quantities having the opposite sign to the imaginary parts of the arguments of the two logarithms.

$$\ln\left(a\left(y-y_1\right)\left(y-y_2\right)\right)\to\ln\left(\left(a-i\epsilon\right)\left(y-y_1\right)\left(y-y_2\right)\right)=\ln(a-i\epsilon)+\ln\left(\left(y-y_1\right)\left(y-y_2\right)\right)$$

$$\ln\left(a\left(y_0-y_1\right)\left(y_0-y_2\right)\right)\to\ln\left(\left(a-i\delta\right)\left(y_0-y_1\right)\left(y_0-y_2\right)\right)=\ln(a-i\delta)+\ln\left(\left(y_0-y_1\right)\left(y_0-y_2\right)\right)$$

Then

$$\begin{aligned}S_3&=\int_0^1 dy\frac{1}{y-y_0}\left[\ln\left(\left(y-y_1\right)\left(y-y_2\right)\right)-\ln\left(\left(y_0-y_1\right)\left(y_0-y_2\right)\right)+\ln(a-i\epsilon)-\ln(a-i\delta)\right]\\&=\int_0^1 dy\frac{1}{y-y_0}\left[\ln\left(\left(y-y_1\right)\left(y-y_2\right)\right)-\ln\left(\left(y_0-y_1\right)\left(y_0-y_2\right)\right)-\eta\left(a-i\epsilon,\frac{1}{a-i\delta}\right)\right]\\&=\int_0^1 dy\frac{1}{y-y_0}\left[\ln\left(\left(y-y_1\right)\left(y-y_2\right)\right)-\ln\left(\left(y_0-y_1\right)\left(y_0-y_2\right)\right)\right]-\eta\left(a-i\epsilon,\frac{1}{a-i\delta}\right)\int_0^1 dy\frac{1}{y-y_0}\end{aligned}$$

When

$$\begin{aligned}&\ln(ab)=\ln a+\ln b+\eta(a,b)\\&\eta(a,b)=2\pi i[\theta(-\mathrm{Im}(a))\theta(-\mathrm{Im}(b))\theta(\mathrm{Im}(ab))-\theta(\mathrm{Im}(a))\theta(\mathrm{Im}(b))\theta(-\mathrm{Im}(ab))]\end{aligned}$$

$$S_3=\int_0^1 dy\frac{1}{y-y_0}\left[\ln\left(\left(y-y_1\right)\left(y-y_2\right)\right)-\ln\left(\left(y_0-y_1\right)\left(y_0-y_2\right)\right)\right]-\eta\left(a-i\epsilon,\frac{1}{a-i\delta}\right)\ln\left(\frac{y_0-1}{y_0}\right)$$

$$\begin{aligned}S_3=&\int_0^1 dy\frac{1}{y-y_0}\Big[\ln\left(y-y_1\right)+\ln\left(y-y_2\right)+\eta\left(-y_1,-y_2\right)\\&-\ln\left(y_0-y_1\right)-\ln\left(y_0-y_2\right)-\eta\left(y_0-y_1,y_0-y_2\right)\Big]-\eta\left(a-i\epsilon,\frac{1}{a-i\delta}\right)\ln\left(\frac{y_0-1}{y_0}\right)\end{aligned}$$

Finally,

$$S_3=\quad R\left(y_0,y_1\right)+R\left(y_0,y_2\right)+\left[\eta\left(-y_1,-y_2\right)-\eta\left(y_0-y_1,y_0-y_2\right)-\eta\left(a-i\epsilon,\frac{1}{a-i\delta}\right)\right]\ln\left(\frac{y_0-1}{y_0}\right)$$

Where R integrals are evaluated as follows

$$R\left(y_0, y_1\right)=\int_0^1 dy \frac{1}{y-y_0}\left\{\ln\left(y-y_1\right)-\ln\left(y_0-y_1\right)\right\}$$

Now we make the change of the variables $y \rightarrow y-y_1$

$$=\int_{-y_1}^{1-y_1} dy \frac{1}{y+y_1-y_0}\left\{\ln y-\ln\left(y_0-y_1\right)\right\}$$

$$=\int_0^{1-y_1} dy \frac{1}{y+y_1-y_0}\left\{\ln y-\ln\left(y_0-y_1\right)\right\}-\int_0^{-y_1} dy \frac{1}{y+y_1-y_0}\left\{\ln y-\ln\left(y_0-y_1\right)\right\}$$

Changing the variables $y=(1-y_1)y'$ and $y=-y_1y'$ respectively. Using partial integration and dilogarithm identity e.g. $\mathrm{Li}_2(x)=-\mathrm{Li}_2(1-x)+\frac{1}{6}\pi^2-\ln(x)\ln(1-x)$. we get

$$\begin{aligned}R\left(y_0, y_1\right)= & \ln\left(\frac{-y_0}{y_1-y_0}\right)\eta\left(-y_1, \frac{1}{y_0-y_1}\right)-\ln\left(\frac{1-y_0}{y_1-y_0}\right)\eta\left(1-y_1, \frac{1}{y_0-y_1}\right) \\ & +\mathrm{Li}_2\left(\frac{y_0}{y_0-y_1}\right)-\mathrm{Li}_2\left(\frac{1-y_0}{y_1-y_0}\right)\end{aligned}$$

2.3 Some Useful Integrals

2.3.1 $C_0(m_H^2, 0, 0; m_W^2, m_W^2, m_W^2)$

This physical expression is important for Higgs decays to two photons calculation through W-boson loop and fermion loops. The previous result is given in [12].
By using Eq. (2.26) with $(p_1+p_2)^2=m_H^2 \rightarrow 2p_1\cdot p_2=m_H^2$, $p_1^2=p_2^2=0$
We have , $a=b=d=0$, $c=-m_H^2$, $e=m_H^2$ and $f=-m_W^2$
We will try to derive it in two different ways.

(I) First : Direct Calculation

$$\begin{aligned}C_0(m_H^2, 0, 0; m_W^2, m_W^2, m_W^2) & =\int_0^1 dx \int_0^x dy \frac{1}{m_H^2 y(1-x)-m_W^2} \\ & =\int_0^1 dx' \int_0^{1-x'} dy \frac{1}{m_H^2 y x'-m_W^2} \\ & =\frac{1}{m_W^2}\int_0^1 dx' \int_0^{1-x'} dy \frac{1}{m_H^2 y x'-m_W^2}\ , \ \tau=\frac{m_H^2}{4m_W^2} \\ & =\frac{-1}{m_W^2}\int_0^1 dx' \int_0^{1-x'} dy \frac{1}{1-4\tau x' y} \\ & =\frac{-1}{m_W^2}\int_0^1 dx' \frac{-\ln\left(1-4\tau x'\left(1-x'\right)\right)}{4\tau x'} \\ & =-\frac{1}{4\tau m_W^2}\int_0^1 dx' \int_0^\tau d\tau' \frac{d}{d\tau'}\frac{-\ln\left(1-4\tau' x'\left(1-x'\right)\right)}{x'} \\ & =-\frac{1}{4\tau m_W^2}\int_0^\tau d\tau' \int_0^1 dx' \frac{4\left(1-x'\right)}{1-4\tau' x'\left(1-x'\right)}\end{aligned}$$

We explore the integral in dx'. We shift $x' \to x' + \frac{1}{2}$

$$\int_0^1 dx' \frac{4\,(1-x')}{1-4\tau' x'\,(1-x')} = \int_{-\frac{1}{2}}^{\frac{1}{2}} dx' \frac{4\left(\frac{1}{2}-x'\right)}{1-4\tau'\left(\frac{1}{4}-x'^2\right)}$$

The domain and the denominator are symmetric to $x' \to -x'$, so we can drop the odd term in the numerator.

$$\begin{aligned}
\int_{-\frac{1}{2}}^{\frac{1}{2}} dx' \frac{2}{1-\tau'+4\tau' x'^2} &= \frac{1}{1-\tau'} \int_{-\frac{1}{2}}^{\frac{1}{2}} dx' \frac{2}{1+\left(2\frac{\sqrt{\tau'}}{\sqrt{1-\tau'}}x'\right)^2} \\
&= \frac{1}{\sqrt{\tau'\,(1-\tau')}} \arctan 2\frac{\sqrt{\tau'}}{\sqrt{1-\tau'}} x' \Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} \\
&= 2\frac{1}{\sqrt{\tau'\,(1-\tau')}} \arctan \frac{\sqrt{\tau'}}{\sqrt{1-\tau'}} = 2\frac{1}{\sqrt{\tau'\,(1-\tau')}} \arcsin\sqrt{\tau'}
\end{aligned}$$

Finally, we can integrate with respect to τ':

$$\begin{aligned}
C_0(m_H^2,0,0;m_W^2,m_W^2,m_W^2) &= -\frac{2}{4\tau m_W^2} \int_0^\tau d\tau' \frac{1}{\sqrt{\tau'\,(1-\tau')}} \arcsin\sqrt{\tau'} \\
&= -\frac{1}{\tau m_W^2} \int_0^\tau \arcsin\sqrt{\tau'} d(\arcsin\sqrt{\tau'}) \\
&= -\frac{2}{4\tau m_W^2} \arcsin^2\sqrt{\tau}
\end{aligned}$$

The arcsin can be analitically continued for, $\tau > 1$, giving

$$f(\tau) = \begin{cases} \arcsin^2(\sqrt{\tau}) & \text{for} \quad \tau \leq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \text{for} \quad \tau > 1 \end{cases}$$

Thus,

$$C_0(m_H^2,0,0;m_W^2,m_W^2,m_W^2) = \frac{-1}{4\tau m_W^2}(2f(\tau)) = \frac{-2}{m_H^2} f(\tau) \tag{2.29}$$

(II) Second : Traditional Technique

$$\begin{aligned}
C_0(m_H^2,0,0;m_W^2,m_W^2,m_W^2) &= \int_0^1 dy \int_y^1 dx \frac{1}{m_H^2 y(1-x) - m_W^2} \\
&= \frac{1}{-m_H^2} \int_0^1 \frac{dy}{y} \ln\left(\frac{-m_W^2}{m_H^2 y(1-y) - m_W^2}\right) \\
&= \frac{1}{-m_H^2} \int_0^1 \frac{dy}{y} \ln\left(\frac{1}{4\tau y^2 - 4\tau y + 1}\right) \;,\; \tau = \frac{m_H^2}{4m_W^2} \\
&= \frac{1}{-m_H^2} \left[\mathrm{Li}_2(1/y_1) + \mathrm{Li}_2(1/y_2) - \int_0^1 \frac{dy}{y} \ln(4\tau y_1 y_2)\right] \;,\; 4\tau y_1 y_2 = 1
\end{aligned}$$

Where y_1 and y_2 are the roots of $4\tau y^2 - 4\tau y + 1$.
Using the dilogarithm identity $\mathrm{Li}_2(z) + \mathrm{Li}_2(z/(z-1)) = -\frac{1}{2}\ln^2(1-z)$, we get

$$C_0(m_H^2, 0, 0; m_W^2, m_W^2, m_W^2) = \frac{-2}{m_H^2} f(\tau)$$

In this case, the denominator is $1/(c + 2\alpha b) \to 1/c = -1/m_H^2 \ (\alpha = 0)$. All η- function and the terms coresponding to $\mathrm{Li}_2\left(y_0/(y_0 - y_i)\right)$ vanish since it is real and have no y_0.

2.3.2 $C_0(m_H^2, 0, m_Z^2; m_W^2, m_W^2, m_W^2)$

This expression is important for Higgs decays to Z-boson and photon calculation .
Using Eq. (2.26) with $(p_1+p_2)^2 = m_H^2 \to 2p_1 \cdot p_2 = m_H^2 - m_Z^2$, $p_1^2 = m_Z^2$ and $p_2^2 = 0$
We have , $a = 0$, $b = -m_Z^2$, $c = m_Z^2 - m_H^2$, $d = 0$, $e = m_H^2$ and $f = -m_W^2$

$$\begin{aligned}
C_0 &= \int_0^1 dx \int_0^x dy \frac{1}{by^2 + cxy + ey + f} = \int_0^1 dy \int_y^1 dx \frac{1}{by^2 + cxy + ey + f} \\
&= \frac{1}{c}\int_0^1 \frac{dy}{y} \ln\left(\frac{by^2 + (c+e)y + f}{(b+c)y^2 + ey + f}\right) \\
&= \frac{1}{c}\left[\int_0^1 \frac{dy}{y}\ln\left(\frac{(y_1 - y)(y_2 - y)}{y_1 y_2}\right) - \int_0^1 \frac{dy}{y}\ln\left(\frac{(y_3 - y)(y_4 - y)}{y_3 y_4}\right) - \int_0^1 \frac{dy}{y}\ln\left(\frac{4\tau_Z y_1 y_2}{4\tau_H y_3 y_4}\right)\right] \\
&= -\frac{1}{c}\left[\mathrm{Li}_2(1/y_1) + \mathrm{Li}_2(1/y_2) - \mathrm{Li}_2(1/y_3) - \mathrm{Li}_2(1/y_4)\right]
\end{aligned}$$

Where $y_1, y_2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \tau_Z^{-1}}$,and $y_3, y_4 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \tau_H^{-1}}$
with $\tau_H = m_H^2/4m_W^2$, $\tau_Z = m_Z^2/4m_W^2$
Finally,

$$4m_W^2 C_0(m_H^2, 0, m_Z^2; m_W^2, m_W^2, m_W^2) = \frac{-2}{\tau_Z - \tau_H}\left[f(\tau_Z) - f(\tau_H)\right] \tag{2.30}$$

Which differ from Djouadi's result [11] by how they define the sign of the dilogarithm function.

3 Minimal Dark Matter-Neutrino Interaction : Simplified Model

Here, we start from the simplified model for DM-neutrino interaction presented in the previous work [1]. They make an assumptions that DM, χ, a mediator, ϕ (or ψ for a fermionic mediator), and the SM fields are the only light degrees of freedom. The rest of new physics particles are heavy and decoupled. In this simplified scenario, the renormalizable interaction between DM and neutrino is given by

$$\mathcal{L}_{int} = \begin{cases} y\bar{\chi}P_L\nu\phi + \text{ h.c.} & \text{fermionic DM,} \\ y\bar{\psi}P_L\nu\chi + \text{ h.c.} & \text{scalar DM} \end{cases} \tag{3.1}$$

Where y is the DM-neutrino coupling constant. In this work, we will assume that DM is not its own antiparticle. To preserve SU(2) gauge invariance, we embed our neutral messenger into a SU(2) doublet with hypercharge $Y = -1/2$

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi \\ \phi^- \end{pmatrix}, \quad \text{fermionic DM} \\ \Psi &= \begin{pmatrix} \psi \\ \psi^- \end{pmatrix}, \quad \text{scalar DM.} \end{aligned} \tag{3.2}$$

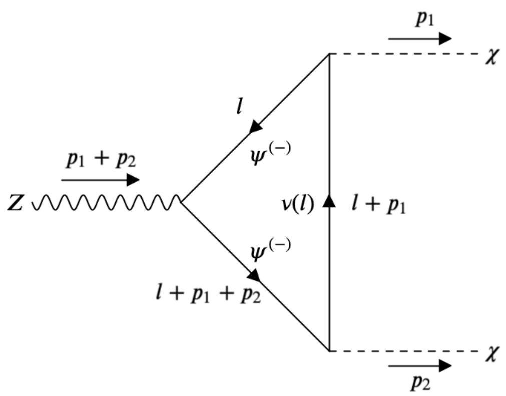
Thus, the SU(2) invariant DM-neutrino interactions become,

$$\mathcal{L}_{int} = \begin{cases} y\bar{\ell}P_R\chi\Phi + \text{h.c.}, & \text{fermionic DM} \\ y\bar{\ell}P_R\Psi\chi + \text{h.c.}, & \text{scalar DM} \end{cases} \tag{3.3}$$

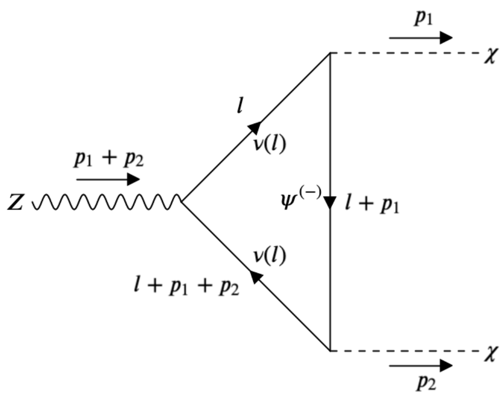
For the scalar DM case, we should introduce Ψ in a vector-like representation to avoid gauge anomaly. Also in the scalar DM case, we can impose Z_2 symmetry in which χ and Ψ are odd while all the SM fields are even to avoid $\Psi - l$ mixing.

4 The Evaluation of the Amplitude

The model in Eq.(3.1) allows us to calculate an invisible decay width of Z-boson decays into two scalar-DMs through fermion-one-loop diagrams. Since gauge-invariant structure of the model implies our model to be renormalizable, this absence of direct couplings of Z-boson to dark matter (χ) at tree-level implies that one-loop contribution to the decay through the possible loops presented in Figure 3 must necessarily be finite (or there is no counter term to absorb the divergence of loop calculations). So, there is no need to renormalize and we expect that the decay width is finite.



(a)



(b)

Figure 3: Possible Feynman diagrams for Z-boson decays into scalar-2DMs

4.1 Loop Calculations

We will calculate the corresponding amplitudes presented in Figure 3 that can be extract to 5 sub-diagrams. Here, we omit the polarization vectors of the incoming Z-boson.

Since the four momenta of the Z-boson is p_1+p_2, we have $2(p_1\cdot p_2)=m_Z^2-2m_\chi^2$ when $p_1^2=p_2^2=m_\chi^2$. Also, the opposite fermion flows in the two diagrams leads to a relative sign between them. For the loops with neutral particles, we have

4.1.1 $I^Z_{\psi^0}$ $(\psi^0-\nu-\psi^0)$

$$\begin{aligned}
I^Z_{\psi^0} &= g^Z_{\psi^0}\int\frac{d^4l}{(2\pi)^4}\frac{\mathrm{Tr}\left[\left(\not{l}+\not{p}_1+\not{p}_2+m_\psi\right)\gamma^\mu\left(\not{l}+m_\psi\right)P_L\left(\not{l}+\not{p}_1+m_\nu\right)P_R\right]}{\left[l^2-m_\psi^2\right]\left[(l+p_1)^2-m_\nu^2\right]\left[(l+p_1+p_2)^2-m_\psi^2\right]}\\
&=\frac{g^Z_{\psi^0}}{2}\int\frac{d^4l}{(2\pi)^4}\frac{\mathrm{Tr}\left[\left(\not{l}+\not{p}_1+\not{p}_2\right)\gamma^\mu\left(\not{l}\right)\left(\not{l}+\not{p}_1\right)\right]+m_\psi^2\,\mathrm{Tr}\left[\gamma^\mu\left(\not{l}+\not{p}_1\right)\right]}{\left[l^2-m_\psi^2\right]\left[(l+p_1)^2-m_\nu^2\right]\left[(l+p_1+p_2)^2-m_\psi^2\right]}
\end{aligned}$$

Here, we find the $\epsilon^{\nu\mu\rho\sigma}p_{1\sigma}p_{2\nu}l_\rho$ term that must be vanish when we shift the momentum l.

$$\begin{aligned}
I^Z_{\psi^0} &= 2g^Z_{\psi^0}\int\frac{d^4l}{(2\pi)^4}\frac{(l+p_1+p_2)^\mu(l\cdot(l+p_1))-(l+p_1)^\mu(l\cdot(l+p_1+p_2))}{\left[l^2-m_\psi^2\right]\left[(l+p_1)^2-m_\nu^2\right]\left[(l+p_1+p_2)^2-m_\psi^2\right]}\\
&+2g^Z_{\psi^0}\int\frac{d^4l}{(2\pi)^4}\frac{l^\mu((l+p_1+p_2)\cdot(l+p_1))+2(l+p_1)^\mu m_\psi^2}{\left[l^2-m_\psi^2\right]\left[(l+p_1)^2-m_\nu^2\right]\left[(l+p_1+p_2)^2-m_\psi^2\right]}
\end{aligned}$$

$$I^Z_{\psi^0}=2g^Z_{\psi^0}\int\frac{d^4l}{(2\pi)^4}\frac{l^2l^\mu+l^2p_2^\mu+2(l\cdot p_1)l^\mu+(p_1^2+p_1\cdot p_2+m_\psi^2)l^\mu-(l\cdot p_2)p_1^\mu+(l\cdot p_1)p_2^\mu+m_\psi^2p_1^\mu}{\left[l^2-m_\psi^2\right]\left[(l+p_1)^2-m_\nu^2\right]\left[(l+p_1+p_2)^2-m_\psi^2\right]} \tag{4.1}$$

Applying Passarino-Veltman's reduction in the section 2.1.1 for each term above , we have

$$\begin{aligned}
\frac{1}{i\pi^2}\int d^dl\frac{l^2l^\mu}{d_1d_2d_3} &= \frac{1}{i\pi^2}\int d^dl\frac{(l^2-m_\psi^2)l^\mu}{d_1d_2d_3}+\frac{1}{i\pi^2}\int d^dl\frac{m_\psi^2l^\mu}{d_1d_2d_3}\\
&=\frac{1}{i\pi^2}\int d^dl\frac{l^\mu}{d_2d_3}+m_\psi^2C^\mu(1,2,3)\\
\frac{1}{i\pi^2}\int d^dl\frac{l^2l^\mu}{d_1d_2d_3} &= p_2^\mu B_1(2,3)-p_1^\mu B_0(2,3)+m_\psi^2C^\mu(1,2,3)
\end{aligned}$$

$$\frac{1}{i\pi^2}\int d^dl\frac{l^2p_2^\mu}{d_1d_2d_3}=\frac{p_2^\mu}{i\pi^2}\int d^dl\frac{(l^2-m_\psi^2)l^\mu}{d_1d_2d_3}+\frac{p_2^\mu}{i\pi^2}\int d^dl\frac{m_\psi^2l^\mu}{d_1d_2d_3}=p_2^\mu B_0(2,3)+m_\psi^2p_2^\mu C_0(1,2,3)$$

$$\begin{aligned}
\frac{1}{i\pi^2}\int d^dl\frac{2(l\cdot p_1)l^\mu}{d_1d_2d_3} &= \frac{1}{i\pi^2}\int d^dl\frac{(d_2-d_1+f_1)l^\mu}{d_1d_2d_3}=\frac{1}{i\pi^2}\int d^dl\left[\frac{l^\mu}{d_1d_3}-\frac{l^\mu}{d_2d_3}z+\frac{f_1l^\mu}{d_1d_2d_3}\right]\\
&=(p_1+p_2)^\mu B_1(1,3)-p_2^\mu B_1(2,3)+p_1^\mu B_0(2,3)+f_1C^\mu(1,2,3)
\end{aligned}$$

$$\frac{p_1^2 + p_1 \cdot p_2 + m_\psi^2}{i\pi^2} \int d^d l \frac{l^\mu}{d_1 d_2 d_3} = (p_1^2 + p_1 \cdot p_2 + m_\psi^2) C^\mu(1,2,3)$$

$$\frac{1}{i\pi^2} \int d^d l \frac{(-l \cdot p_2) p_1^\mu}{d_1 d_2 d_3} = \frac{-p_1^\mu}{2i\pi^2} \int d^d l \frac{(d_3 - d_2 + f_2) l^\mu}{d_1 d_2 d_3} = \frac{-p_1^\mu}{2} (B_0(1,2) - B_0(1,3) + f_2 C_0(1,2,3))$$

$$\frac{1}{i\pi^2} \int d^d l \frac{(l \cdot p_1) p_1^\mu}{d_1 d_2 d_3} = \frac{p_2^\mu}{2i\pi^2} \int d^d l \frac{(d_2 - d_1 + f_1) l^\mu}{d_1 d_2 d_3} = \frac{p_2^\mu}{2} (B_0(1,3) - B_0(2,3) + f_1 C_0(1,2,3))$$

Where we omit the constant μ for simplicity. Thus, Eq. (4.1) becomes

$$\begin{aligned}\frac{16\pi^2 I^Z_{\psi^0}}{2ig^Z_{\psi^0}} &= (m_\psi^2 + m_\nu^2 - m_\chi^2 + \frac{m_Z^2}{2}) C^\mu(1,2,3) + \left[\frac{(2m_\psi^2 + f_1) p_2^\mu}{2} + \frac{(2m_\psi^2 - f_2) p_1^\mu}{2}\right] C_0(1,2,3) \\ &+ \frac{p_2^\mu}{2} B_0(2,3) - \frac{p_1^\mu}{2} B_0(1,2) + (p_1 + p_2)^\mu B_1(1,3) + \frac{(p_1+p_2)^\mu}{2} B_0(1,3)\end{aligned}$$

Where $f_1 = m_\nu^2 - m_\psi^2 - m_\chi^2$, $f_2 = m_\psi^2 + m_\chi^2 - m_\nu^2 - m_Z^2$
and $B_1(1,3) = \frac{1}{2(p_1+p_2)^2}[(f_1+f_2)B_0(1,3)+A_0(1)-A_0(3)]$; $A_0(1) = A_0(3)$ $(m_1 = m_3 = m_\psi)$
Then,

$$\begin{aligned}\frac{16\pi^2 I^Z_{\psi^0}}{2ig^Z_{\psi^0}} &= (m_\psi^2 + m_\nu^2 - m_\chi^2 + \frac{m_Z^2}{2}) C^\mu(1,2,3) + \frac{p_2^\mu}{2} B_0(2,3) - \frac{p_1^\mu}{2} B_0(1,2) \\ &+ \frac{1}{2}\left[(m_\psi^2 + m_\nu^2 - m_\chi^2) p_2^\mu + (m_\nu^2 + m_Z^2 - m_\chi^2 + m_\psi^2) p_1^\mu\right] C_0(1,2,3)\end{aligned}$$

$C^\mu(1,2,3)$ can be reduced by using Eq.(2.6) - Eq.(2.8).

$$\begin{aligned}C^\mu(1,2,3) = &\frac{-p_1^\mu}{4m_\chi^2 - m_Z^2}\left[(m_\psi^2 + 3m_\chi^2 - m_\nu^2 - m_Z^2) C_0(1,2,3) + B_0(1,2) - B_0(1,3)\right] \\ &\frac{-p_2^\mu}{4m_\chi^2 - m_Z^2}\left[(m_\nu^2 - m_\psi^2 + m_\chi^2) C_0(1,2,3) + B_0(1,3) - B_0(2,3)\right]\end{aligned}$$

with $B_0(1,2) = B_0(2,3)$, we have

$$\begin{aligned}I^Z_{\psi^0} = \frac{ig^Z_{\psi^0}}{16\pi^2}&\Bigg[\frac{2m_\psi^2 + 2m_\nu^2 - 2m_\chi^2 + m_Z^2}{4m_\chi^2 - m_Z^2}\Big\{\big((m_Z^2 + m_\nu^2 - 3m_\chi^2 - m_\psi^2) p_1^\mu + (m_\psi^2 - m_\chi^2 - m_\nu^2) p_2^\mu\big) C_0(1,2,3) \\ &+ (p_2 - p_1)^\mu (B_0(1,2) - B_0(1,3))\Big\} + \left[(m_\psi^2 + m_\nu^2 - m_\chi^2) p_2^\mu + (m_\nu^2 + m_Z^2 - m_\chi^2 + m_\psi^2) p_1^\mu\right] C_0(1,2,3) \\ &+ (p_2 - p_1)^\mu B_0(1,2)\Bigg] \qquad (4.2)\end{aligned}$$

Where $C_0(1,2,3) = C_0(m_Z^2, m_\chi^2, m_\chi^2; m_\psi^2, m_\nu^2, m_\psi^2)$, $B_0(1,2) = B_0(2,3) = B_0(m_\chi^2; m_\nu^2, m_\psi^2)$
and $B_0(1,3) = B_0(m_Z^2, m_\psi^2, m_\psi^2)$

4.1.2 I^Z_ν $(\nu - \psi^0 - \nu)$

$$
\begin{aligned}
I^Z_\nu &= -g^Z_\nu \int \frac{d^4l}{(2\pi)^4} \frac{\mathrm{Tr}\left[\left(\not{l} + \not{p}_1 + \not{p}_2 + m_\nu\right)\gamma^\mu P_L \left(\not{l} + m_\nu\right) P_R \left(\not{l} + \not{p}_1 + m_\psi\right) P_L\right]}{[l^2 - m_\nu^2]\left[(l+p_1)^2 - m_\psi^2\right]\left[(l+p_1+p_2)^2 - m_\nu^2\right]} \\
&= -2g^Z_{\psi^0} \int \frac{d^4l}{(2\pi)^4} \frac{l^2 l^\mu + l^2 p_2^\mu + 2(l\cdot p_1)l^\mu + (p_1^2 + p_1\cdot p_2)l^\mu - (l\cdot p_2)p_1^\mu + (l\cdot p_1)p_2^\mu}{[l^2 - m_\nu^2]\left[(l+p_1)^2 - m_\psi^2\right]\left[(l+p_1+p_2)^2 - m_\nu^2\right]}
\end{aligned}
$$

Applying Passarino-Veltman reduction, we get

$$
\begin{aligned}
I^Z_\nu = -\frac{ig^Z_\nu}{16\pi^2}&\Bigg[\frac{2m_\psi^2 - 2m_\chi^2 + m_Z^2}{4m_\chi^2 - m_Z^2}\Bigg\{\left((m_Z^2 + m_\psi^2 - 3m_\chi^2 - m_\nu^2)p_1^\mu + (m_\nu^2 - m_\chi^2 - m_\psi^2)p_2^\mu\right)C_0(1,2,3) \\
&+ (p_2-p_1)^\mu(B_0(1,2) - B_0(1,3))\Bigg\} + \left[(m_\psi^2 + m_\nu^2 - m_\chi^2)p_2^\mu + (m_\psi^2 + m_Z^2 - m_\chi^2 - m_\nu^2)p_1^\mu\right]C_0(1,2,3) \\
&+ (p_2-p_1)^\mu B_0(1,2)\Bigg] \qquad (4.3)
\end{aligned}
$$

Where $C_0(1,2,3) = C_0(m_Z^2, m_\chi^2, m_\chi^2; m_\nu^2, m_\psi^2, m_\nu^2)$, $B_0(1,2) = B_0(2,3) = B_0(m_\chi^2; m_\nu^2, m_\psi^2)$ and $B_0(1,3) = B_0(m_Z^2, m_\nu^2, m_\nu^2)$

The divergence cancel between $I^Z_{\psi^0}$ and I^Z_ν where $g^Z_{\psi^0} = g^Z_\nu = m_Z y^2/v$ through the contributions from $B_0(1,2)$ (the last term) and among each contributions from $B_0(1,2) - B_0(1,3)$.

To check the results, contract with $(p_1+p_2)_\mu$ (momentum of the Z-boson). The results must vanish due to gauge invariance (satisfy Ward identities).

For the loop with charged particles, we have

4.1.3 $I^Z_{\psi^-}$ $(\psi^- - l - \psi^-)$

$$
\begin{aligned}
I^Z_{\psi^-} &= g^Z_{\psi^-} \int \frac{d^4l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(l + \not{p}_1 + p_2 + m_\psi)\gamma^\mu (l + m_\psi) P_L (l + p_1 + m_l) P_R\right]}{\left[l^2 - m_\psi^2\right]\left[(l+p_1)^2 - m_l^2\right]\left[(l+p_1+p_2)^2 - m_\psi^2\right]} \\
I^Z_{\psi^-} &= \frac{ig^Z_{\psi^-}}{16\pi^2}\Bigg[\frac{2m_\psi^2 + 2m_l^2 - 2m_\chi^2 + m_Z^2}{4m_\chi^2 - m_Z^2}\Bigg\{\left((m_Z^2 + m_l^2 - 3m_\chi^2 - m_\psi^2)p_1^\mu + (m_\psi^2 - m_\chi^2 - m_l^2)p_2^\mu\right)C_0(1,2,3) \\
&+ (p_2-p_1)^\mu(B_0(1,2) - B_0(1,3))\Bigg\} + \left[(m_\psi^2 + m_l^2 - m_\chi^2)p_2^\mu + (m_l^2 + m_Z^2 - m_\chi^2 + m_\psi^2)p_1^\mu\right]C_0(1,2,3) \\
&+ (p_2-p_1)^\mu B_0(1,2)\Bigg] \qquad (4.4)
\end{aligned}
$$

Where $C_0(1,2,3) = C_0(m_Z^2, m_\chi^2, m_\chi^2; m_\psi^2, m_l^2, m_\psi^2)$, $B_0(1,2) = B_0(2,3) = B_0(m_\chi^2; m_l^2, m_\psi^2)$ and $B_0(1,3) = B_0(m_Z^2, m_\psi^2, m_\psi^2)$

Since the Z- boson can couple to both left- and right-handed charged lepton. We can separate it into two (left and right) contributions. We have,

4.1.4 $I^Z_{e_L}$ $(l_L(e_L) - \psi^- - l_L(e_L))$

$$I^Z_{e_L} = -g^Z_{e_L} \int \frac{d^4l}{(2\pi)^4} \frac{\mathrm{Tr}\left[(l + p_1 + p_2 + m_l)\,\gamma^\mu P_L\,(l + m_l)\,P_R\,(l + p_1 + m_\psi)\,P_L\right]}{\left[l^2 - m_l^2\right]\left[(l + p_1)^2 - m_\psi^2\right]\left[(l + p_1 + p_2)^2 - m_l^2\right]}$$

$$\begin{aligned}
I^Z_{e_L} = &-\frac{ig^Z_{e_L}}{16\pi^2}\Bigg[\frac{2m_\psi^2 - 2m_\chi^2 + m_Z^2}{4m_\chi^2 - m_Z^2}\Bigg\{\big((m_Z^2 + m_\psi^2 - 3m_\chi^2 - m_l^2)p_1^\mu + (m_l^2 - m_\chi^2 - m_\psi^2)p_2^\mu\big)C_0(1,2,3) \\
&+ (p_2 - p_1)^\mu(B_0(1,2) - B_0(1,3))\Bigg\} + \left[(m_\psi^2 + m_l^2 - m_\chi^2)p_2^\mu + (m_\psi^2 + m_Z^2 - m_\chi^2 - m_l^2)p_1^\mu\right]C_0(1,2,3) \\
&+ (p_2 - p_1)^\mu B_0(1,2)\Bigg]
\end{aligned} \tag{4.5}$$

Where $C_0(1,2,3) = C_0(m_Z^2, m_\chi^2, m_\chi^2; m_l^2, m_\psi^2, m_l^2)$, $B_0(1,2) = B_0(2,3) = B_0(m_\chi^2; m_l^2, m_\psi^2)$ and $B_0(1,3) = B_0(m_Z^2, m_l^2, m_l^2)$

The divergence cancel between $I^Z_{\psi^-}$ and $I^Z_{e_L}$ where $g^Z_{\psi^-} = g^Z_{e_L} = 2m_Z y^2\left(-1/2 + s_w^2\right)/v$ through the contributions from $B_0(1,2)$ (the last term) and among each contributions from $B_0(1,2) - B_0(1,3)$.

Again, we can check the results by contracting with $(p_1 + p_2)_\mu$. The results must vanish due to gauge invariance.

4.1.5 $I^Z_{e_R}$ $(l_R(e_R) - \psi^- - l_R(e_R))$

$$\begin{aligned}
I^Z_{e_R} &= -g^Z_{e_R} \int \frac{d^4l}{(2\pi)^4} \frac{\mathrm{Tr}\left[\left(\not{l} + \not{p}_1 + \not{p}_2 + m_l\right)\gamma^\mu P_R\left(\not{l} + m_l\right)P_R\left(\not{l} + \not{p}_1 + m_\psi\right)P_L\right]}{\left[l^2 - m_l^2\right]\left[(l + p_1)^2 - m_\psi^2\right]\left[(l + p_1 + p_2)^2 - m_l^2\right]} \\
&= -2g^Z_{e_R} \int \frac{d^4l}{(2\pi)^4} \frac{m_l^2 (l + p_1)^\mu}{\left[l^2 - m_l^2\right]\left[(l + p_1)^2 - m_\psi^2\right]\left[(l + p_1 + p_2)^2 - m_l^2\right]}
\end{aligned}$$

$$-\frac{16\pi^2 I^Z_{e_R}}{2im_l^2} = C^\mu(1,2,3) + p_1^\mu C_0(1,2,3)$$

Applying Passarino-Veltman reduction, we get

$$\begin{aligned}
I^Z_{e_R} = &-\frac{ig^Z_{e_R}}{16\pi^2}\left(\frac{2m_l^2}{4m_\chi^2 - m_Z^2}\right)\left[\left((m_\chi^2 + m_\psi^2 - m_l^2)p_1^\mu + (m_l^2 - m_\chi^2 - m_\psi^2)p_2^\mu\right)C_0(1,2,3)\right. \\
&\left. + (p_2 - p_1)^\mu(B_0(1,2) - B_0(1,3))\right]
\end{aligned} \tag{4.6}$$

Where $g^Z_{e_R} = 2m_Z y^2\left(s_w^2\right)/v$, $C_0(1,2,3) = C_0(m_Z^2, m_\chi^2, m_\chi^2; m_l^2, m_\psi^2, m_l^2)$, $B_0(1,2) = B_0(2,3) = B_0(m_\chi^2; m_l^2, m_\psi^2)$ and $B_0(1,3) = B_0(m_Z^2, m_l^2, m_l^2)$

4.1.6 Evaluating $C_0(m_Z^2, m_\chi^2, m_\chi^2; m^2, M^2, m^2)$

In our case, three-point function have $m_1 = m_3 = m$ and $m_2 = M$ with $p_1^2 = p_2^2 = m_\chi^2$ and $(p_1+p_2)^2 = m_Z^2$. We can derive an analytic expressions in the following.
Using the results in the section 2.2.3, we have

$$\begin{aligned} a &= b = -m_\chi^2 \\ c &= -2p_1 \cdot p_2 = 2m_\chi^2 - m_Z^2 \\ d &= m^2 - M^2 + m_\chi^2 \\ e &= M^2 - m^2 + m_Z^2 - m_\chi^2 \\ f &= -m^2 \end{aligned} \tag{4.7}$$

Solving the equation $b\alpha^2 + c\alpha + a = 0$ (choosing one of the roots of the equation),we have

$$\alpha = \frac{2m_\chi^2 - m_Z^2 + m_Z\sqrt{m_Z^2 - 4m_\chi^2}}{2m_\chi^2} \tag{4.8}$$

We get the general result as,

$$C_0 = \frac{1}{(c+2\alpha b)}\left[S_3\left(y_{01}, y_{11}, y_{21}\right) + S_3\left(y_{02}, y_{12}, y_{22}\right) - S_3\left(y_{03}, y_{13}, y_{23}\right)\right] \tag{4.9}$$

Where

$$S_3(a,b,c) = R(a,b) + R(a,c) + [\eta(-b,-c) - \eta(a-b, a-c)]\ln\left(\frac{a-1}{a}\right)$$

and

$$R(a,b) = \ln\left(\frac{a}{a-b}\right)\eta\left(-b, \frac{1}{a-b}\right) - \ln\left(\frac{1-a}{b-a}\right)\eta\left(1-b, \frac{1}{a-b}\right) + \mathrm{Li}_2\left(\frac{a}{a-b}\right) - \mathrm{Li}_2\left(\frac{1-a}{b-a}\right) \tag{4.10}$$

When

$$y_0 = -\frac{d+\alpha e}{c+2\alpha a}, \qquad y_{01} = y_0 + \alpha, \qquad y_{02} = -\frac{y_0}{\alpha}, \qquad y_{03} = \frac{y_0}{1-\alpha}$$

$$\begin{aligned} y_{11,21} &= \frac{M^2 - m^2 + m_\chi^2 \pm \lambda^{1/2}(M^2, m^2, m_\chi^2)}{2m_\chi^2} \\ y_{12,22} &= \frac{m^2 - M^2 + m_\chi^2 \pm \lambda^{1/2}(M^2, m^2, m_\chi^2)}{2m_\chi^2} \\ y_{13,23} &= \frac{1 \pm \sqrt{1 - 4m^2/m_Z^2}}{2} \end{aligned}$$

All η functions vanish if α and all the masses m_i are real ($m_Z > 2m_\chi$). α is real in on-shell decay. In this case, the three-point function associates with 12-dilogarithm functions.

5 Invisible Decay Width of Z-boson to 2DMs

Now, we collect all of the amplitudes and express it in the rest frame of decaying particle (Z-boson). The four momenta of the external particles are ,

$$(p_1+p_2)^\mu = (m_Z,0,0,0), \qquad p_1^\mu = (E,0,0,p) \qquad \text{and} \qquad p_2^\mu = (E,0,0,-p)$$

When $E = m_Z/2$ and $p = \sqrt{m_Z^2 - 4m_\chi^2}/2$. Contracting with polarization vectors of Z-boson, we have

$$\epsilon_{i\mu}(p_1+p_2)^\mu = 0, \qquad \epsilon_{i\mu}p_1^\mu = -p \qquad \text{and} \qquad \epsilon_{i\mu}p_2^\mu = p$$

When $\epsilon_{1\mu} = (0,1,0,0), \epsilon_{2\mu} = (0,0,1,0)$ and $\epsilon_{3\mu} = (0,0,0,1)$

Thus, the amplitude becomes

$$\begin{aligned}
\mathcal{M} &= \epsilon_{i\mu}\left(I^Z_{\psi^0} + I^Z_\nu + I^Z_{\psi^-} + I^Z_{e_L} + I^Z_{e_R}\right)^\mu \\
\mathcal{M} &= \left(\frac{2im_Zy^2p}{16\pi^2v}\right)\frac{1}{2}\Bigg[\frac{2m_\psi^2+2m_\nu^2-2m_\chi^2+m_Z^2}{4m_\chi^2-m_Z^2}\Big\{\big(2m_\psi^2+2m_\chi^2-2m_\nu^2-m_Z^2\big)\times \\
&\quad \times C_0(m_Z^2,m_\chi^2,m_\chi^2;m_\psi^2,m_\nu^2,m_\psi^2) + 2B_0(m_\chi^2;m_\nu^2,m_\psi^2) - 2B_0(m_Z^2,m_\psi^2,m_\psi^2)\Big\} \\
&\quad - m_Z^2C_0(m_Z^2,m_\chi^2,m_\chi^2;m_\psi^2,m_\nu^2,m_\psi^2) + \frac{2m_\chi^2-2m_\psi^2-m_Z^2}{4m_\chi^2-m_Z^2}\Big\{\big(2m_\nu^2+2m_\chi^2-2m_\psi^2-m_Z^2)\big)\times \\
&\quad \times C_0(m_Z^2,m_\chi^2,m_\chi^2;m_\nu^2,m_\psi^2,m_\nu^2) + 2B_0(m_\chi^2;m_\nu^2,m_\psi^2) - 2B_0(m_Z^2,m_\nu^2,m_\nu^2)\Big\} \\
&\quad + (2m_\nu^2-m_Z^2)C_0(m_Z^2,m_\chi^2,m_\chi^2;m_\nu^2,m_\psi^2,m_\nu^2)\Bigg] \\
&\quad + \frac{2im_Zy^2p}{16\pi^2v}\left(-\frac{1}{2}+s_w^2\right)\Bigg[\frac{2m_\psi^2+2m_l^2-2m_\chi^2+m_Z^2}{4m_\chi^2-m_Z^2}\Big\{\big(2m_\psi^2+2m_\chi^2-2m_l^2-m_Z^2\big)\times \\
&\quad \times C_0(m_Z^2,m_\chi^2,m_\chi^2;m_\psi^2,m_l^2,m_\psi^2) + 2B_0(m_\chi^2;m_l^2,m_\psi^2) - 2B_0(m_Z^2,m_\psi^2,m_\psi^2)\Big\} \\
&\quad - m_Z^2C_0(m_Z^2,m_\chi^2,m_\chi^2;m_\psi^2,m_l^2,m_\psi^2) + \frac{2m_\chi^2-2m_\psi^2-m_Z^2}{4m_\chi^2-m_Z^2}\Big\{\big(2m_l^2+2m_\chi^2-2m_\psi^2-m_Z^2\big)\times \\
&\quad \times C_0(m_Z^2,m_\chi^2,m_\chi^2;m_l^2,m_\psi^2,m_l^2) + 2B_0(m_\chi^2;m_l^2,m_\psi^2) - 2B_0(m_Z^2,m_l^2,m_l^2)\Big\} \\
&\quad + (2m_l^2-m_Z^2)C_0(m_Z^2,m_\chi^2,m_\chi^2;m_l^2,m_\psi^2,m_l^2)\Bigg] + \frac{2im_Zy^2p}{16\pi^2v}s_w^2\left(\frac{2m_l^2}{4m_\chi^2-m_Z^2}\right)\times \\
&\quad \times\Big[\big(2m_\chi^2+2m_\psi^2-2m_l^2\big)C_0(m_Z^2,m_\chi^2,m_\chi^2;m_l^2,m_\psi^2,m_l^2) + 2B_0(m_\chi^2;m_l^2,m_\psi^2) - 2B_0(m_Z^2,m_l^2,m_l^2)\Big]
\end{aligned} \tag{5.1}$$

We rewrite above expression as,

$$\mathcal{M} = \left(\frac{2im_Zy^2p}{16\pi^2v}\right)\mathcal{F} \tag{5.2}$$

In the rest frame of decaying particle (Z-boson), the decay rates for two-body final states with equal masses (m_χ) is given by

$$\Gamma(Z \to \chi\chi) = \frac{1}{16\pi m_Z^2}(m_Z^2 - 4m_\chi^2)^{1/2} \left|\sum_{\text{gen}} \mathcal{M}\right|^2 \times \frac{1}{3}$$

Where we assume that for all of the generations of lepton have the same coupling constant y. So, we can collect all of the contributions from any generations of lepton in the amplitude to enhance the decay width of the process. Also, we have included the factor 1/3 for averaging over all polarization states of Z-boson.
Thus we find the following decay width:

$$\Gamma(Z \to \chi\chi) = \frac{y^4 G_F}{6144\sqrt{2}\pi^5}(m_Z^2 - 4m_\chi^2)^{3/2} \left|\sum_{\text{gen}} \mathcal{F}\right|^2 \tag{5.3}$$

In figures below, we display the decay width $\Gamma(Z \to \chi\chi)$ as a function of m_χ (mass of DM) for mass of the mediator $m_\psi = 20,\ 50$ and 90 GeV when we let the coupling $y = 1$, mass of Z-boson: $m_Z = 91.1876$ GeV and small neutrino masses : $m_\nu \approx 1$ eV . Also, for χ to be stable, we must have $m_\chi \leq m_\psi$.

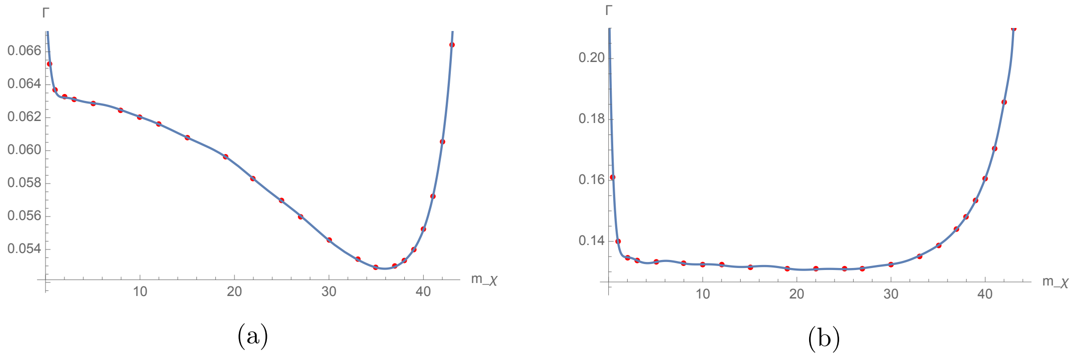


Figure 4: The decay width for the decay $Z \to \chi\chi$ as a function of m_χ with (a) $m_\psi = 20$ GeV and (b) $m_\psi = 50$ GeV.

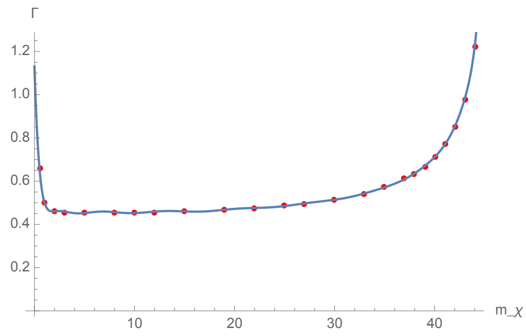


Figure 5: The decay width for the decay $Z \to \chi\chi$ as a function of m_χ with $m_\psi = 90$ GeV.

6 Conclusions and Discussions

In this work, we have considered an extension of the DM-neutrino interaction scenario, introducing interactions to the dark sector via a $SU(2)$ gauge symmetry with its corresponding vector boson (Z-boson). The dark sector in scalar-DM case consist of a dark scalar χ, fermion mediator Ψ and lepton doublets (l) which constitute the DM relics. Therefore the model allow us to study Z-boson decay to 2DMs and we calculate the decay width of this process.

The amplitude is finite and respects gauge invariance as we expect from gauge-invariant structure of the model to be renormalize because there have no tree-level couplings of DM χ to the Z boson. For most terms presented in this amplitude are in the form of scalar three point functions with different massive internal lines that leads to 12-dilogarithm functions plus a collection of simple logarithm functions associated with η-functions (we can neglect the η-functions if we consider only for the physical process $m_Z \geq 2m_\chi$). The rest of them are in terms of two-point scalar functions which also have an analytic expression.

In Figures 4 and 5, the decay width increase when the mass of mediators (m_ψ) is increased but we find the divergent behavior when we reach $m_\chi = m_Z/2$. For the physical process, this divergence should not occur. Instead, it should smoothly reach zero when $m_\chi = m_Z/2$. So, there must be a mistaken in the calculation of the amplitude that we haven't identified.

We can check the results of the three-point function by comparing the numerical values with the other results, e.g. Denner's result [4] and Oldenborgh's result [5] which differ from t'Hooft and Veltman result by how they shift the integrands to bring it into symmetric form. Also, we can check the results by using the another solution of the α to evaluate the three-point functions. The three-point function should not depend on how we choose the α parameter in Eq.(2.27). The another way to check the results is by comparing the general result given in section 2.2.3 and try to use them to evaluate the physical result presented in section 2.3.

This calculation can be further employed to calculate direct detection cross-section of the same model scenario when DM scatter with the nucleon of the detector by exchanging the Z-boson particles.

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