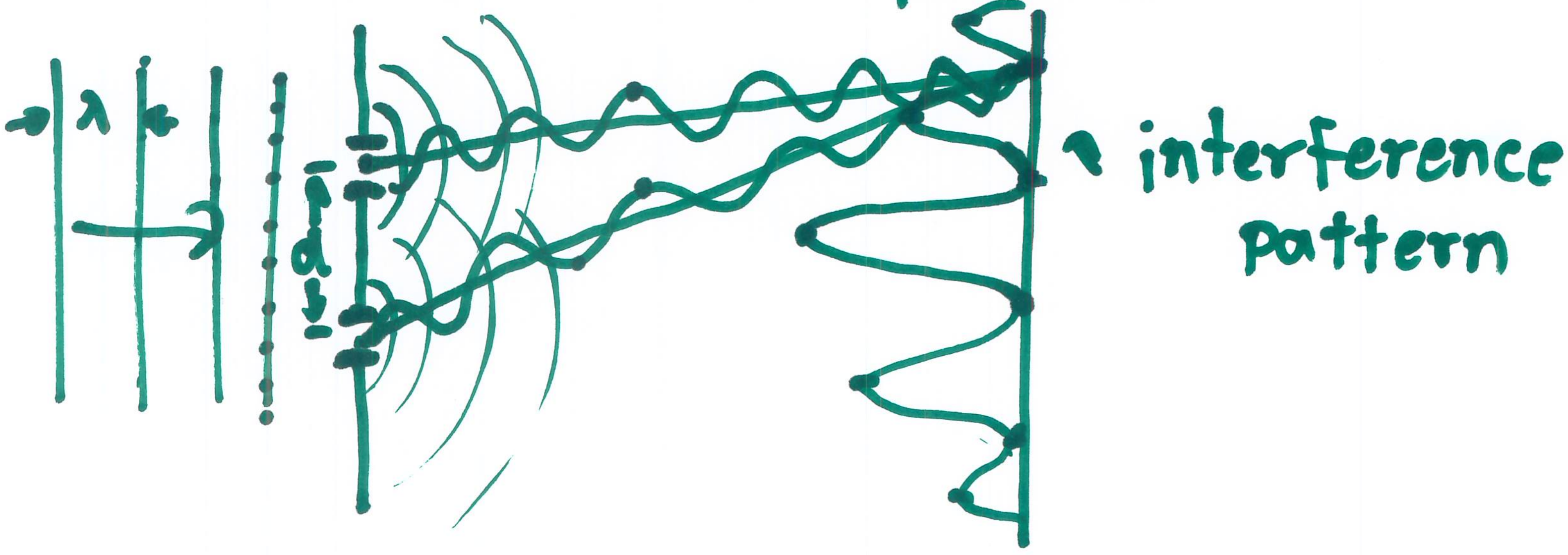
Optics

1. Interference การแทรกสอด
2. Diffraction การเลี้ยวเบน
3. Polarization

1. Interference.

Double slit interference



หลักการที่ทำให้เกิด interference.

1. Huygen's principle: แต่ละจุดบนหน้าคลื่นที่ไม่โดยกีดขวางจะทำหน้าที่เป็นแหล่งกำเนิดคลื่นใหม่

2. Superposition : เราสามารถรวม amplitude ของคลื่นที่เวลา t และตำแหน่ง $\vec{r}$ ใดๆ ได้

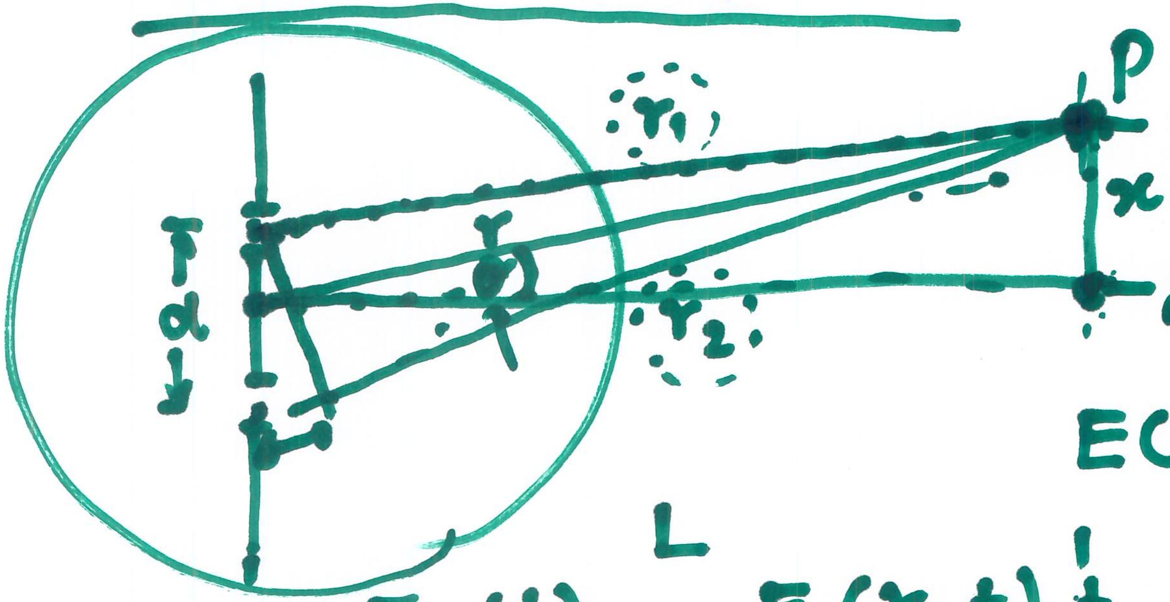
อยากอธิบาย

1. Double slit
2. Multiple-slit
3. Single slit

การแทรกสอดจะเกิดได้เมื่อ.

1. แหล่งกำเนิดอาพันธ์ (coherent sources)
2. มีความถี่เดียวกัน (แสงสีเดียวกัน)

Double slit interference.



สมมุติฐานว่า amplitude E_0 ไม่ขึ้นกับ r

สมการคลื่น

$$E(r,t) = E_0 \cos(\omega t - kr)$$

จะหา $E_p(t) = E(r_1,t) + E(r_2,t)$ ✓

$$= E_0 \cos(\omega t - kr_1) + E_0 \cos(\omega t - kr_2)$$

จุดสว่าง ถ้า $I = 4I_0 \Rightarrow \cos^2(\delta/2) = 1$

$$\frac{\delta}{2} = 0, \pi, 2\pi, 3\pi, \ldots.$$

$\frac{\delta}{2}$ [illegible] $\frac{k}{2}(r_2 - r_1) = n\pi \quad ; n = 0, 1, 2, 3, \ldots$

$$\Downarrow$$

เรารู้ว่า $\lambda = \frac{2\pi}{k} \Rightarrow k = \frac{2\pi}{\lambda}$

$$\frac{\cancel{2}\cancel{\pi}}{\lambda} \cdot \frac{1}{\cancel{2}}(r_2 - r_1) = n\cancel{\pi}$$

$$\Rightarrow \boxed{r_2 - r_1 = n\lambda}$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= E_0 \cdot 2\cos\left(\omega t - \underbrace{k\left(\frac{r_1+r_2}{2}\right)}\right)\cos\left(\underbrace{\frac{k}{2}(r_2-r_1)}_{=\delta/2}\right)$$

ให้ $\tilde{\phi} = \frac{k}{2}(r_1+r_2)$ $\quad \delta = k(r_2-r_1)$

$$\Rightarrow E_p(t) = 2E_0\cos\left(\frac{\delta}{2}\right)\cos\left(\omega t - \tilde{\phi}\right)$$

$$I \propto E_p^2(t) = \underline{\underline{4}}\,E_0^2\,\underline{\underline{\cos^2\left(\frac{\delta}{2}\right)}}\,\cos^2\left(\boxed{\omega} t - \tilde{\phi}\right)$$

สั่นเร็วมา ตาเราไม่
สามารถแยกแยะได้.

$$\rightarrow I = 4I_0\cos^2\left(\frac{\boxed{\delta}}{2}\right)$$

จุดมืด $I = 0$

จุดแสงสว่างที่สุด $I = 4I_0$

แถบมืด

$$\cos^2 \frac{\delta}{2} = 0 \Rightarrow I = 0$$

$$\frac{\delta}{2} = \frac{\pi}{2}, 3 \cdot \frac{\pi}{2}, 5 \cdot \frac{\pi}{2}, \ldots, = (2n+1)\frac{\pi}{2}$$

$$n = 0, 1, 2, \ldots$$

$$\frac{\delta}{2} = (2n+1)\frac{\pi}{2}$$

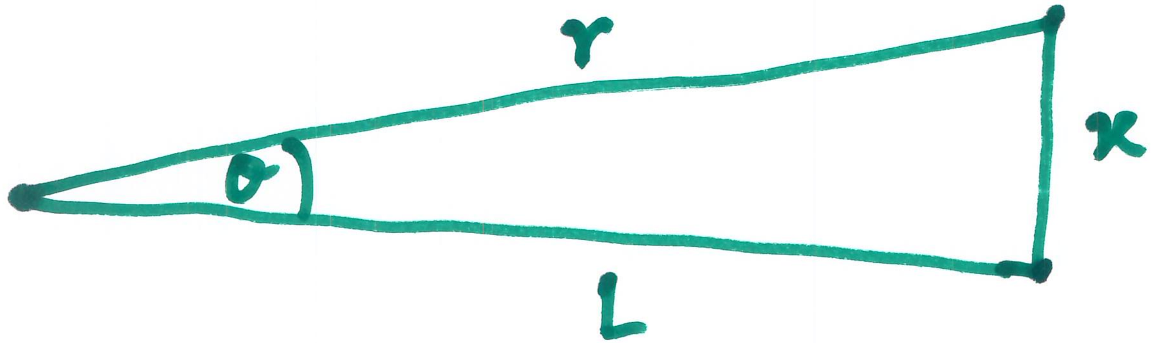
$$\frac{k}{2}(r_2 - r_1) = (2n+1)\frac{\pi}{2}$$

$$\boxed{r_2 - r_1 = \left(\frac{2n+1}{2}\right)\lambda}$$

$$\boxed{r_2 - r_1 \cong d\sin\theta \cong d \cdot \frac{x}{L} = \left(\frac{2n+1}{2}\right)\lambda}$$

ตำแหน่งแถบ
มืด

จาก



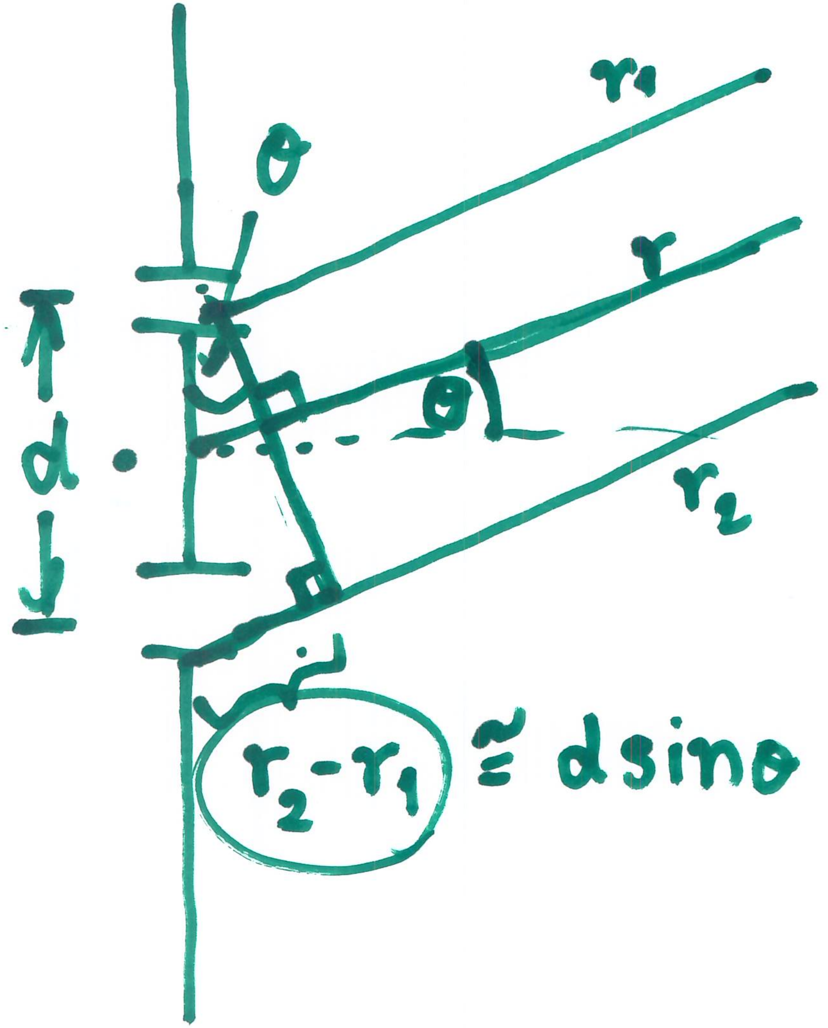
$\sin\theta = \frac{x}{r}$ แต่ถ้า $\theta \ll 1$

$\Rightarrow r \sim L \quad \Rightarrow \sin\theta \cong \frac{x}{L}$

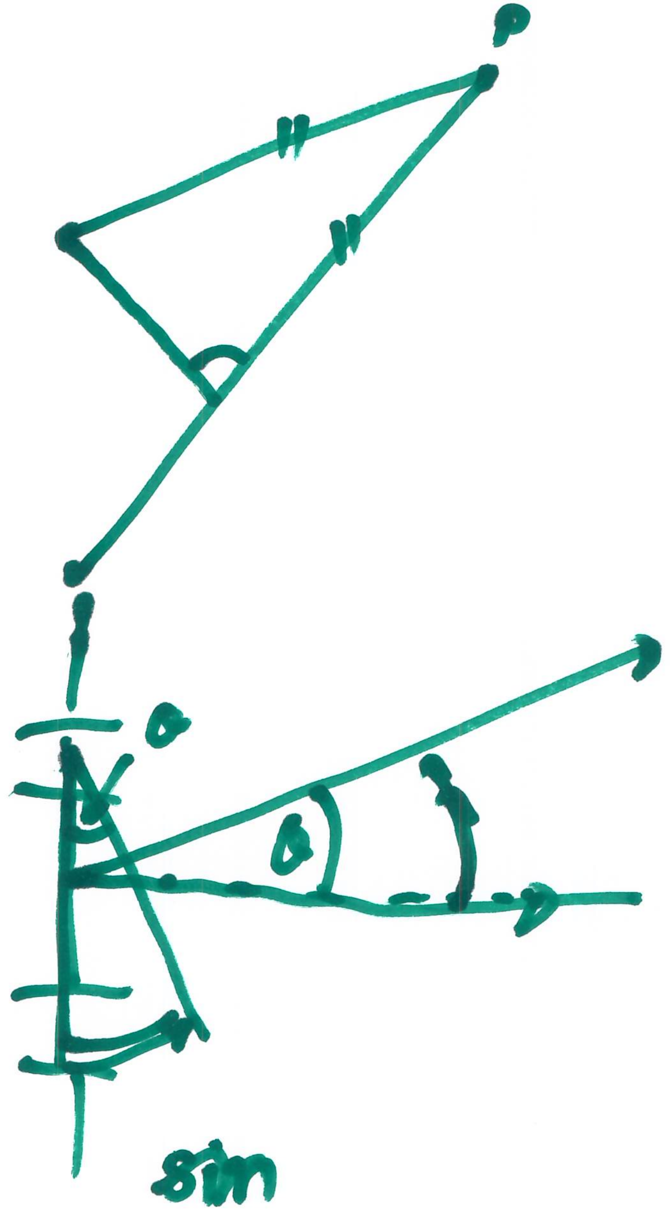
$$\Rightarrow \boxed{r_2 - r_1 \cong d\sin\theta \cong d\cdot\frac{x}{L}} = n\lambda$$

ถ้าเป็นแถบสว่าง

หา $r_2 - r_1$ ในรูปที่ง่ายต่อการวัด



$\boxed{r_2 - r_1} \cong d\sin\theta$



ถ้า P ห่างจาก slit มากๆ

$\Rightarrow \quad r_2 - r_1 \cong \underline{d\sin\theta}$

จากสมการของ Double slit

$$I = 4I_0\cos^2\left(\frac{\delta}{2}\right)$$

$$\frac{\delta}{2} = \frac{k}{2}(r_2 - r_1) = \frac{\cancel{2}\pi}{\lambda}\cdot\frac{1}{\cancel{2}}\cdot d\sin\theta = \frac{\pi d\sin\theta}{\lambda}$$

$$\Rightarrow \boxed{I = 4I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)}$$

ถ้า $N = 2$

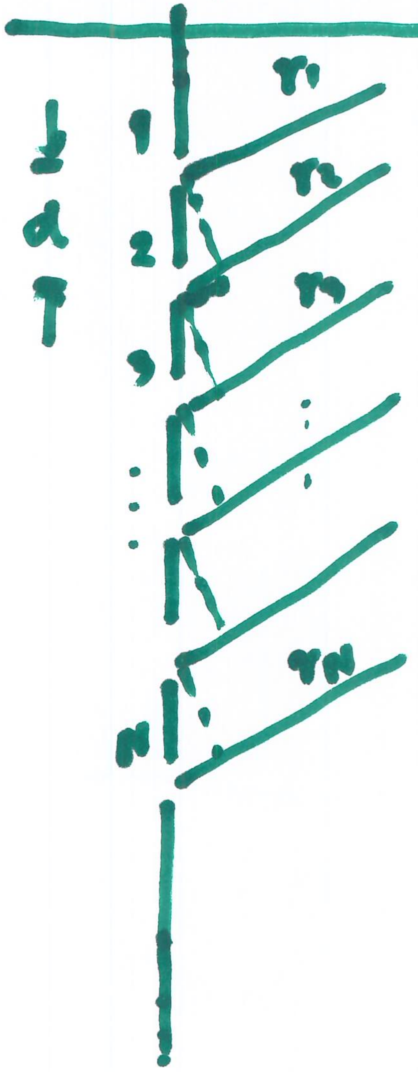
$$I_p = I_0 \left[\frac{\sin(2(\delta/2))}{\sin(\delta/2)} \right]^2$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$= I_0 \left[\frac{2 \cancel{\sin(\delta/2)} \cos(\delta/2)}{\cancel{\sin(\delta/2)}} \right)^2$$

$$I_p = 4 I_0 \cos^2 \delta/2$$ ← douple slit interference.

Multiple-slit interference.



ใน double slit.

$$\underset{\substack{\uparrow\\ \text{phase difference}}}{\delta} = \frac{2\pi d\sin\theta}{\lambda}$$

$$E_p(t) = E_0\Big[\cos(\omega t - \overset{\substack{\phi_0\\ \downarrow}}{kr_1}) + \cos(\omega t - \overset{\substack{\phi_0+\delta\\ \downarrow}}{kr_2})$$

$$+ \cos(\omega t - \underset{\substack{\uparrow\\ \phi_0+2\delta}}{kr_3}) + \ldots + \cos(\omega t - \underset{\substack{\uparrow\\ \phi_0+(N-1)\delta}}{kr_N}$$

$$E_p(t) = E_0[\cos(\omega t - \phi_0) + \cos(\omega t - \phi_0 - \delta) + \cos(\omega t - \phi_2 - 2\delta)$$

$$+ \cdots + \cos(\omega t - \phi_0 - (N-1)\delta)]$$

$$= E_0 \sum_{n=0}^{N-1} \cos(\omega t - \phi_0 - n\delta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

~~$E_0 \sum_{n=0}^{N-1}$~~

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$E_p(t) = E_0 \sum_{n=0}^{N-1} \frac{1}{2}\left(e^{i(\omega t - \phi_0 - n\delta)} + e^{-i(\omega t - \phi_0 - n\delta)}\right)$$

$$= E_0 \sum_{n=0}^{N-1} \left[\boxed{\underbrace{\frac{1}{2} e^{i(\omega t - \phi_0)}} \cdot e^{-in\delta}} \cancel{+ \frac{1}{2} e^{-i(\omega t \pm \phi_0)} \cdot e^{-in}} \right.$$

$$\left. + \underbrace{\frac{1}{2} e^{-i(\omega t - \phi_0)}} \cdot e^{+in\delta} \right]$$

$$\sum_{n=0}^{N-1} \underbrace{\cdot \frac{1}{2} e^{i(\omega t - \phi_0)}} \cdot e^{-in\delta} = \frac{1}{2} e^{i(\omega t - \phi_0)} \underbrace{\sum_{n=0}^{N-1} \left(\overset{r}{\boxed{e^{-i\delta}}} \right)^n}$$

$$\sum_{n=0}^{N\to\infty} r^n = \frac{1}{1-r} \Rightarrow 1 + r + r^2 + r^3 + \dots = \frac{1}{1-r}$$

ถ้า $r < 1$

$$1 + r + r^2 + r^3 + \cdots + r^N = \frac{1 - r^{N+1}}{1 - r}$$ ✓

$$\sum_{n=0}^{N-1} \left(e^{-i\delta}\right)^n = 1 + e^{-i\delta} + e^{-2i\delta} + e^{-3i\delta} + \cdots + e^{-(N-1)i\delta}$$

$$= \frac{1 - e^{-iN\delta}}{1 - e^{-i\delta}}$$

$$E_p(t) = E_0 \left[\frac{1}{2} e^{i(\omega t \mp \phi_0)} \cdot \left[\frac{1 - e^{-iN\delta}}{1 - e^{-i\delta}} \right] + \frac{1}{2} e^{-i(\omega t - \phi_0)} \cdot \left[\frac{1 - e^{iN\delta}}{1 - e^{i\delta}} \right) \right]$$

$$= E_0 \left[\frac{1}{2} e^{i(\omega t - \phi_0)} \frac{e^{-iN\delta/2}}{e^{-i\delta/2}} \left(\frac{e^{iN\delta/2} - e^{-iN\delta/2}}{e^{i\delta/2} - e^{-i\delta/2}} \right) \right.$$

$$\left. + \frac{1}{2} e^{-i(\omega t + \phi_0)} \frac{e^{iN\delta/2}}{e^{i\delta/2}} \left(\frac{e^{-iN\delta/2} - e^{iN\delta/2}}{e^{-i\delta/2} - e^{i\delta/2}} \right) \right]$$

$$\Rightarrow E_p(t) = E_0 \cos(\omega t - \phi_0 - (N-1)\delta/2) \cdot \frac{\sin(N\delta/2)}{\sin(\delta/2)}$$

$$I_p \propto E_p^2$$

$$\boxed{I_p = I_0 \left[\frac{\sin(N\delta/2)}{\sin(\delta/2)} \right]^2}$$

$$\frac{N\delta}{2} = 0, \pi, 2\pi, ..$$

$$\delta = \frac{2n\pi}{N}$$

จาก $E_p(t) = E_0\left[\frac{1}{2}e^{i(\omega t-\phi_0)}\cdot e^{-iN\delta/2+i\delta/2}\left(\frac{e^{iN\delta/2}-e^{-iN\delta/2}}{e^{i\delta/2}-e^{-i\delta/2}}\right)\right.$

$$\left.+\frac{1}{2}e^{-i(\omega t-\phi_0)}\cdot e^{iN\delta/2-i\delta/2}\left(\frac{e^{-iN\delta/2}-e^{iN\delta/2}}{e^{-i\delta/2}-e^{i\delta/2}}\right)\right]$$

$$= \frac{E_0}{2}\left[e^{i(\omega t-\phi_0-(N-1)\delta/2)}\cdot\frac{\cancel{2i}\sin(N\delta/2)}{\cancel{2i}\sin(\delta/2)}\right.$$

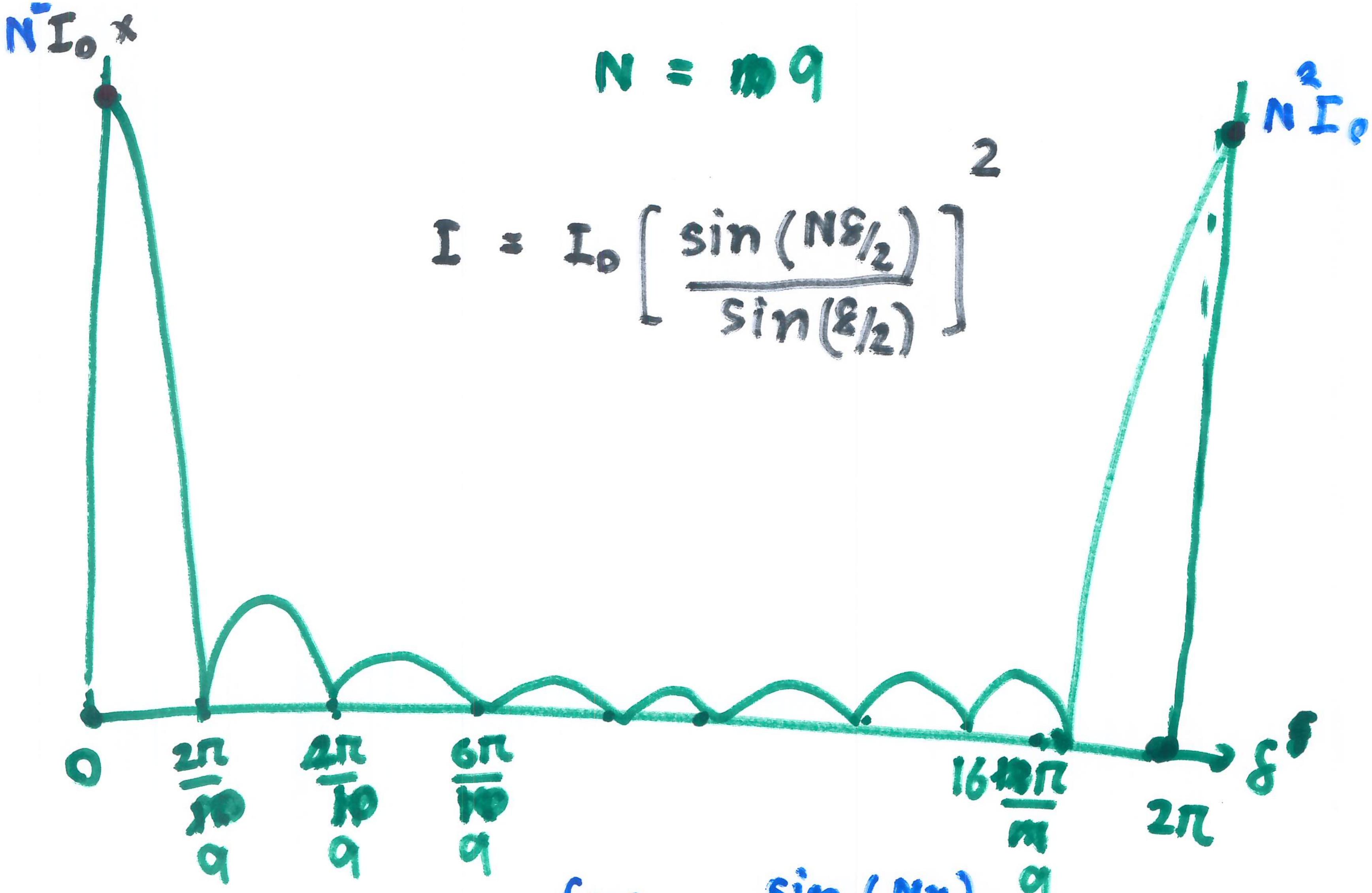
$$\left.+e^{-i(\omega t-\phi_0-(N-1)\delta/2)}\cdot\frac{\cancel{-2i}\sin(N\delta/2)}{\cancel{-2i}\sin(\delta/2)}\right]$$

$$= E_0\cdot\frac{\sin(N\delta/2)}{\sin(\delta/2)}\underbrace{\frac{1}{2}\left[e^{i(\omega t-\phi_0-(N-1)\delta/2)}+e^{-i(\omega t-\phi_0-(N-1)\delta/2)}\right]}_{=\cos(\omega t-\phi_0-(N-1)\delta/2)}$$

$$\Rightarrow \quad E_p(t) = E_0\frac{\sin(N\delta/2)}{\sin(\delta/2)}\cos(\omega t-\phi_0-(N-1)\delta/2)$$

N = ~~10~~ 9

$$I = I_0\left[\frac{\sin(N\delta/2)}{\sin(\delta/2)}\right]^2$$



$$f(x) = \frac{\sin(Nx)}{\sin x} \;;\; x = \delta/2$$

$$\lim_{x\to 0} f(x) = \cancel{\lim_{x\to 0}\,\text{[illegible]}} = \lim_{x\to 0}\frac{N\cos(Nx)}{\cos x} = N$$

Multiple slit N = 11

ระหว่าง principle maxima n=0 กับ n = 1