A 555 Timer IC Chaotic Circuit: Chaos in a Piecewise Linear System With Stable but No Unstable Equilibria

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Abstract

The 555 timer IC is a well-known integrated circuit having been widely used as a pulse generator. We propose a circuit containing a single 555 IC, an LED, an inductor, two capacitors, and three resistors that exhibits chaos. The IC serves as a hysteretic switch causing the system to be alternately attracted to each of two stable equilibria. Ours appears to be the first chaotic circuit governed by piecewise-linear equations that have stable but no unstable equilibria. Also unique is the sensitive dependence on initial conditions resulting only from a square-root map. This arises from the grazing impact of two voltages. The circuit is also unusual in exhibiting periodic orbits with a Farey tree structure and a transition to chaos via a period-adding cascade. This and other bifurcations seen experimentally are in agreement with those of the governing three-dimensional ODEs with hysteretic conditions. Inclusion of three extra resistors in the circuit lowers the characteristic frequency so that the bifurcations can be detected via the LED without the need of an oscilloscope. The circuit is therefore also suitable for experimentation by electronics hobbyists with limited resources.

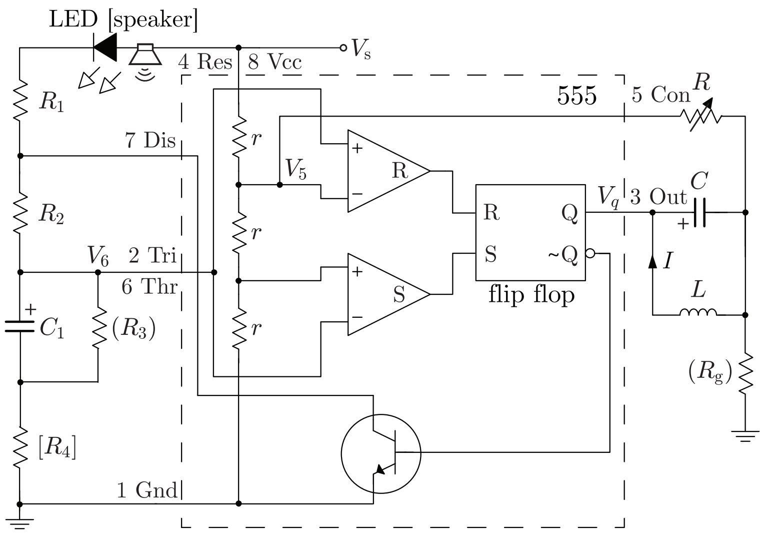

Diagram of the full circuit. In the (slightly simpler) minimal circuit, for which the LED flashes too quickly for the bifurcations to be studied by eye, components in parentheses are removed and those in square brackets are replaced by shorts.

Background

A system is chaotic if it has sensitive dependence on initial conditions (SDIC) while the system remains bounded for all time. This means that after starting the system from two very similar initial conditions, the two trajectories then grow part exponentially in the short term but remain within a finite region in the long term. As a result, the behaviour of a chaotic system can look random although the governing equations are entirely deterministic.

A state or set of states that a dissipative system tends towards after a long time is known as an attractor. The simplest of these are the stable equilibrium and stable periodic orbit. However, if the system is chaotic then it will instead tend towards a so-called strange attractor which is aperiodic and has a fractal structure. A system can also have one or more unstable equilibria or period orbits. If the system starts on one of these, it will stay there, but if it starts arbitrarily close to one, it will move away from it. An example is a pencil balanced on its tip. In the most studied chaotic systems, the SDIC arises from the existence of such unstable orbits.

Most so-called chaotic systems do not exhibit chaos for all values of the parameters. Instead, the type of attractor seen varies with the parameter values. Such a change in the nature of an attractor is called a bifurcation and a plot of the points on the attractors against a parameter value is called a bifurcation diagram. Most of the chaotic systems that have been analysed show a period-doubling route to chaos: as a parameter is varied an attractor of one particular period becomes unstable and the system jumps to an attractor with twice the period. This occurs in a roughly geometric series of ever decreasing intervals of the parameter value, and after the infinite period limit is reached, chaos occurs. For some systems where the governing equations have a discontinuity it is also possible to find a period-adding route to chaos in which the attractor periods increase by a constant before abruptly switching to a chaotic attractor.

Electronic circuits are an ideal tool for demonstrating bifurcations and chaos. They are generally easy to construct and the state of the system can be monitored using an oscilloscope (or, if the characteristic frequency is low enough, an LED) as a parameter (normally a resistance) is varied.

A classic way to achieve chaos is by perturbing a nonlinear oscillation with a periodic signal. An example is periodically moving the pivot of a compound pendulum up and down; the pendulum then swings chaotically for some ranges of the driving frequency. We choose what would seem like one of the simplest ways to do this electronically: our nonlinear oscillator is the standard 555 astable multivibrator circuit and it is perturbed by the output from an LCR circuit. It turns out that chaos is difficult to achieve in this system; the values of the components need to be chosen carefully to meet the conditions for what turns out to be chaos with an unusual source of SDIC. It is for this reason that this relatively simple design of a chaotic circuit has been overlooked for so long.

Key results

- A chaotic circuit which is quite easy to construct and in which bifurcations and chaos can be seen without needing an oscilloscope.
- The piecewise-linear governing equations are simple 1st or 2nd order ordinary differential equations which can be solved analytically up to each switching event (from one set of equations to another).
- The equations have stable equilibria but these are never reached owing to the switching condition being met before this can occur.
- The behaviour of the system can be explained using empirical maps whose parameters are obtained from simulation.
- Chaos with SDIC resulting only from a square-root singularity in the derivative of the map describing the system.
- The circuit can be used to study discontinuity-induced bifurcations such as the period-adding cascade leading to chaos and the Farey-tree structure of some of the periodic orbits in the bifurcation diagram.
- The systems shows hysteresis as a result of more than one attractor existing for the same values of the parameters.
- The circuit effectively generates sequences of integers; each integer is the number of regularly spaced pulses (seen as flashes of the LED which correspond to a particular switching condition being met) before there is a gap. When in a chaotic mode, the integers appear to be random and we therefore have ‘digital chaos’ without digitization.

Related resources

[1] Sprott JC (2003) *Chaos and Time-Series Analysis*, Oxford University Press.
[2] Sprott JC, Thio WJC (2022) *Elegant Circuits: Simple Chaotic Oscillators*, World Scientific.