Reviews on Magnetism and Induction

SCPY152, Second Semester 2021-22

Udom Robkob, Physics-MUSC

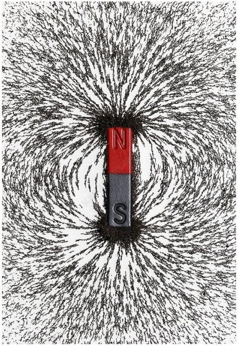
January 11, 2022

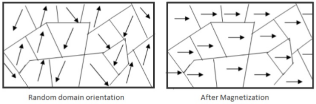
Reviewed Topics

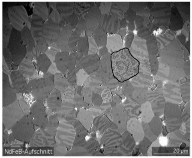
1. Nature of magnetic field
2. Magnetic field from current carrying wire
3. Ampere's law
4. Induction and Lenz's law
5. Faraday's law
6. Electromotive force
7. Inductors
8. Complex impedance
9. AC circuits

Nature of magnetic field

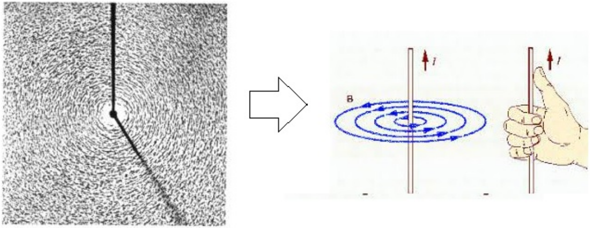
- We observe magnetic field from permanent magnet
- Permanent magnet is *ferromagnetic material*, consist of large domains aligned magnetic moment (magnetization)



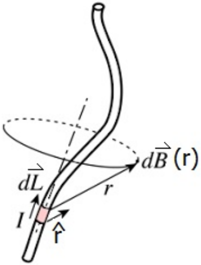




- Magnetic field is also generated from current carrying wire



- The magnetic field strength is calculated by Biot-Savart law



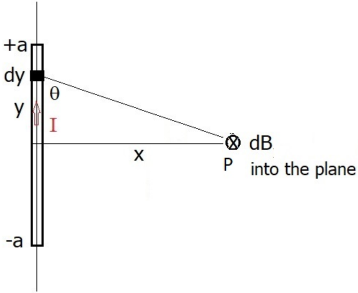
$$d\vec{B}(r) = \frac{\mu_0}{4\pi}\frac{Id\vec{L} \times \hat{r}}{r^2} \qquad (1)$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{L} \times \hat{r}}{r^2} \qquad (2)$$

$d\vec{L}$ got direction from the flow of current I

$\mu_0 = 1.257 \times N/A^2$ is vacuum permeability

- Magnetic field at P from finite wire



$$dB_x = \frac{\mu_0 I}{4\pi}\frac{\sin\theta\, dy}{x^2+y^2}$$

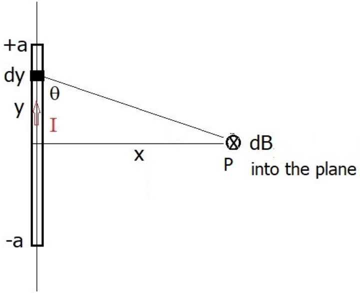
$$B_x = \int_{-a}^{a} dB_x$$

$$= \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{dy}{(x^2+y^2)^{3/2}}$$

$$= \frac{\mu_0 I x}{4\pi}\left[\frac{2a}{x^2\sqrt{x^2+a^2}}\right]$$

$$\xrightarrow[a\to\infty]{} \frac{\mu_o I}{2\pi x}, \quad \text{(Infinite wire)}$$

- Magnetic field at P from finite wire



$$dB_x = \frac{\mu_0 I}{4\pi} \frac{\sin\theta dy}{x^2 + y^2}$$

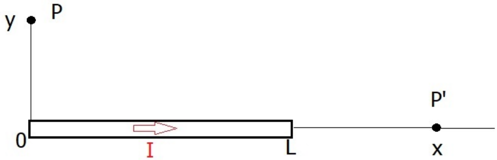
$$B_x = \int_{-a}^{a} dB_x$$

$$= \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{dy}{(x^2 + y^2)^{3/2}}$$

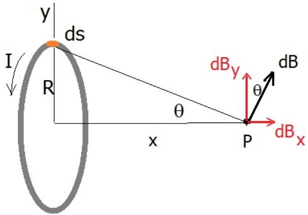
$$= \frac{\mu_0 I x}{4\pi} \left[\frac{2a}{x^2\sqrt{x^2 + a^2}} \right]$$

$$\xrightarrow[a\to\infty]{} \frac{\mu_o I}{2\pi x}, \text{ (Infinite wire)}$$

- *Exercise:* magnetic field at P,P' from finite wire



- Magnetic field at P from current loop



$$dB = \frac{\mu_0 I}{4\pi} \frac{ds}{x^2 + R^2}$$

$$dB_x = dB \sin\theta$$

$$= \frac{\mu_0 I}{4\pi} \frac{Rds}{(x^2 + R^2)^{3/2}}$$

$$B_P = \oint dB_x$$

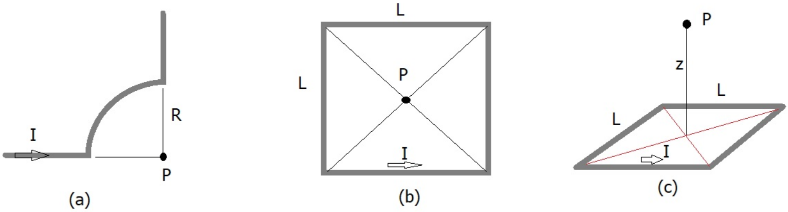
$$= \frac{\mu_0 I}{4\pi} \frac{R}{(x^2 + R^2)^{3/2}} \oint ds$$

$$B_P = \frac{\mu_0 I}{4\pi} \frac{R(2\pi R)}{(x^2 + R^2)^{3/2}} = \frac{\mu_0 I}{2} \frac{R^2}{(x^2 + R^2)^{3/2}}$$

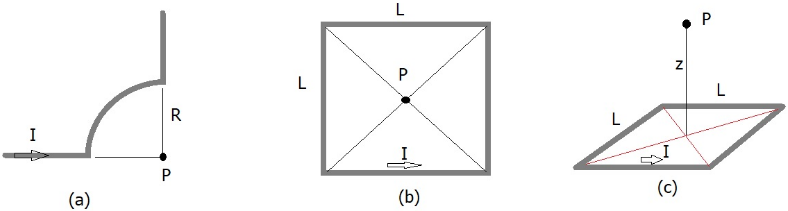
$$B_P \xrightarrow[x \to 0]{} \frac{\mu_0 I}{2R} \text{ at the center of the loop}$$

$$B_P = \frac{\mu_0 N I}{2R} \text{ at the center of N loops}$$

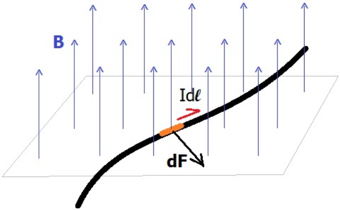
- *Exercise:* find the magnetic field (strength and direction) at P from the following wires



- *Exercise:* find the magnetic field (strength and direction) at P from the following wires



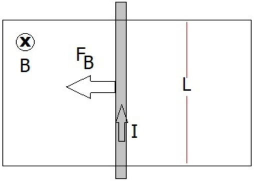
- Magnetic force on current carrying wire



$$d\vec{F} = Id\vec{l} \times \vec{B}$$

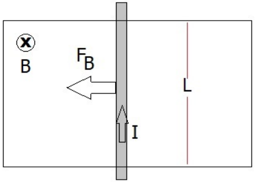
$$\vec{F} = I \int d\vec{l} \times \vec{B}$$

- Magnetic force on straight line



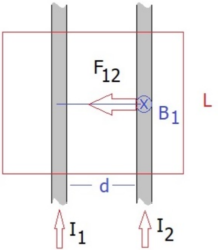
$$F_B = ILB$$

- Magnetic force on straight line



$$F_B = ILB$$

- Magnetic force between parallel straight lines

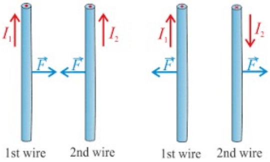


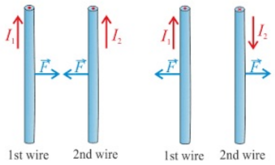
$$B_1 = \frac{\mu_0 I_1}{2d}$$

$$F_{12} = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2d}$$

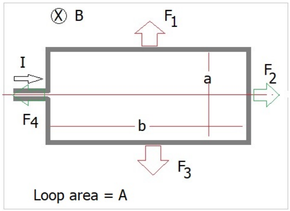
$$\frac{F_{12}}{L} = \frac{\mu_0 I_1 I_2}{2d}, \quad F_{21} = F_{12}$$

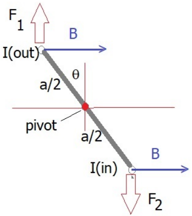
In opposite direction.


I_1
I_2
$\vec{F}$
$\vec{F}$
1st wire
2nd wire
I_1
I_2
$\vec{F}$
$\vec{F}$
1st wire
2nd wire



- Magnetic force on loop wire



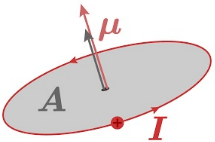


$$F_2 = -F_4 = IaB \rightarrow F_2 + F_4 = 0$$

$$F_1 = -F_2 = IbB \rightarrow F_1 + F_3 = 0$$

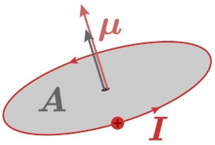
$$\text{Net torque from } F_1, F_3: \ \tau = IabB\sin\theta \rightarrow \vec{\tau} I\vec{A} \times \vec{B}$$

- Magnetic moment from current loop



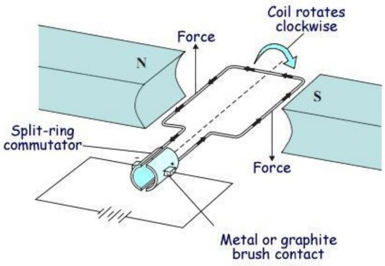
$$\vec{\mu} = I\vec{A} + RHR$$
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

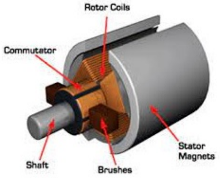
- Magnetic moment from current loop



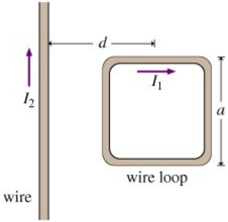
$$\vec{\mu} = I\vec{A} + RHR$$
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

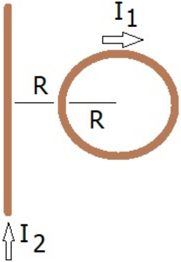
- DC electric motor





- *Exercise:* Determine magnetic force on following current loop I_1



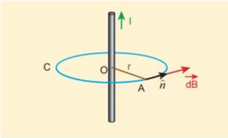


Ampere's law

- From the magnetic field from infinite wire, we may imagine a closed loop around the wire and pass through the point P we have to determine the magnetic field. The magnetic field will lie along the circular loop, and we can say that

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I$$

where $\oint d\vec{l}$ is closed Ampere's loop.

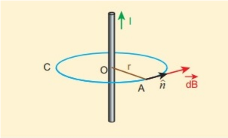


Ampere's law

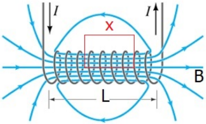
- From the magnetic field from infinite wire, we may imagine a closed loop around the wire and pass through the point P we have to determine the magnetic field. The magnetic field will lie along the circular loop, and we can say that

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I$$

where $\oint d\vec{l}$ is closed Ampere's loop.



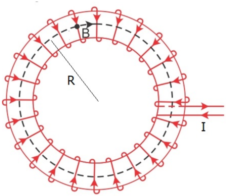
- Magnetic field inside solenoid of N turns



$$Bx = \mu_0 \frac{x}{L} NI$$

$$B = \mu_0 \frac{N}{L} I = \mu_0 n I, n = \frac{N}{L}$$

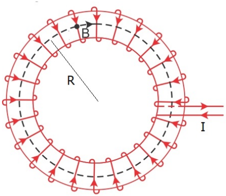
- Magnetic field inside a toroid of N turns and radius R



$$B(2\pi R) = \mu_0 NI$$

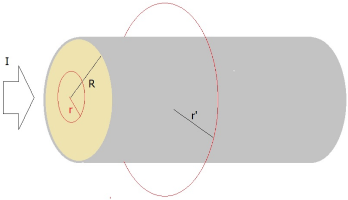
$$B = \frac{\mu_0 NI}{2\pi R}$$

- Magnetic field inside a toroid of N turns and radius R



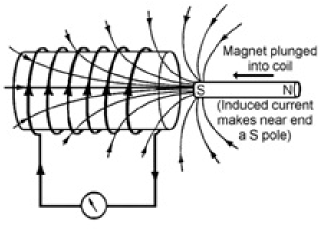
$$B(2\pi R) = \mu_0 NI$$
$$B = \frac{\mu_0 NI}{2\pi R}$$

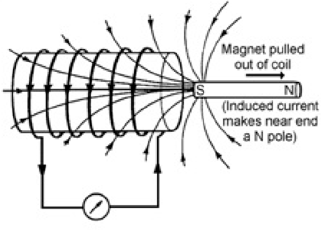
- *Exercise:* Magnetic field inside (r) and outside (r') a conductor of radius R



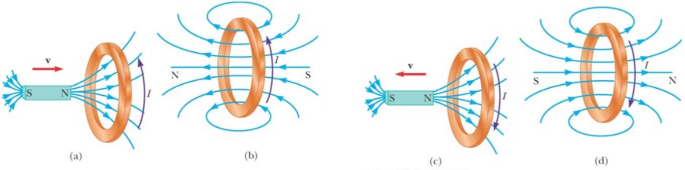
Induction

- Electromagnetic induction experiment





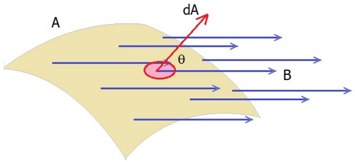
- Lenz law: *the induced current flow in direction to compensate magnetic flux reduced or increased through the loop wire*



- Faraday's law: *the electromotive force (to drive the induced current in the loop wire) is proportional to the rate of change of magnetic flux through the loop*

$$\mathcal{E}_{loop\ \ wire} \propto \frac{d\Phi_B}{dt} = -\frac{d\Phi_B}{dt} \tag{3}$$

- Magnetic flux



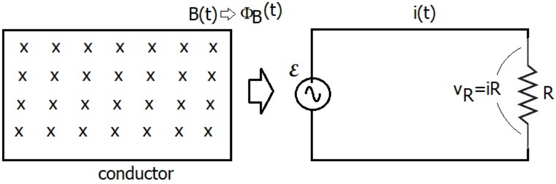
$$d\Phi_b = \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

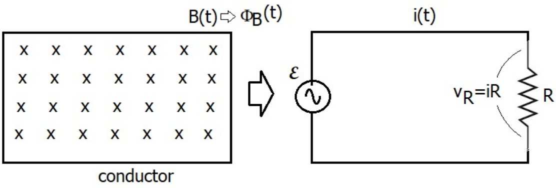
$$0 = \oint \vec{B} \cdot d\vec{A}_{closed}$$

Zero net flux through closed surface, since we have no magnetic charge. This is *Gauss's law of magnetic field*.

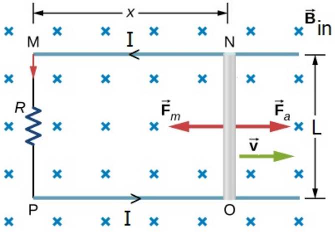
- Electromotive force (emf) $\mathcal{E}$ force current to flow around the loop wire, it behaves like a source of potential different



- Electromotive force (emf) $\mathcal{E}$ force current to flow around the loop wire, it behaves like a source of potential different

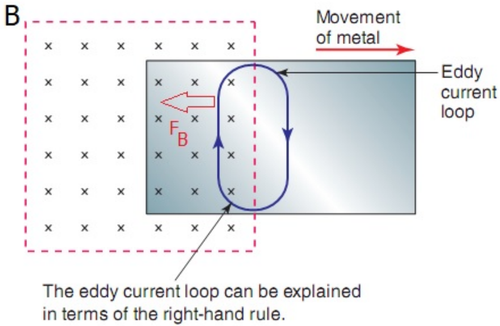


- Motional emf $\mathcal{E}_m$

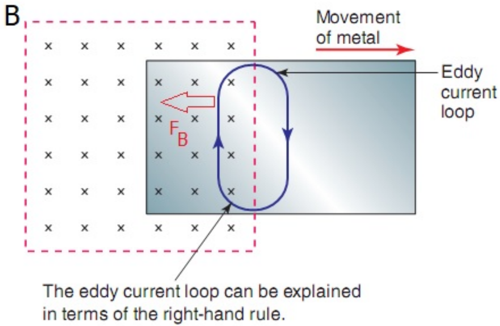


$$\mathcal{E}_m = -\frac{d\Phi_B}{dt} = -\frac{BLdx}{dt}$$
$$= -BLv = -IR$$
$$\rightarrow I = \frac{BLv}{R} \text{ clockwise}$$
$$F_m = ILB = \frac{vB^2L^2}{R}$$
$$F_a = -F_m$$

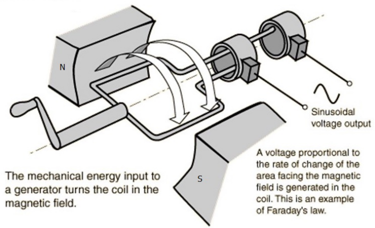
- Eddy current loop



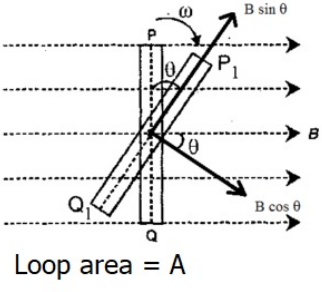
- Eddy current loop



- Alternating current / voltage generator



- Rotational loop wire emf (area A and N turns)



$$\Phi_B(t) = NAB\cos\theta(t)$$

$$\mathcal{E}(t) = -\frac{d\Phi_B(t)}{dt} = NAB\omega\sin\theta(t)$$

$$\mathcal{E}(t) = \mathcal{E}_0\sin(\omega t)$$

$\theta(t) = \omega t$ and $\mathcal{E}_0 = NAB\omega$.

Alternating current

$$i(t) = \frac{\mathcal{E}(t)}{R} = I_0\sin(\omega t),\;\; I_0 = \frac{\mathcal{E}_0}{R}$$