Reviews on Electricity and Basic Electronics

SCPY152, Second Semester 2021-22

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Reviewed Topics

1. Electric charge and Coulomb force
2. Electric field
3. Gauss's law
4. Electric potential
5. Capacitors
6. Electronic circuit
7. Basic DC circuits

Electric charge and Coulomb force

- We observe electric charge in nature from static charge and lightning

Electric charge and Coulomb force

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- Charles Coulomb was studied force between electric charge, he was observed that there two kinds of them and were assigned to be *the positive charge* $+Q$ and *negative charge* $-Q$.
- Charge is measured in *Coulomb: C*, the smallest charge is of electron and proton

$$q_e = -e, \ \ q_p = +e, \ \ e = 1.6 \times 10^{-19} C$$

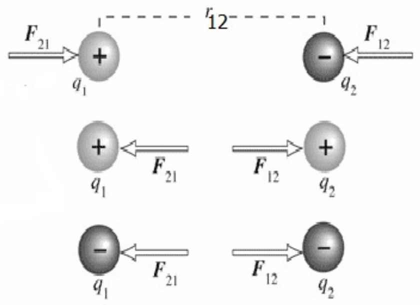
This was observed by Robert A. Millikan in his oil-drop experiment (1909)

- The force between two charges is proportional to the charges and inverse distance square

$$F_{12} \propto \frac{Q_1 Q_2}{r_{12}^2} \rightarrow F_{12} = K \frac{Q_1 Q_2}{r_{12}^2}$$

$K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 N/m^2 \cdot C^2$ is a constant.
($\epsilon_0 = 8.854 \times 10^{-12} Farad/m$ is vacuum dielectric permittivity)

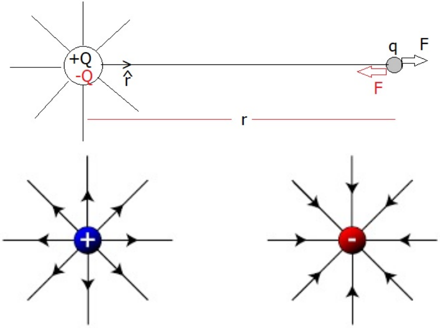
- The force is *attractive* between the different charges and *repulsive* between the same charges



Electric Field

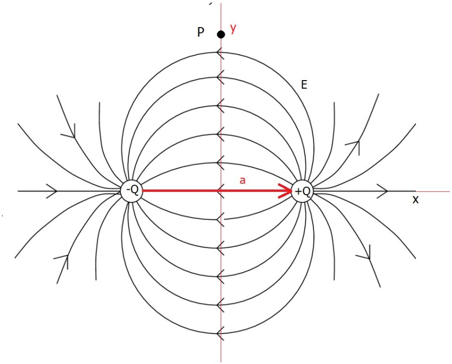
- From the viewpoint of *action at a distance*, we may think of a source charge $Q_1 = Q$ emits the electric force field in to all direction, and this feels by a test charge $Q_2 = q$ at a distance r from the source. From the Coulomb's law we will have

$$\vec{F} = K\frac{Qq}{r^2}\hat{r} = q\vec{E} \to \vec{E}(r) = \frac{KQ}{r^2}\hat{r}$$



- Electric field from multiple charges at any point P is a vector summation $\vec{E} = \sum_i \vec{E}_i$
- Electric field from electric dipole $\vec{d} = Q\vec{a} = Qa\hat{x}$

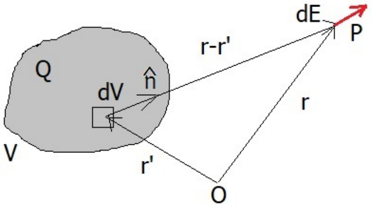
$$\vec{E}_P(y) = -\frac{Kd}{y^3}\hat{x}, \quad y >> a$$



- Electric field from charge distribution, with uniform charge density $\rho_c = \frac{Q}{V} \to dq = \rho dV$. The electric field at any point P is

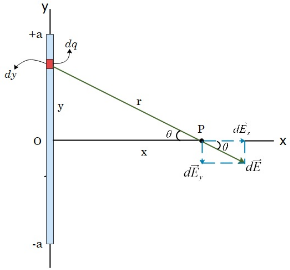
$$d\vec{E} = K\frac{\hat{n}dq}{|r-r'|^2} = \rho_c K\frac{\hat{n}dV}{|r-r'|^2}$$

$$\vec{E}_P = K\rho_c \int_V \frac{\hat{n}dV}{|r-r'|^2} \tag{1}$$



V must have geometrical shapes in order to be integrable!

- Electric field from finite line charge $\lambda = Q/2a,\ dq = \lambda dy$



$$d\vec{E}_P = dE_x\hat{x} + dE_y\hat{y}$$

$$\int_{-a}^{+a} dy dE_y = 0$$

$$dE_x = K\lambda\frac{\cos\theta dy}{x^2+y^2}$$

$$= K\lambda\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$E_x = K\lambda x\int_{-a}^{+a}\frac{dy}{(x^2+y^2)^{3/2}} = K\lambda x\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{-a}^{+a}$$

$$= \frac{K\lambda}{x}\frac{2a}{\sqrt{x^2+a^2}} = \frac{KQ}{x\sqrt{x^2+a^2}} \xrightarrow[a\to 0]{} \frac{KQ}{x^2},\ \vec{E}_P = E_x\hat{x}$$

Gauss's Law

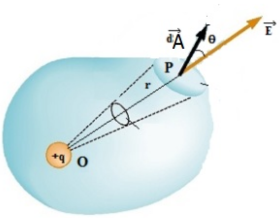
- Even we have determined the electric field from charge distributed geometrical object, it is quite complicate to do the volume integration.
- We can find an electric field from charge distributed geometrical object at some particular point by using geometrical method. It is called *Gauss's law*.
- Let us start by determining electric field at P at a distance r from a point charge $+Q$, we find that

$$\vec{E}_P = \frac{KQ}{r^2}\hat{r} \equiv \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

- We can realize that $4\pi r^2\hat{r} = \vec{A}(r)$ is a surface of sphere of radius r cover the point charge Q. Then we can write

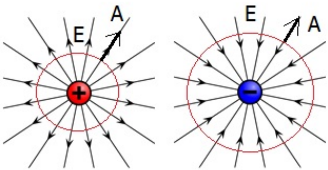
$$\vec{E}_P \cdot \vec{A}(r) \equiv \oint \vec{E}_P \cdot d\vec{A}(r) = \frac{Q}{\epsilon_0}$$

- The generic expression of Gauss's law is *the electric* $\vec{E}$ *that pass through the surface* $\vec{A}$ *is equal to the charge* Q *enclosed by that surface divided by* ϵ_0

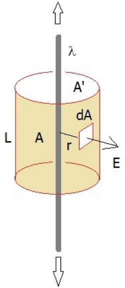


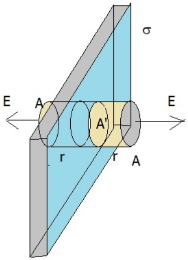
Note that $\vec{A}$ is called *Gauss's surface*, and the quantity $\oint \vec{E} \cdot \vec{A} = \Phi_E$ is called *electric flux* that pass through Gauss's surface.

It is $(+ve)\Phi_E$ from $+Q$ and $(-ve)\Phi_E$ from $-Q$.



- Electric field E at a distance r from infinite line and plate of charged insulators





$$\Phi_E = 2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\Phi_E = E2\pi rL = \frac{\lambda L}{\epsilon_0}$$

$$\rightarrow E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

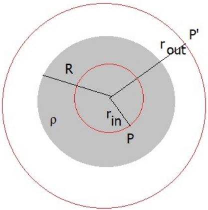
- Electric field at P(in) and P'(out) of charged insulating sphere of radius R, $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

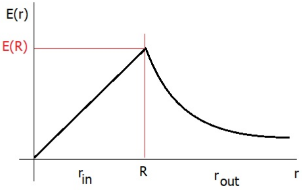
$$E_{in} 4\pi r_{in}^2 = \frac{\rho \frac{4}{3}\pi r_{in}^3}{\epsilon_0}$$

$$E_{in} = \frac{Q r_{in}}{4\epsilon_0 R^3}$$

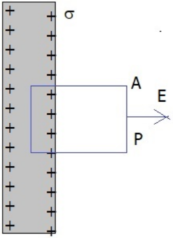
$$E_{out} 4\pi r_{out}^2 = \frac{\rho \frac{4}{3}\pi R^3}{\epsilon_0}$$

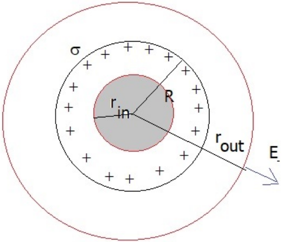
$$\rightarrow E_{out} = \frac{Q}{4\pi\epsilon_0 r_{out}^2}$$





- Electric field from conducting plate σ and conducting sphere σ





$$EA = \frac{\sigma A}{\epsilon_0}$$

$$\rightarrow E = \frac{\sigma}{\epsilon_0}$$

uniform field

$$E_{in} 4\pi r_{in}^2 = 0 \rightarrow E_{in} = 0$$

$$E_{out} 4\pi r_{out}^2 = \frac{\sigma 4\pi R^2}{\epsilon_0}$$

$$\rightarrow E_{out} = \frac{Q}{4\pi\epsilon_0 r_{out}^2}$$

point charge field

Electric force on charged particles

- Electric force on a point charge q, in uniform electric field $\vec{E}$

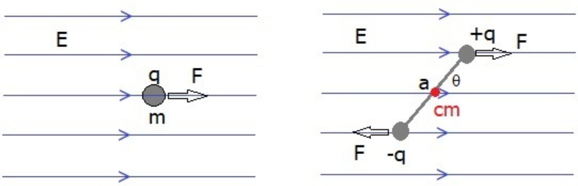
$$\vec{F} = q\vec{E} = m\vec{a} \to \vec{a} = \frac{q}{m}\vec{E} \tag{2}$$

, it produces a torque on the dipole as

- Electric force on electric dipole $\vec{d} = q\vec{a}$

$$\vec{\tau} = \vec{a} \times q\vec{E} = I_0\vec{\alpha} \to \tau = aqE\sin\theta \simeq aqE\theta = -I_0\ddot{\theta}$$
$$\to \ddot{\theta} + \frac{aqE}{I_0}\theta = 0 \tag{3}$$

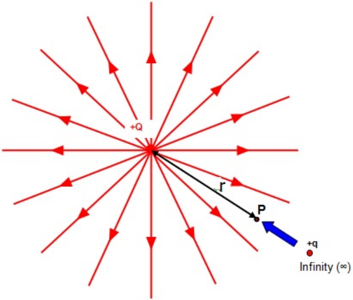
The dipole will oscillate by the torque with angular frequency $\omega = \sqrt{aqE/I_0}$.



Electric potential

- Let us determine work done on test charge q in bringing from infinity to a point P at distance r from the source charge $+Q$

$$W_{\infty r} = \int_{\infty}^{r} \vec{F}_{ext} \cdot d\vec{r}' = -KQq \int_{r}^{\infty} \frac{dr'}{r'^2} = \frac{KQq}{r}$$
$$= U(r) - U(\infty) \rightarrow U(r) = \frac{KQq}{r} \qquad (4)$$



- Electric potential of the source charge Q is defined to be this work done per unit test charge q

$$V(r) = \frac{W_{\infty r}}{q} = \frac{KQ}{r}$$

  The sign of V depend on the sign of source charge Q, and it is scalar quantity. Note about its dimension
  $[U] = J \rightarrow [V] = J/C = Volt : V$
- The electric field at this point can be derived from the *gradient* of the potential as

$$\vec{E}(r) = -\nabla_r V(r) = -\hat{r}\frac{d}{dr}V(r) = \frac{KQ}{r^2}\hat{r}$$

- Similarly, the Coulomb force on a test charge can be derived from the gradient of the electric potential energy

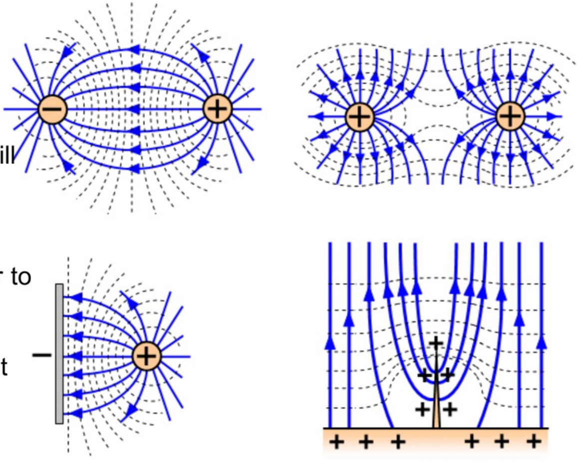
$$\vec{F}(r) = -\nabla_r U(r) = \frac{KQq}{r^2}\hat{r}$$

- Equi-potential lines = contours of constant potential V and electric field (point down hill)

E field points downhill

Downhill is always perpendicular to level

Conductors at rest are equipotential

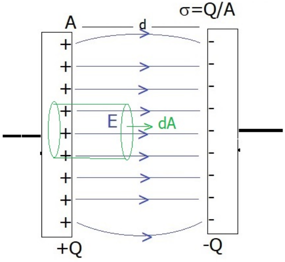


(resourcefulphysics.org)

Capacitor

- Capacitor is a device use to store electric charge, its structure compose of two conducting plates of surface area A placed at a distance d. When a charge $+Q$ placed on one plate, it will uniformly distribute the inner surface ($\sigma = Q/A$), and induces negative charge $-Q$ on the inner surface of opposite plate.
- The electric field $\vec{E}$ between the plate is determined from Gauss's law to be

$$\vec{E}\cdot d\vec{A} = EdA = \frac{dQ}{\epsilon_0} \to E = \frac{1}{\epsilon_0}\frac{dQ}{dA} = \frac{\sigma}{\epsilon_0}$$



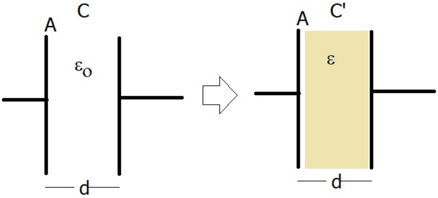
The electric field is uniform and potential different between the two plates is

$$\Delta V = Ed = \frac{d\sigma}{\epsilon_0}$$

- Capacitance C (the ability of storing charge of the capacitor) is defined from

$$Q = C\Delta V \to C = \frac{\epsilon_0 A}{d}, \ [C] = Farad : F$$

- Note that C is increased by A and reduced by d
- On the other hand we can increase C by replacing ϵ_0 with ϵ (dielectric constant of some dielectric medium)



$$C' = \frac{\epsilon A}{d} = \frac{\epsilon}{\epsilon_0}\frac{\epsilon_0 A}{d} = \kappa C$$

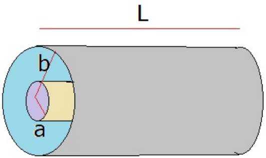
Note that the dielectric constant $\kappa = \epsilon/\epsilon_0 > 1$

- Dielectric constant

Dielectric constants for various materials at 25°C

Material	Dielectric constant κ
Vacuum	1.0
Air at 1.0 atm	1.0005
Polystyrene	2.6
Paper	3.5
Pyrex glass	4.7
Porcelain	6.5
Nerve membrane	7.0
Silicon	12.0
Ethanol	25.0
Water	78.5

- Capacitance of coaxial cylindrical capacitor



$$E = \frac{q}{2\pi\varepsilon_0 Lr}. \tag{26-12}$$

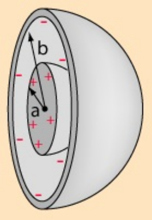
Substitution of this result into Eq. 26-6 yields

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right), \tag{26-13}$$

where we have used the fact that here $ds = -dr$ (we integrated radially inward). From the relation $C = q/V$, we then have

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor)}. \tag{26-14}$$

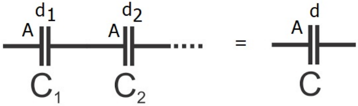
- Capacitance of co-center spherical capacitor



$$C = \frac{4\pi\epsilon_0}{1/a - 1/b}$$

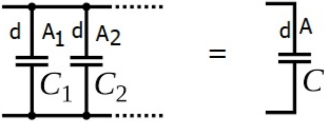
- Connection of capacitors, what is its total capacitance?
  - series connection (increase distance d/fixed plate area A)

$$d = d_1 + d_2 + ... = \epsilon_0 A \left(\frac{1}{C_1} + \frac{1}{C_2} + .. \right) = \frac{\epsilon_0 A}{C}$$
$$\rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

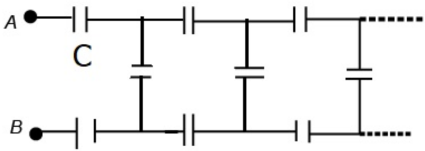


  - Parallel connection (increase plate area A/fixed distance d)

$$A = A_1 + A_2 + ... = \frac{\epsilon_0}{d} (C_1 + C_2 + ...) = \frac{\epsilon_0 C}{d}$$
$$\rightarrow C = C_1 + C_2 + ...$$



- Exercise: What is capacitance C_{AB} of this connection of capacitors? (All capacitors have capacitance C)



Electric field energy

- Determine parallel plate capacitor with area A and plate distance d, its capacitance is $C = \epsilon_0 A/d$. The electric field between the two plates will be $E = \sigma/\epsilon_0, \sigma = Q/A$, where Q is a storing charge.
- The work done on this capacitor in charging process is

$$dW = Vdq \rightarrow W = \int_0^Q Vdq = \frac{1}{C}\int_0^Q qdq = \frac{1}{2C}Q^2 \equiv U \quad (5)$$

$$Q = EA/\epsilon_0 \rightarrow U = \frac{1}{2}\epsilon_0 E^2 vol., \;\; vol. = Ad \quad (6)$$

$$u = \frac{U}{vol.} = \frac{1}{2}\epsilon_0 E^2 \quad (7)$$

The energy put into the capacitor stored in form of electric field inside the, with the above energy density.

Electric circuit

- Electrical conduction in (isotropic) matter, by definition, is described by conduction equation

$$\vec{j} = \sigma \vec{E}$$

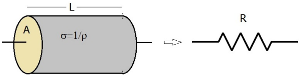
Note that $[\vec{j}] = C/s \cdot m^2 = A/m^2$ is current density, $(C/s = Ampere : A)$ The electric field $[\vec{E}] = V/m$, and σ is electrical conductivity of matter, and $[\sigma] = A/V \cdot m$. We also use $\rho = 1/\sigma$, an electrical resistivity

Material	Electrical Resistivity (Ω x cm)	Electrical Conductivity (Ω^{-1} x cm^{-1})
Cu	0.034 x10^{-5}	29 x 10^5
Fe	32.54 x 10^{-5}	0.031 x 10^5
Ag	0.36 x 10^{-5}	2.8 x 10^5
Al	0.03 x 10^{-5}	33.3 x 10^5
Ni	0.046 x 10^{-5}	21.7 x 10^5
Cu-Fe	33.37 x 10^{-5}	0.030 x 10^5
Cu-Ag	2.71 x 10^{-5}	0.37 x 10^5
Al-Ni	0.564 x 10^{-5}	1.77 x 10^5

- Electrical conduction in terms of current I and potential different V

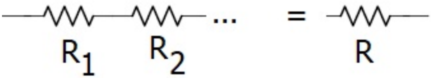
$$I = \vec{j} \cdot \vec{A} = \sigma \vec{A} \cdot \vec{E} = \sigma A E = \sigma A \frac{V}{L} = \frac{A}{\rho L} V$$
$$\rightarrow I = \frac{V}{R}, \quad R = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

We derive *Ohm's law*, with a defined resistance R in Ohm (Ω)

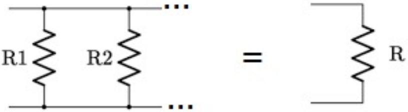


Note that R increase by L and ρ, but reduce by σ and A.

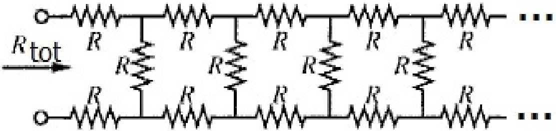
- Connections of resistors
  - series connection:
    $L = L_1 + L_2 + ... = \sigma A(R_1 + R_2 + ..) \equiv \sigma AR, R = R_1 + R_2 + ...$



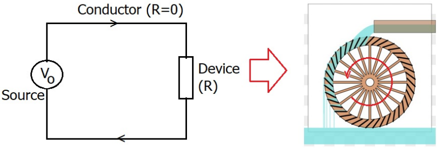
- Connection of resistors (cont.)
  - parallel connection:
    $$A = A_1 + A_2 + \ldots = \frac{L}{\sigma}\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right) \equiv \frac{L}{\sigma R}, \;\; \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$



- Exercise: total resistance of infinite resistors connection



- Electrical circuit



- Circuit theory
  - Ohm's law $V = IR$
  - Kirchhoff law
    - set loop currents for all loops circuit
    - sum of all dropped voltages = sum of all sources appear in the loop circuit

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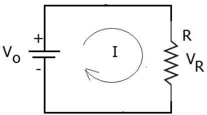
- Direct current or DC circuit: a simple circuit

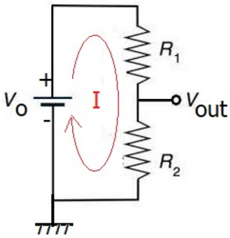


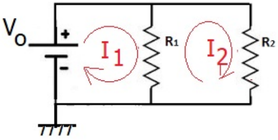
Loop current is I, dropped voltage on R is

$$V_R = IR$$

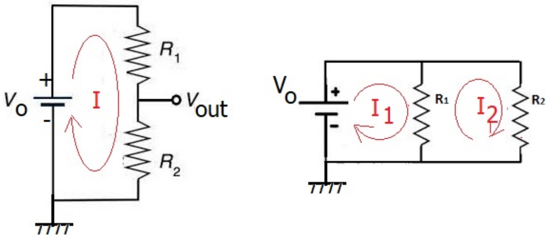
$$V_0 = V_R = IR \to I = \frac{V_0}{R}$$

- Voltage divider and current divider circuits

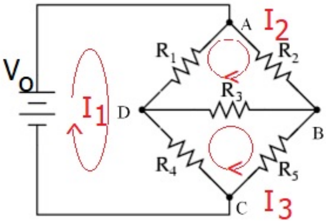




- Voltage divider and current divider circuits

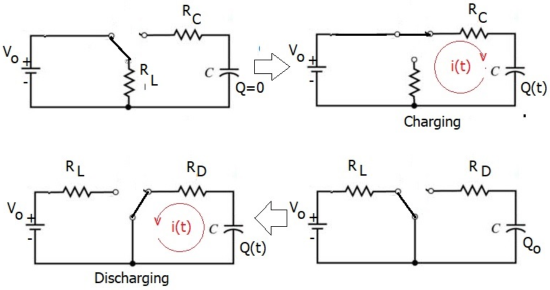


- Resistance bridge circuit



Bridge balance condition: $V_D = V_B \rightarrow I_2 = I_3, \frac{R_1}{R_2} = \frac{R_4}{R_5}$

- Transient circuits of capacitor charging/discharging



- Capacitor charging:

$$V_0 = i(t)R_C + \frac{1}{C}q(t) \rightarrow \frac{dQ(t)}{dt} + \frac{1}{R_C C}Q(t) - \frac{V_0}{R_C} = 0$$

$$q(t) = Q(t) - CV_0 \rightarrow \frac{dq(t)}{dt} + \frac{1}{R_C C}q(t) = 0$$

$$\frac{dq(t)}{q(t)} = -\frac{1}{R_C C}dt \rightarrow q(t) = q(0)e^{-t/R_C C}$$

$$Q(t) = Q_0(1 - e^{-t/R_C C}), \ Q_0 = CV_0$$

- Capacitor charging (cont.) charging time $\tau_C = R_C C$,

$$Q(t) = Q_0 \left(1 - e^{t/\tau_C}\right)$$

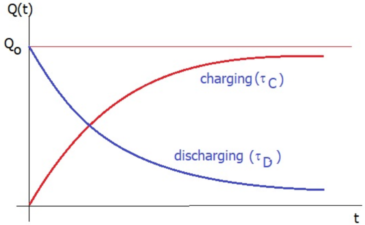
- Capacitor discharging

$$V_C(t) = \frac{1}{C} Q(t) = i(t) R_D = -R_D \frac{dQ(t)}{dt}$$

$$\rightarrow \frac{dQ(t)}{Q(t)} = \frac{1}{R_D C} dt \rightarrow Q(t) = Q_0 e^{-t/R_D C}$$

Discharging time $\tau_D = R_D C$

$$Q(t) = Q_0 r^{-t/\tau_D}$$



- Electric power dissipate on a resistor, using Ohm's law

$$\begin{aligned} P &= IV \; [Amp \cdot volt = \frac{C}{s} \cdot volt = \frac{J}{s} = watt] \qquad (8) \\ \rightarrow P &= I^2 R = \frac{V^2}{R} \qquad (9) \end{aligned}$$