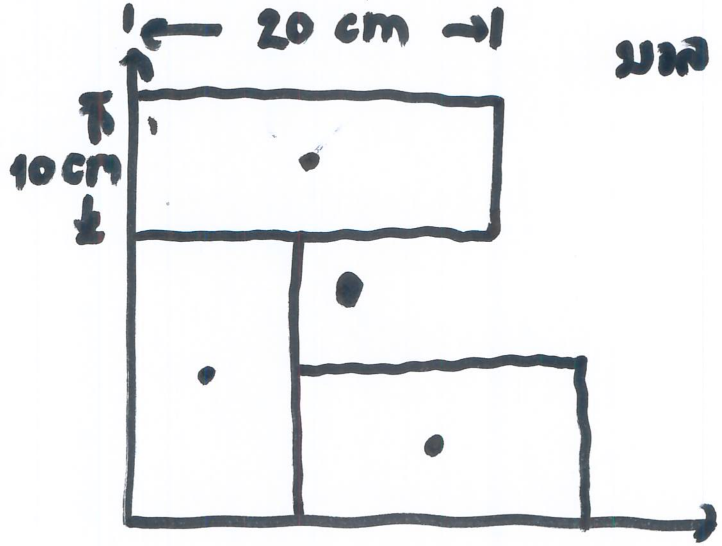
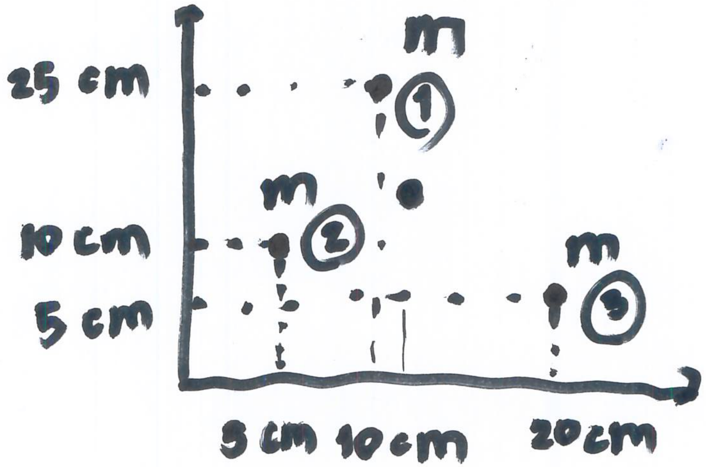
①



มวล m จงหาจุดศูนย์กลางมวล.

$$\vec{r}_{cm} = \frac{1}{M}\sum m_i \vec{r}_i$$

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$



$$\vec{r}_{cm} = \frac{1}{M}\sum m_i \vec{r}_i$$

$$= \frac{1}{3m}\Big(m\,(10\text{ cm}\,\hat{x} + 25\text{ cm}\,\hat{y}) + m\,(5\text{ cm}\,\hat{x} + 10\text{ cm}\,\hat{y}) + m\,(20\text{ cm}\,\hat{x} + 5\text{ cm}\,\hat{y})\Big)$$

$$= \frac{1}{3\cancel{m}} \cdot \cancel{m} \cdot (35\text{ cm}\,\hat{x} + 40\text{ cm}\,\hat{y})$$

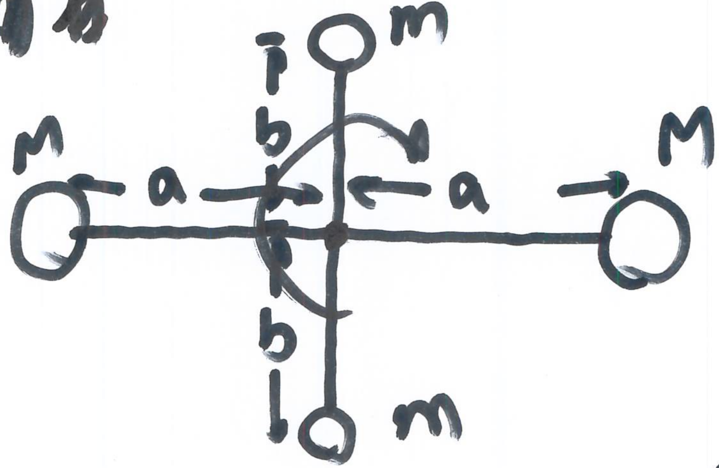
$$\vec{r}_{cm} = \frac{1}{3}(35\text{ cm}\,\hat{x} + 40\text{ cm}\,\hat{y})$$

② โมเมนต์ความเฉื่อย

$I = \sum m_i r_i^2$ $\quad$ $I = \int r^2 dm$

ต้องรู้จุดหมุน $\quad$ ระยะจากจุดหมุน

2)



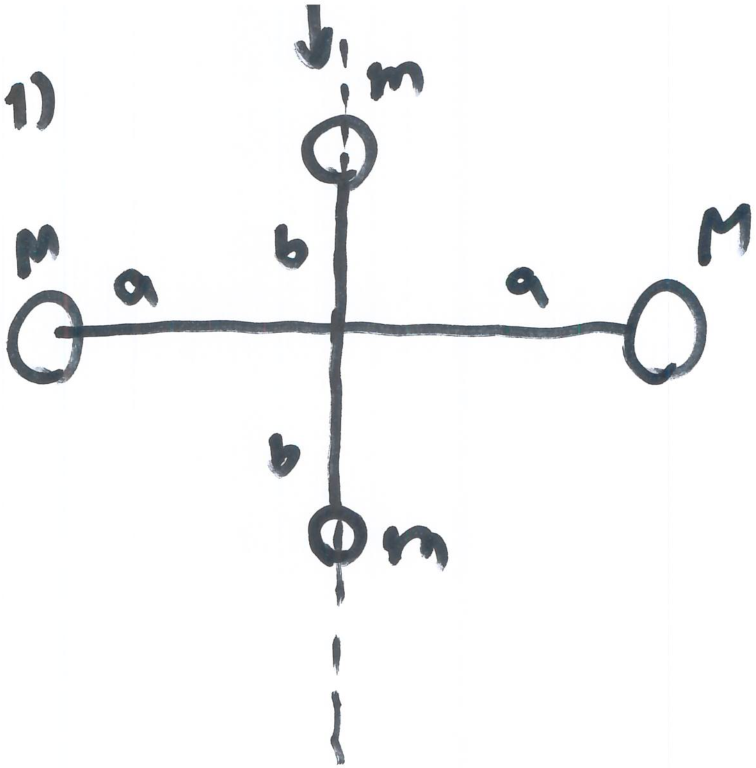
$I_m = \sum m_i r_i^2$

$= mb^2 + mb^2 = 2mb^2$

$I_M = Ma^2 + Ma^2 = 2Ma^2$

$I_{รวม} = I_m + I_M = 2(mb^2 + Ma^2)$

1)



Top view

$$I_M = 2Ma^2$$

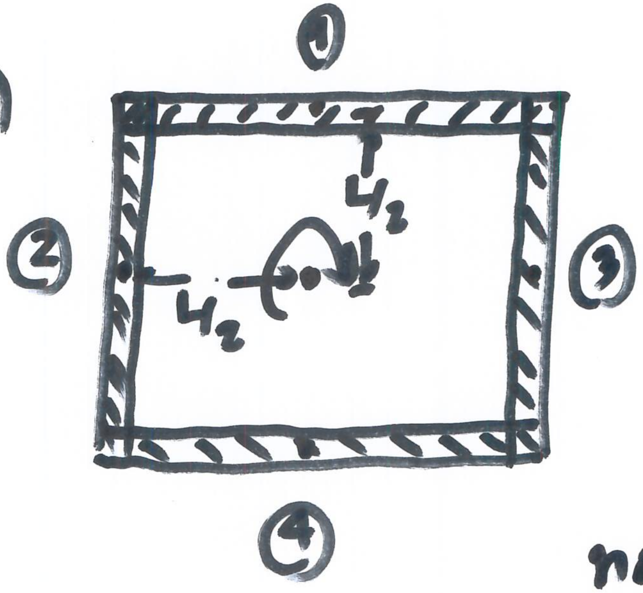
พลังงานจลน์ $K.E. = \frac{1}{2} I\omega^2$

1) $K.E. = \frac{1}{2}(2Ma^2)\omega^2$

2) $K.E = \frac{1}{2}(2mb^2 + 2Ma^2)\omega^2.$

② โมเมนต์ความเฉื่อย.

a)



มวล M ยาว L

จงหา $I = ?$

รู้ว่า I_{cm} ของแท่งยาว L มวล M

$$I_{cm} = \frac{1}{12}ML^2$$

ทฤษฎีแกนขนาน

$$I = I_{cm} + md^2$$

d ระยะระหว่างจุดหมุนกับจุดศูนย์กลางมวล.

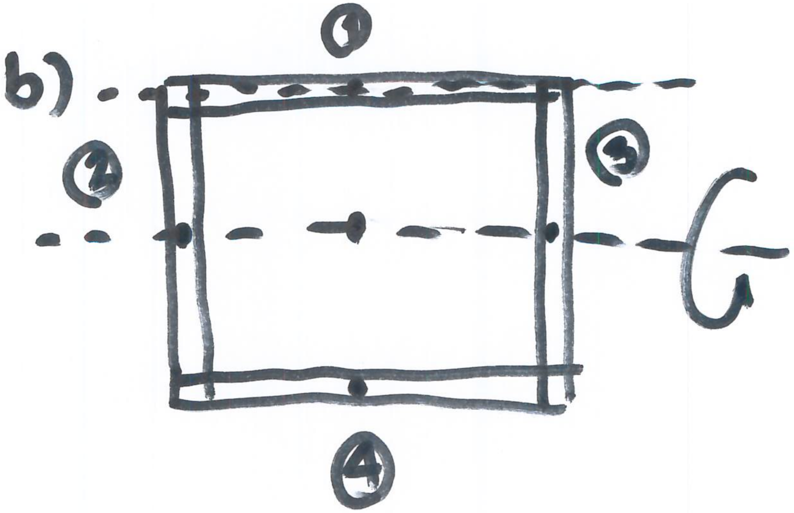
$$I_{รวม} = I_{①} + I_{②} + I_{③} + I_{④}$$

$$I_{①} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \left(\frac{1}{12} + \frac{1}{4}\right)ML^2 = \frac{1}{3}ML^2$$

$$I_{②} = I_{③} = I_{④} = I_{①}$$

$$I_{รวม} = 4I_{①} = \frac{4}{3}ML^2$$

b)



จงหา $I = ?$

$$I_{2} = I_{cm} = \frac{1}{12}ML^2$$

$$I_{3} = I_{cm} = \frac{1}{12}ML^2$$

$$I_{1} = I_{cm} + md^2 = \cancel{\frac{1}{12}ML^2} \quad 0$$

$$= \frac{1}{2}MR^2 + M\left(\frac{L}{2}\right)^2 = \cancel{\frac{1}{2}MR^2} + \frac{1}{4}ML^2$$

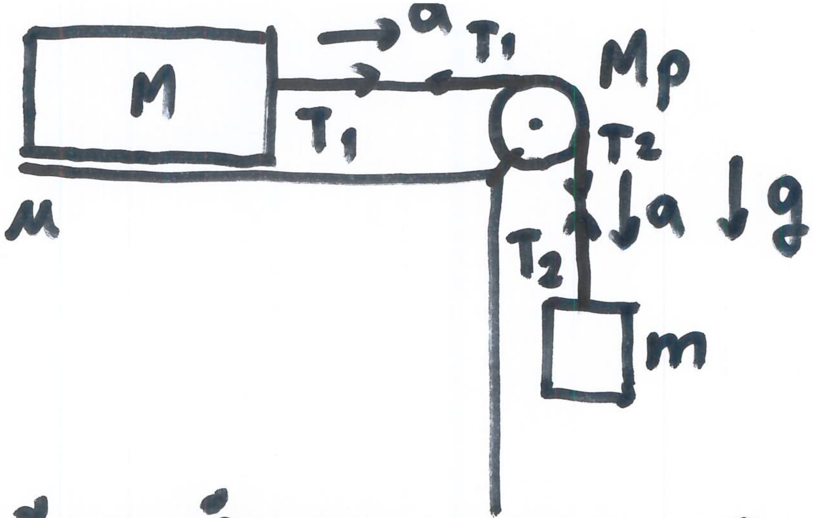
$R << L$

$$= \frac{1}{4}ML^2$$

$$I_{4} = I_{1} = \frac{1}{4}ML^2$$

$$I_{\text{รวม}} = \frac{1}{4}ML^2 + \frac{1}{12}ML^2 + \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{2}{3}ML^2$$

3



ถ้าไม่มีแรงเสียดทาน รอกไม่มีมวล

$$\Sigma\vec{F} = m\vec{a}$$

$$F = mg$$

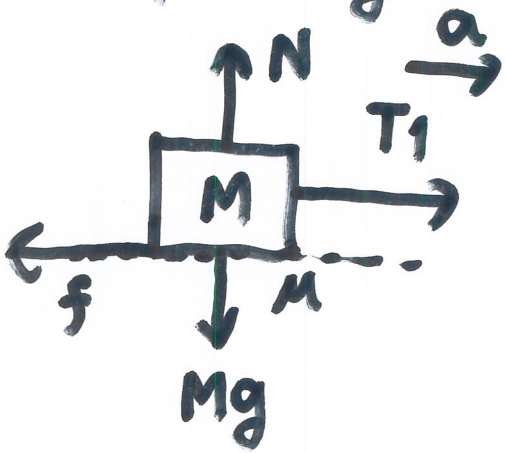
ถ้ามีแรงเสียดทาน และรอกมีมวล จงหา $a = ?$ ถ้าระบบเคลื่อนที่

$$mg = (m+M)a$$

$$\boxed{a = \frac{mg}{m+M}}$$ ความเร่ง

เขียน free body diagram 3 อัน

M :



แนวราบ : $\Sigma\vec{F} = m\vec{a}$

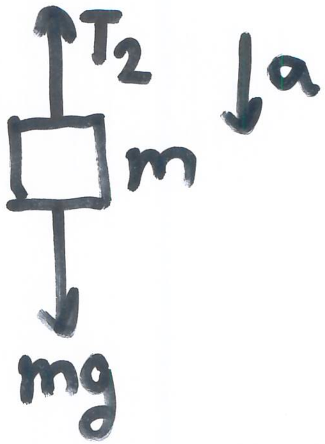
$$T_1 - f = Ma \quad ; f = \mu N$$

แนวตั้ง

$$N - Mg = 0 \Rightarrow N = Mg$$

$$T_1 - \mu M g = Ma \quad \cdots \quad (1)$$

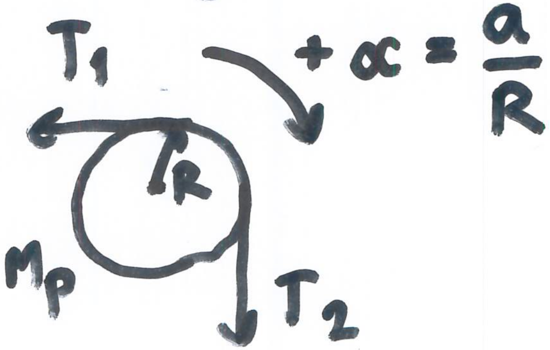
m :



$$mg - T_2 = ma \quad \cdots \quad (2)$$

$$\Sigma \vec{\tau} = I\alpha \quad ; \quad I = \frac{1}{2} M_p R^2$$

Mp :



$$T_2 \cancel{R} - T_1 \cancel{R} = \frac{1}{2} M_p R^{\cancel{2}} \cdot \frac{a}{\cancel{R}}$$

$$T_2 - T_1 = \frac{1}{2} M_p a \quad \cdots \quad (3)$$

(1) + (2)

$$T_1 - \mu M g + mg - T_2 = Ma + ma$$

$$(m - \mu M)g - (T_2 - T_1) = (m + M)a$$

ใช้สมการ (3)

$$(m - \mu M)g - \frac{1}{2}M_p a = (m + M)a$$

$$\left(m + M + \frac{1}{2}M_p\right)a = (m - \mu M)g$$

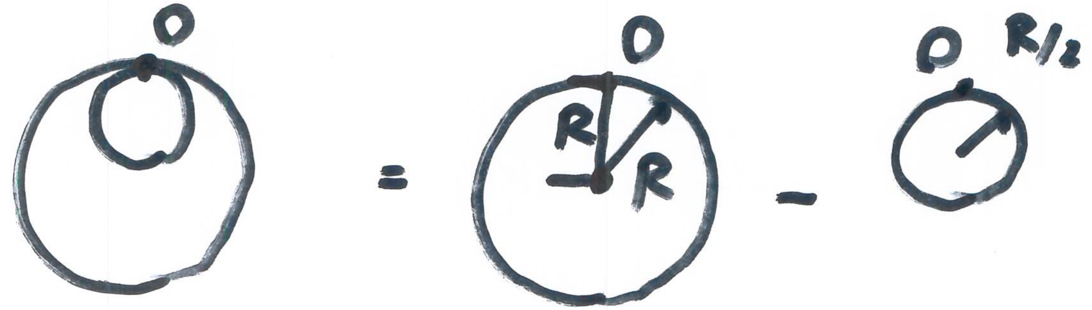
$$a = \frac{(m - \mu M)g}{m + M + \frac{1}{2}M_p}$$

ถ้า $\mu = 0$ $M_p = 0$

$$a = \frac{mg}{m + M}$$

④

a) จงหา $I = ?$ ให้ความหนาแน่นเชิงพื้นที่ σ



$$I_{\text{ที่เหลือ}} = I_R - I_{R/2}$$

$$I_R = I_{cm} + md^2 = \frac{1}{2}M_R R^2 + M_R R^2 = \frac{3}{2}M_R R^2$$

$$M_R = \sigma \cdot \pi R^2$$

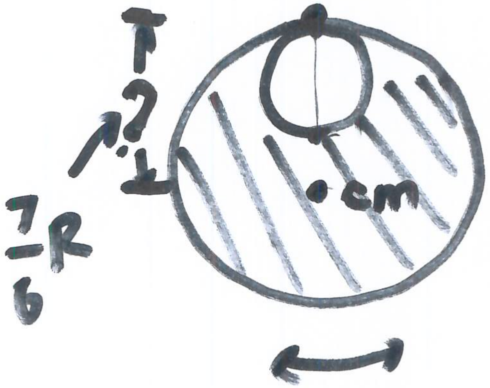
$$\Rightarrow \quad I_R = \frac{3}{2} \cdot \sigma \pi R^4$$

$$I_{R/2} = I_{cm} + md^2 = \frac{1}{2}M_{R/2}\left(\frac{R}{2}\right)^2 + M_{R/2}\left(\frac{R}{2}\right)^2 = \frac{3}{8}M_{R/2}R^2$$

$$M_{R/2} = \sigma \cdot \pi \frac{R^2}{4} \quad \Rightarrow \quad I_{R/2} = \frac{3}{32}\sigma \pi R^4$$

$$I_{\text{ที่เหลือ}} = \left(\frac{3}{2} - \frac{3}{32}\right) 6\pi R^4 = \boxed{\frac{45}{32} 6\pi R^4}$$

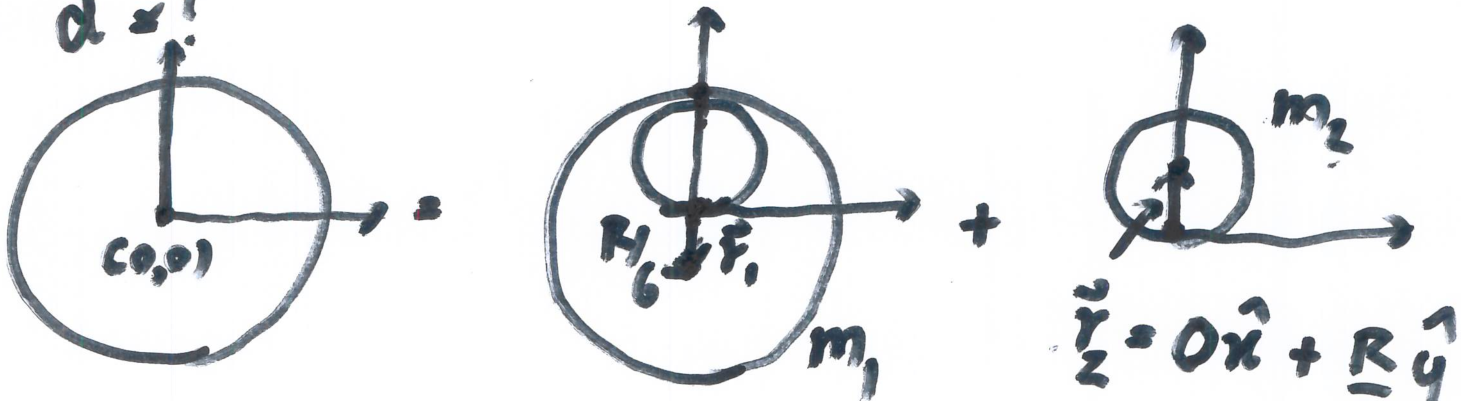
b) Physical pendulum



$$\boxed{\omega = \sqrt{\frac{mgd}{I}}}$$

ระยะระหว่างจุดศูนย์กลางมวล กับจุดหมุน

หา $d = ?$



$$\vec{r}_2 = 0\hat{x} + \frac{R}{2}\hat{y}$$

$$\vec{r}_{cm} = \frac{1}{M}\sum m_i \vec{r}_i$$

$$0 = \frac{1}{M}\left(m_1\vec{r}_1 + m_2 \frac{R}{2}\hat{y}\right)$$

$$\Rightarrow \quad m_1 \vec{r}_1 = - m_2 \frac{R}{2} \hat{y}$$

$$\vec{r}_1 = - \frac{m_2}{m_1} \frac{R}{2} \hat{y}$$

$$\boxed{m_1 = \frac{3}{4} \sigma \pi R^2} \qquad \Rightarrow \quad \frac{m_2}{m_1} = \frac{\sigma \pi R^2}{\cancel{4}} \cdot \frac{\cancel{4}}{3\, \cancel{\sigma \pi R^2}} = \frac{1}{3}$$

$$m_2 = \frac{\sigma \pi R^2}{4}$$

$$\Rightarrow \quad \vec{r}_1 = - \frac{R}{6} \hat{y}$$

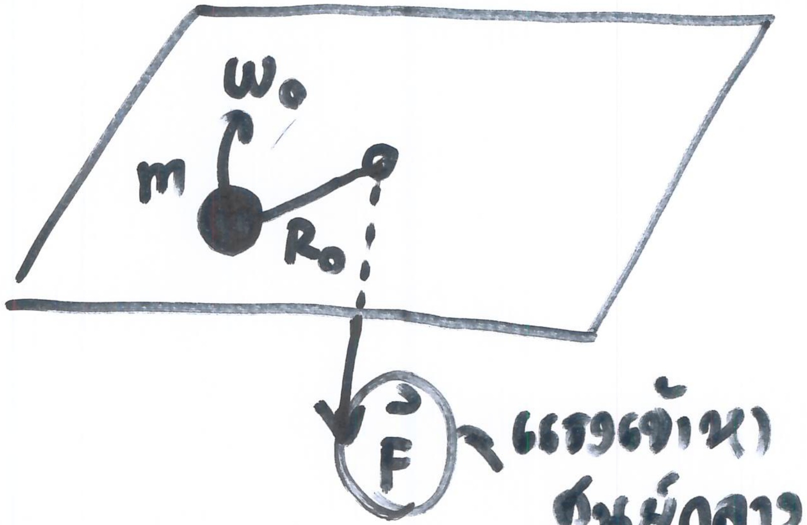
$$\Rightarrow \quad d = R + \frac{R}{6} = \boxed{\frac{7R}{6}.}$$

$$w = \sqrt{\frac{\frac{3}{4}\,\cancel{6\pi}\,\cancel{R^{2}} \cdot g \cdot \frac{7\cancel{R}}{6}}{\frac{45}{32}\,\cancel{6\pi}\,R\,\cancel{R^{4}}}}$$

$$= \sqrt{\frac{\cancel{3}}{\cancel{4}} \cdot \frac{7}{\cancel{6}_{2}} \cdot \frac{\cancel{32}^{\cancel{16}\,4}}{45} \cdot \frac{g}{R}}$$

$$\boxed{\omega = \sqrt{\frac{28}{45}\,\frac{g}{R}}}$$

ตัวอย่าง



หลังจากดึง $R_0 \rightarrow r$.

ในระบบนี้ $\Sigma\vec{\tau} = 0$

$\Rightarrow$ มีการอนุรักษ์โมเมนตัมเชิงมุม

$$L_{ก่อน} = L_{หลัง}$$

$$I_0\omega_0 = I\omega$$

$$mR_0^2\omega_0 = mr^2\omega$$

$$\Rightarrow \omega = \frac{R_0^2}{r^2}\omega_0$$

พลังงานเปลี่ยนไปเท่าไหร่

$$\Delta K.E. = -\frac{1}{2}I_0\omega_0^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mr^2\left(\frac{R_0^2}{r^2}\omega_0\right)^2 - \frac{1}{2}\overset{mR_0^2}{I_0}\omega_0^2$$

$$= \frac{1}{2}m\frac{R_0^4}{r^2}\omega_0^2 - \frac{1}{2}mR_0^2\omega_0^2$$

$$\Delta K.E. = \frac{1}{2} m R_0^2 \omega_0^2 \underbrace{\left(\frac{R_0^2}{r^2} - 1 \right)}_{>0} \quad ; \; r < R_0$$

หางานที่ทำกับมวล m.

$$W = \int \vec{F} \cdot d\vec{s}$$

แรงเข้าหาศูนย์กลาง ที่ทำให้เกิดงาน

$$F = -m\omega^2 r = -m \frac{R_0^4}{r^4} \omega_0^2 \cdot r = -m \frac{R_0^4}{r^3} \omega_0^2$$

$$d\vec{s} = dr$$

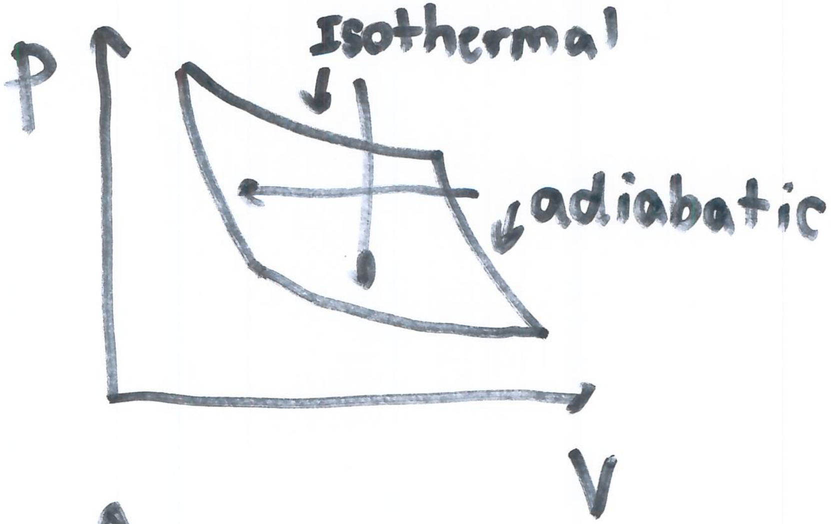
$$W = -m R_0^4 \omega_0^2 \int_{R_0}^{r} \frac{1}{r^3} \, dr = -m R_0^4 \omega_0^2 \left[\frac{-1}{2r^2} \right]_{R_0}^{r}$$

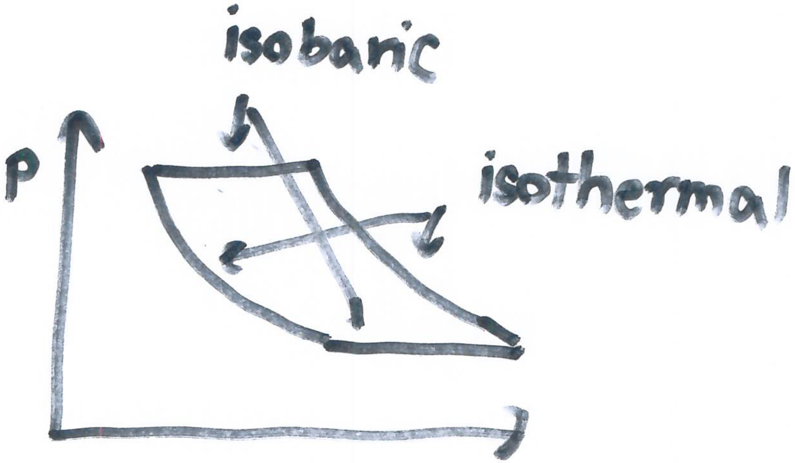
$$= -m R_0^4 \omega_0^2 \left(-\frac{1}{2} \cdot \frac{1}{r^2} - \left(-\frac{1}{2} \cdot \frac{1}{R_0^2} \right) \right.$$

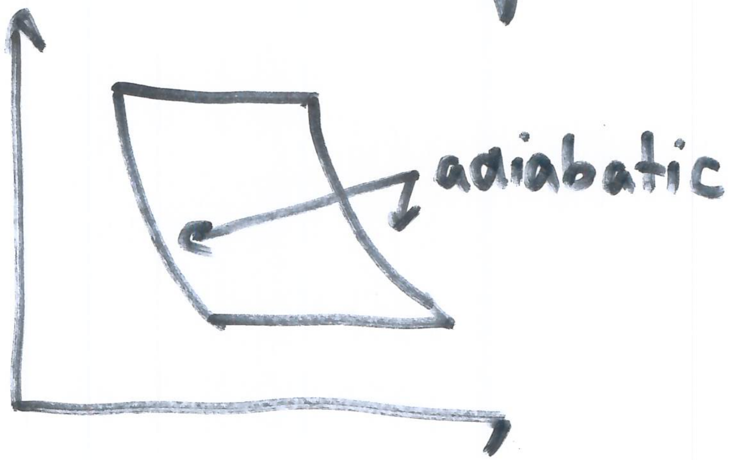
$$W = +\frac{1}{2} m R_0^4 \omega_0^2 \left(\frac{1}{r^2} - \frac{1}{R_0^2} \right)$$

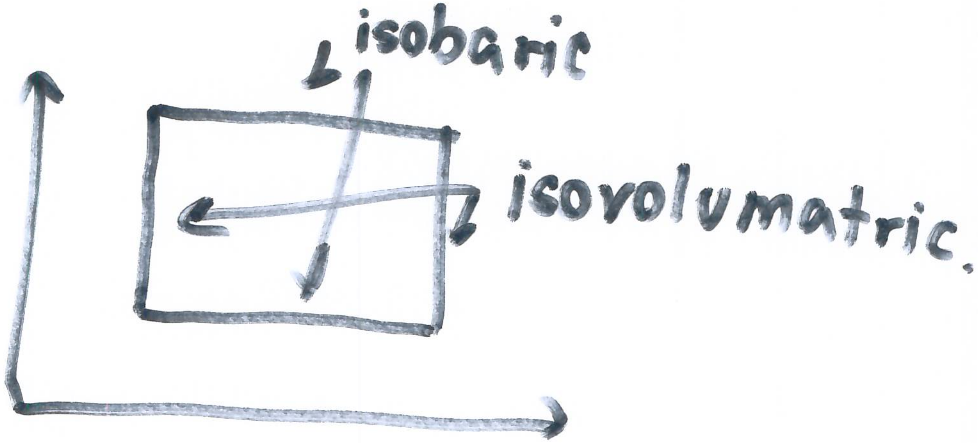
$$\boxed{W = \frac{1}{2} m R_0^2 \omega_0^2 \left(\frac{R_0^2}{r^2} - 1 \right)}$$

Thermodynamics

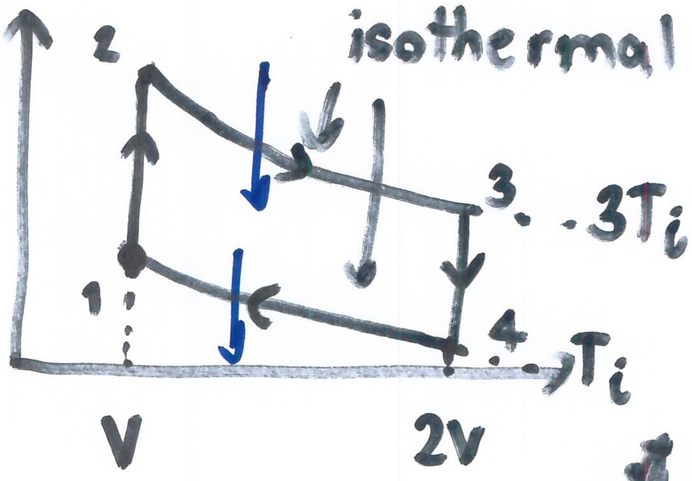








ตัวอย่าง



ก๊าซอุดมคติ โมเลกุลเดี่ยว n โมล จงหา Q ที่ระบบของก๊าซได้รับ ถ้าครบ 1 รอบ แล้วระบบกลับมาที่ 1

$\Rightarrow \Delta E = 0$

$$\Delta E = Q + W$$

$$\Rightarrow Q = -W$$

จะรู้ Q ได้ ต้องคำนวณหา W.

$$W = -\int P\,dv.$$

จาก ① → ② $\quad W = 0$

③ → ④ $\quad W = 0$

$$W_{isothermal} = nRT \ln\left(\frac{V_i}{V_f}\right)$$

② → ③ $\quad V_i = V \quad V_f = 2V \quad T = 3T_i$

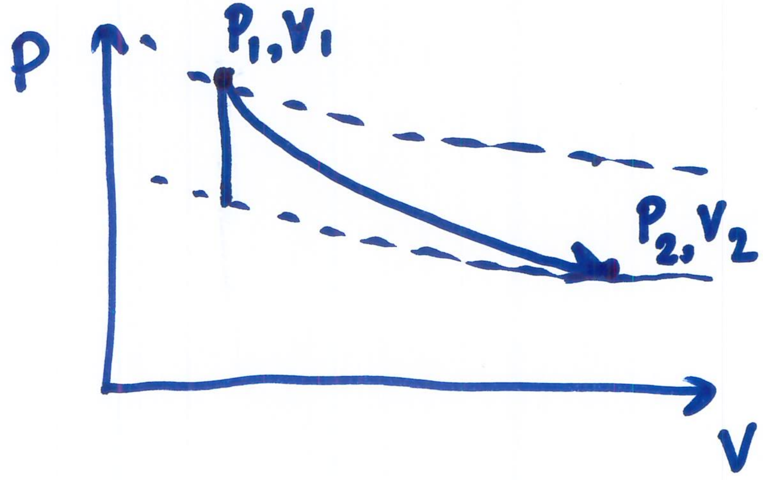
$$W_{② \to ③} = nR3T_i \ln\left(\frac{V}{2V}\right) = 3nRT_i \underbrace{\ln\left(\frac{1}{2}\right)}_{-\ln 2}$$

$$= -3nRT_i \ln 2$$

④ → ① $\quad V_i = 2V \quad V_f = V \quad T = T_i$

$$W_{④ \to ①} = nRT_i \ln\left(\frac{2V}{V}\right) = nRT_i \ln 2$$

ตัวอย่าง



a) จงหา entropy ของก๊าซ ให้ C_v เป็นความจุความร้อนของก๊าซที่ปริมาตรคงที่

$$dS = \frac{đQ}{T}$$

$$đQ = dE - dW = dE + p\,dV$$

$$dE = nC_v\,dT$$

$$\Rightarrow \quad đQ = nC_v\,dT + p\,dV$$

รู้ equation of state.

$$PV = nRT \quad \Rightarrow \quad T = \frac{PV}{nR}.$$

$$PdV + VdP = nRdT$$

$$dT = \frac{1}{nR}\left(Pdv + vdp\right)$$

$$đQ = \cancel{n}C_v \cdot \frac{1}{\cancel{n}R}\left(Pdv + vdp\right) + PdV$$

$$= Pdv\left(\frac{C_v}{R} + 1\right) + vdp \cdot \frac{C_v}{R}$$

$$dS = \frac{đQ}{T} = \frac{Pdv\left(\frac{C_v}{R} + 1\right) + vdp \cdot \frac{C_v}{R}}{PV/nR}$$

$$= nR\left(\frac{dv}{V}\underbrace{\left(\frac{C_v}{R} + 1\right)}_{C_P/R} + \frac{dP}{P} \cdot \frac{C_v}{R}\right)$$

$$\frac{C_v}{R} + 1 = \frac{C_v + R}{R} = \frac{C_p}{R}$$

$$ds = n\left(c_p \cdot \frac{dv}{V} + c_v \frac{dp}{P}\right)$$

$$= nc_v\left(\underbrace{\frac{c_p}{c_v}}_{=\gamma\,=\,\frac{5}{3}\ \text{โมเลกุลเดี่ยว.}} \frac{dv}{V} + \frac{dp}{P}\right)$$

$$= nc_v\left(\gamma \frac{dv}{V} + \frac{dp}{P}\right)$$

$$S = \int ds = nc_v\left(\gamma \int_{v_1}^{v_2} \frac{dv}{V} + \int_{p_1}^{p_2} \frac{dp}{P}\right)$$

$$= nc_v\left(\gamma \ln\left(\frac{v_2}{v_1}\right) + \ln\left(\frac{p_2}{p_1}\right)\right)$$

$$W_{\text{รวม}} = -3nRT_i \ln 2 + nRT_i \ln 2$$

$$= -2nRT_i \ln 2$$

$$Q = -W_{\text{รวม}} = \boxed{2nRT_i \ln 2.}$$

$$= nC_v \left(\cancel{\gamma} \ln \left(\left(\frac{V_2}{V_1}\right)^{\gamma} \cdot \frac{P_2}{P_1} \right) \right)$$

$$\boxed{\Delta S = nC_v \ln \left(\frac{P_2 V_1^{\gamma}}{P_1 V_2^{\gamma}} \right)}$$

b) ถ้า ΔS = คงที่ เกิดอะไรขึ้น $\Rightarrow dQ = 0$

$\Rightarrow$ adiabatic process.

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$$