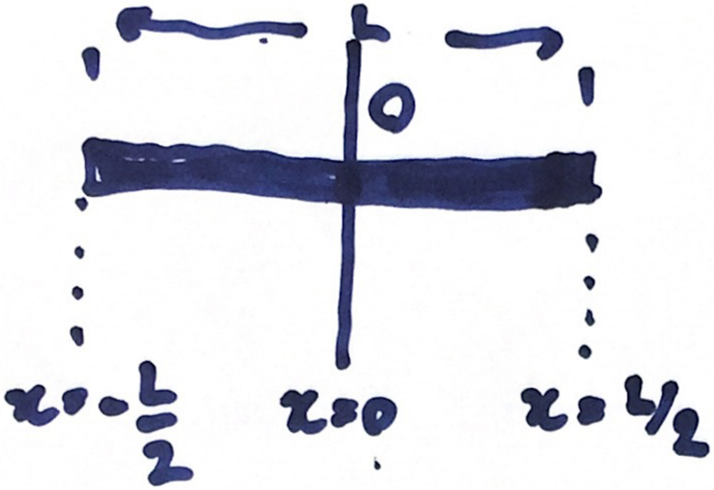


$$\boxed{I = \frac{1}{12} M L^2}$$

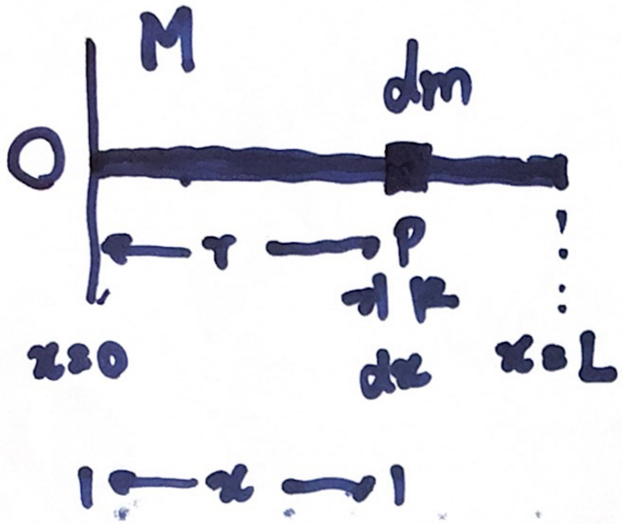
$$I = \int_{-L/2}^{L/2} x^2 \cdot \frac{M}{L}\, dx$$

$$= \frac{M}{L} \cdot \frac{x^3}{3} \Big|_{-L/2}^{L/2}$$

$$= \frac{M}{L}\frac{1}{3}\left(\frac{L^3}{8} - \left(- \frac{L^3}{8} \right) \right)$$

$$= \frac{M}{L} \cdot \frac{1}{3} \cdot \frac{L^3}{4}$$

$$= \frac{1}{12} M L^2$$



$$I = \sum_{i=1}^{n} m_i r_i^2 = \int r^2 \, dm$$

$$dm = ?$$

$$dm = \frac{M}{L} dx \quad ; \quad r = x$$

$$\Rightarrow I = \int_0^L x^2 \cdot \frac{M}{L} dx$$

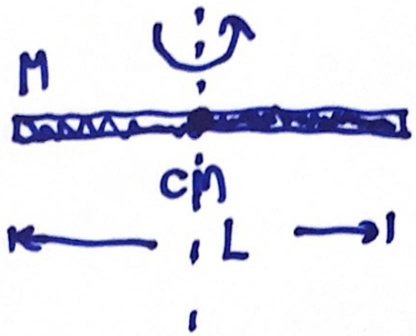
$$= \frac{M}{L} \int_0^L x^2 dx = \frac{M}{L} \cdot \frac{x^3}{3} \Big|_0^L$$

$$= \frac{M}{L} \cdot \frac{1}{3} L^3 - \cancel{\frac{M}{L} \cdot \frac{1}{3} 0}$$

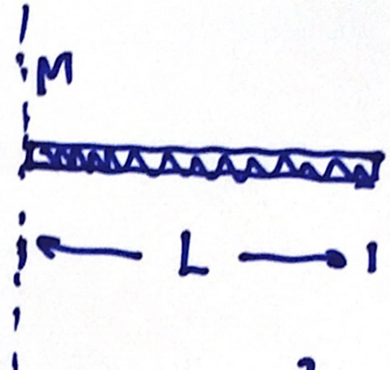
$$\boxed{I = \frac{1}{3} M L^2}$$

SCPY 156

โมเมนต์ความเฉื่อย



$$I_{CM} = \frac{1}{12}ML^2$$



$$I = \frac{1}{3}ML^2$$

ทฤษฎีแกนขนาน

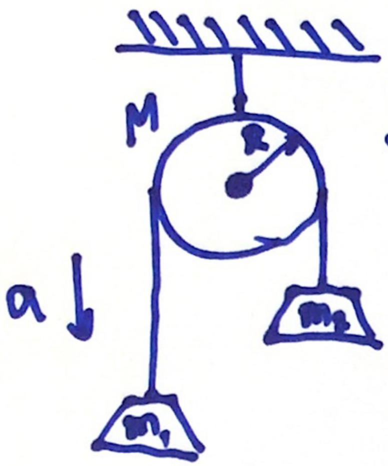
$$I = I_{CM} + Md^2$$

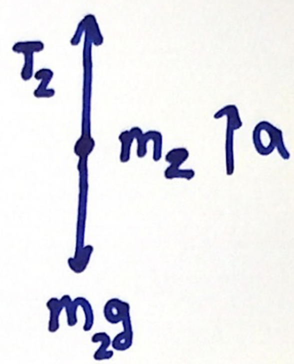
$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2$$

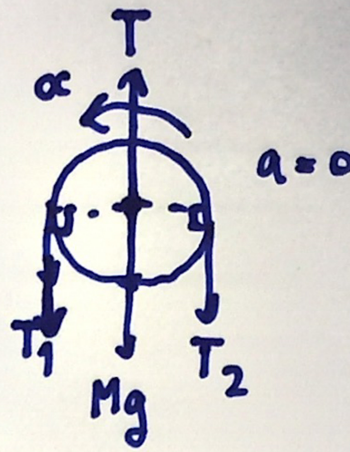
$$I = \frac{1}{3}ML^2$$

รอก

จงหา $a = ?$







$m_1 > m_2$

$$\Sigma F = m_1 a$$
$$m_1 g - T_1 = m_1 a$$
$$T_1 = m_1 g - m_1 a$$

$$\Sigma F = m_2 a$$
$$T_2 - m_2 g = m_2 a$$
$$T_2 = m_2 a + m_2 g$$

$$\Sigma F = 0$$
$$T - Mg - T_1 - T_2 = 0$$
$$\Sigma \tau = I\alpha$$
$$T_1 R - T_2 R = I\alpha$$

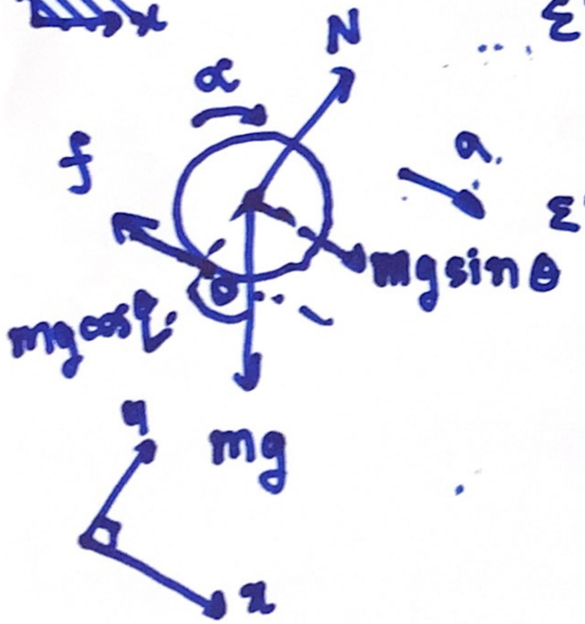
แต่ $\alpha = \frac{a}{R}$

$$T_1 R - T_2 R = \frac{Ia}{R} \quad \ldots (1)$$
$$T_1 - T_2 = Ia/R^2$$

จาก (1)

$$m_1 g - m_1 a - m_2 g - m_2 a = \frac{Ia}{R^2}$$
$$\frac{Ia}{R^2} + m_1 a + m_2 a = m_1 g - m_2 g$$
$$a\left(m_1 + m_2 + \frac{I}{R^2}\right) = (m_1 - m_2) g$$
$$a = (m_1 - m_2) g / (m_1 + m_2 + I/R^2)$$

ท่อกลิ้งลงทางเอียง จงหา $a = ?$



$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma F_x = m a_x$$

$$mg\sin\theta - f = ma \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$\Sigma \tau = I\alpha$$

$$\alpha = \frac{a}{R}$$

$$fR = \frac{Ia}{R}$$

$$f = \frac{Ia}{R^2} \quad \ldots (2)$$

จาก (1) และ (2)

$$mg\sin\theta - \frac{Ia}{R^2} = ma$$

ให้ $I = \frac{1}{2}mR^2$

$$\cancel{m}g\sin\theta - \frac{1}{2}\cancel{m}\cancel{R^2}\frac{a}{\cancel{R^2}} = \cancel{m}a$$

$$g\sin\theta = a + \frac{1}{2}a$$

$$\frac{3}{2}a = g\sin\theta$$

$$\Leftarrow \quad \boxed{a = \frac{2}{3}g\sin\theta}$$

มีการอนุรักษ์พลังงาน.

$$\Delta P.E = \Delta K.E$$

$$\cancel{m}gh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

ถ้ากลิ้งไม่ไถล

$$\omega = \frac{v}{R}$$

$$= \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\cdot\frac{1}{2}\cancel{m}\cancel{R^2}\cdot\frac{v^2}{\cancel{R^2}}$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2$$

$$gh = \frac{3}{4}v^2$$

$$v = \sqrt{\frac{4}{3}gh}$$

$$a = \frac{2}{3}g\sin\theta$$

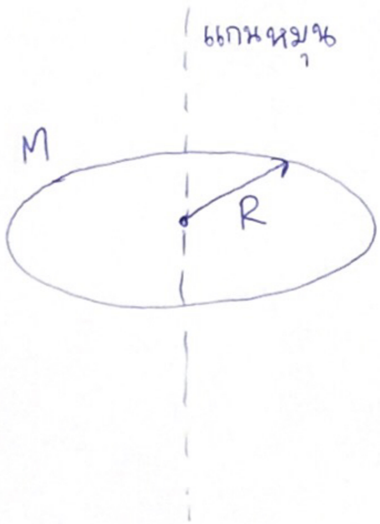
$$v_f^2 = \cancel{v_i^2} + 2as$$

$$v^2 = 2\cdot\frac{2}{3}g\cancel{\sin\theta}\cdot\frac{h}{\cancel{\sin\theta}}$$

$$v^2 = \frac{4}{3}gh$$

$$v = \sqrt{\frac{4}{3}gh}$$

การคำนวณโมเมนต์ความเฉื่อยของวัตถุวงกลม



$$I = \int r^2 \, dm$$

ความหนาแน่นต่อหน่วยพื้นที่

จากรูป $\quad dm = (2\pi r \, dr) \cdot \frac{M}{\pi R^2}$

พื้นที่ของบริเวณที่แรเงาในรูป

$$\Rightarrow I = \int_0^R r^2 \cdot 2\cancel{\pi} r \cdot \frac{M}{\cancel{\pi} R^2} \, dr$$

$$= \frac{2M}{R^2} \int_0^R r^3 \, dr$$

$$= \frac{2M}{R^2} \left. \frac{r^4}{4} \right|_0^R$$

$$= \frac{\cancel{2}M}{\cancel{R^2}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2}$$

$$\boxed{I = \frac{1}{2} MR^2}$$