Reviews on Inductor, AC-Circuits, Electromagnetic Waves, and Visible light

SCPY152, Second Semester 2021-22

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Topics

1. Inductor
2. Magnetic field energy
3. Complex impedance and AC circuits
4. RLC resonance
5. Electromagnetic waves
6. Visible light

Inductor

- Consider a solecnoid of N turn, length L (so that $n = N/L$) and cross section area A. When a current $i(t)$ pass through the wire the magnetic field produced inside $B(t) = \mu_0 n i(t)$ and magnetic flux through the solenoid is

$$\Phi_B(t) = B(t)A = \mu_0 n A i(t)$$

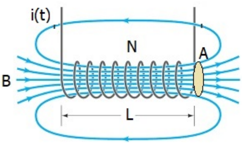
The change in time of this flux induces emf inside the wire as

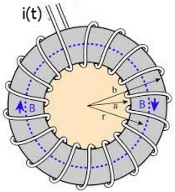
$$\mathcal{E} = -N\frac{d\Phi_B(t)}{dt} = -\mu_0 nNA\frac{di(t)}{dt} \equiv -V_L(t) \quad (1)$$

$$V_L(t) = \mu_0 nNA\frac{di(t)}{dt} = L\frac{di(t)}{dt} \quad (2)$$

$$\rightarrow L = \mu_0 nNA = \mu_0 n^2 vol. \quad (3)$$

L is a solenoid inductance, in unit of [Henri: H], and $vol. = LA$ is the volume of the solenoid.



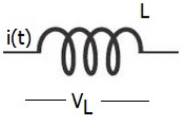


- Toriodal inductance (on N turns, radius r, and cross section area A)

$$B(t) = \frac{\mu_0 N i(t)}{2\pi r} \rightarrow \Phi_B(t) = B(t)A = \mu_0 N A i(t) \quad (4)$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{\mu_0 N^2 A}{2\pi r}\frac{di(t)}{dt} \equiv -V_L(t) \rightarrow L = \frac{\mu_0 N^2 A}{2\pi r}$$

- Inductor symbol



Magnetic field energy

- The rate of electrical energy (power) dissipate into the solenoid inductor is

$$\mathcal{P} = iV_L = Li\frac{di}{dt} \equiv \frac{dU}{dt} \rightarrow U(t) = \frac{1}{2}Li^2 \tag{5}$$

- Rewrite in terms of the magnetic field strength B of the solenoid

$$\mathcal{P} = -i\mathcal{E} = Li\frac{di}{dt} = \frac{dU}{dt} \rightarrow U = \frac{1}{2}Li^2 \tag{6}$$

$$i = \frac{B}{\mu_0 n} \rightarrow U = \frac{1}{2}\frac{LB^2}{\mu_0^2 n^2} = \frac{1}{2\mu_0}B^2 vol. \tag{7}$$

$$u = \frac{U}{vol.} = \frac{1}{2\mu_0}B^2 \tag{8}$$

This is the energy density of the magnetic field strength B

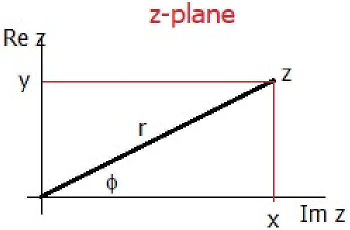
Complex impedance

- complex number

$$z = x + iy \rightarrow Re(z) = x, Im(z) = y, i^2 = -1 \quad (9)$$

- Polar form

$$z = re^{i\phi},\ \ r = \sqrt{x^2 + y^2}, \phi = \tan^{-1}\frac{y}{x}$$
$$x = r\cos\phi, y = r\sin\phi \quad (10)$$



- Complex impedance Z

$$Z = R + iY, \;\; R - resistance, \;\; Y - admittance \qquad (11)$$

- Ohm's law for AC current, with sinusoidal current

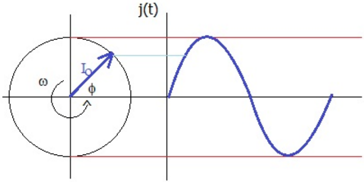
$$j(t) = I_0 \sin(\omega t) = Im\left(I_0 e^{i\omega t}\right) \qquad v_Z = jZ \qquad (12)$$

$$v_R = jZ_R = jR \qquad \rightarrow Z_R = R \qquad (13)$$

$$v_L = jZ_L = L\frac{dj}{dt} = i\omega Lj \qquad \rightarrow Z_L = i\omega L \qquad (14)$$

$$v_C = \frac{1}{C}q(t) = \frac{1}{C}\int j(t)dt = \frac{1}{i\omega C}j \qquad \rightarrow Z_C = \frac{1}{i\omega C} \qquad (15)$$

- Phasor of AC current

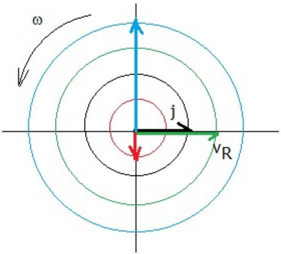


- Phasor of v_Z, with $i = e^{i\pi/2}$ and $-i = e^{-i\pi/2}$

$$v_R = jZ_R = I_0 R e^{i\omega t} = V_{0R} e^{i\omega t}, \;\; V_{0R} = I_0 R \tag{16}$$

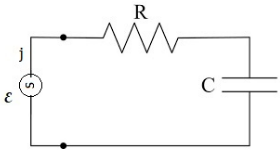
$$v_C = jZ_C = \frac{-i}{\omega C} I_0 e^{i\omega t} = V_{0C} e^{i\omega t - i\pi/2}, \;\; V_{0C} = \frac{I_0}{\omega C} \tag{17}$$

$$v_L = jZ_L = i\omega L I_0 e^{i\omega t} = V_{0L} e^{i\omega t + i\pi/2}, \;\; V_{0L} = \omega L I_0 \tag{18}$$



We observe that v_R is in-phase, v_C has lacking-phase, and v_L has leading-phase to the current $j(t)$

- Let we have the current source $j(t) = I_0 e^{i\omega t}$
- Let us determine the RC circuit



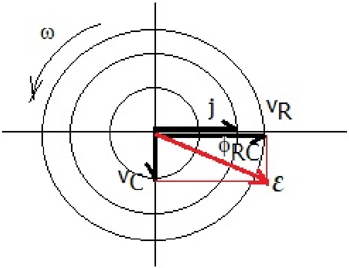
$$\mathcal{E} = v_R + v_C = (R - \frac{i}{\omega C})j$$

$$R - \frac{i}{\omega C} = \sqrt{R^2 + \frac{1}{(\omega C)^2}} e^{-i\phi_{RC}}$$

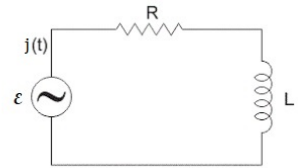
$$\phi_{RC} = \tan^{-1}\frac{1}{\omega CR}$$

$$\mathcal{E}(t) = \mathcal{E}_0 e^{i\omega t - i\phi_{RC}}$$

$$\mathcal{E}_0 = I_0\sqrt{R^2 + \frac{1}{(\omega C)^2}}$$



- Let us determine the RL circuit



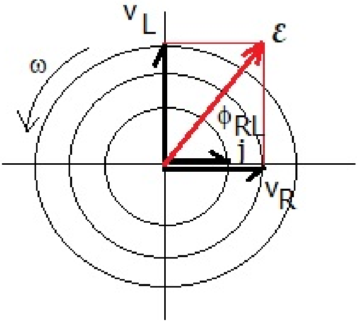
$$\mathcal{E} = v_R + v_L = (R + i\omega L)j$$

$$(R + i\omega L) = \sqrt{R2 + (\omega L)^2} e^{i\phi_{RL}}$$

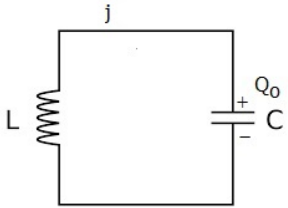
$$\phi_{RL} = \tan^{-1} \frac{\omega L}{R}$$

$$\mathcal{E} = \mathcal{E}_0 e^{i\omega t + i\phi_{RL}}$$

$$\mathcal{E}_0 = I_0 \sqrt{R^2 + \omega^2 L^2}$$



- Let us determine the LC circuit, start from a full stored capacitor



$$v_L + v_C = 0 \to i\omega L j - \frac{i}{\omega C} j = 0$$

$$(\omega^2 - \frac{1}{LC})j = 0,\ \omega = \frac{1}{\sqrt{LC}} \equiv \omega_0$$

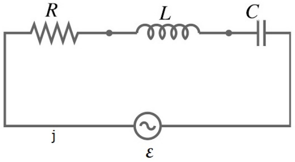
Simple oscillation of charge $q(t)$ and the discharge current $j(t) = -dq(t)/dt$

$$0 = v_C + v_L = \frac{1}{C}q(t) + L\frac{dj(t)}{dt} = \frac{1}{c}q(t) - L\frac{d^2q(t)}{dt^2}$$

$$\to \frac{d^2q}{dt^2} - \omega_0^2 q = 0 \to q(t) = Q_0 e^{i\omega_0 t}$$

RLC resonance

- Let us determine the series RLC circuit



$$\begin{aligned}\mathcal{E} &= v_R + v_L + v_C \\ &= [R + i(\omega L - \frac{1}{\omega C})]j \\ &\equiv \mathcal{E}_0 e^{i\omega t + i\phi_{RLC}} \\ \mathcal{E}_0 &= I_0 Z_0\end{aligned}$$

where

$$Z_0 = \sqrt{R^2 + (\omega L - 1/\omega C)^2}, \text{ and } \phi_{RLC} = \tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$$

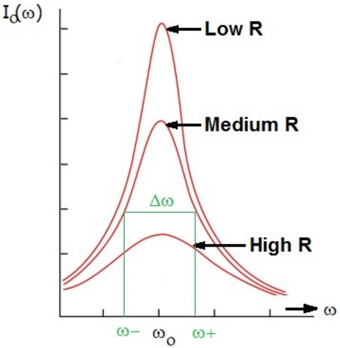
- Let $\mathcal{E}_0$ is fixed, we observe that $Z_0 = Z_0(\omega)$ and $I_0 = I_0(\omega)$

$$I_0(\omega) = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \rightarrow I_{0,max}(\omega_0) = \frac{\mathcal{E}_0}{R}$$

- We observe that there will be a maximum power deposit on R, with the mean power is

$$\bar{P}_{R,max} = \frac{1}{2} I_{0,max}^2 R = \frac{\mathcal{E}_0^2}{2R}$$

- This is a *resonance* condition, the resonance peak of the current $I_0(\omega)$ is



- The *quality* of the peak is determined from the *full width half-maximum peak* $\Delta\omega = \omega_+ - \omega_-$, which is

$$\rightarrow \frac{\mathcal{E}_0^2}{4R} = \frac{1}{2}\frac{\mathcal{E}_0^2 R}{R^2 + (\omega L - 1/\omega C)^2}$$

$$2 = 1 + \frac{(\omega L - 1/\omega C)^2}{R^2} \rightarrow \left(\frac{L}{R\omega}\right)^2 (\omega^2 - \omega_0^2)^2 = 1$$

$$\rightarrow \omega_\pm^2 - \omega_0^2 = \pm\frac{R\omega_\pm}{L} \tag{19}$$

Solving for $\omega_\pm$, we get

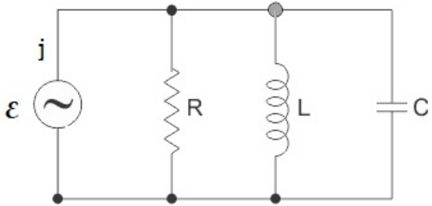
$$\omega_\pm = \pm\frac{R}{2L} + \sqrt{(R/4L)^2 + \omega_0^2} \tag{20}$$

$$\rightarrow \Delta\omega = \omega_+ - \omega_- = \frac{R}{L} \tag{21}$$

- The quality Q factor is defined to be

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R}, \;\; R_{low} \rightarrow high\;\; Q, R_{high} \rightarrow low\;\; Q$$

- *Exercise:* Resonance of parallel RLC circuit (ω_0, Q)



Electromagnetic waves

- Electromagnetic equations

$$\begin{aligned}
\text{Gauss's law} \quad & \oint \vec{E} \cdot \vec{A} = \frac{q}{\epsilon_0}, \ \oint \vec{B} \cdot d\vec{A} = 0 && (22) \\
\text{Ampere's law} \quad & \oint \vec{B} \cdot d\vec{l} = \mu_0 I && (23) \\
\text{Faraday's law} \quad & \mathcal{E} = \oint_{\partial A} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \\
& = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A} && (24)
\end{aligned}$$

Electromagnetic waves

- Electromagnetic equations

$$\text{Gauss's law} \quad \oint \vec{E} \cdot \vec{A} = \frac{q}{\epsilon_0}, \ \oint \vec{B} \cdot d\vec{A} = 0 \tag{22}$$

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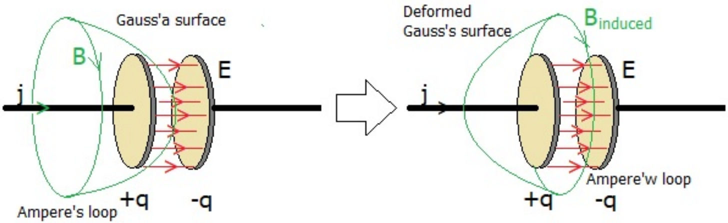
$$\text{Faraday's law} \quad \mathcal{E} = \oint_{\partial A} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A} \tag{24}$$

- Ampere-Maxwell's law



- Ampere-Maxwell's law (cont.)

$$\vec{B}_{induced} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \frac{d}{dt} \int_A \vec{E} \cdot d\vec{A} \qquad (25)$$

- Approved electromagnetic equations

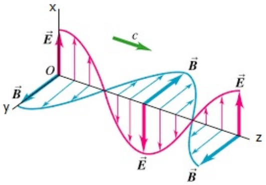
$$\text{Gauss's law} \qquad \oint \vec{E} \cdot \vec{A} = \frac{q}{\epsilon_0}, \ \oint \vec{B} \cdot d\vec{A} = 0 \qquad (26)$$

$$\text{Faraday's law} \qquad \oint_{\partial A} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A} \qquad (27)$$

$$\text{Ampere--Maxwell's law} \qquad \oint_{\partial A} \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \epsilon_0 \frac{d}{dt} \int_A \vec{E} \cdot d\vec{A} \qquad (28)$$

This set of equations is called *Maxwell's equation* in integral form.

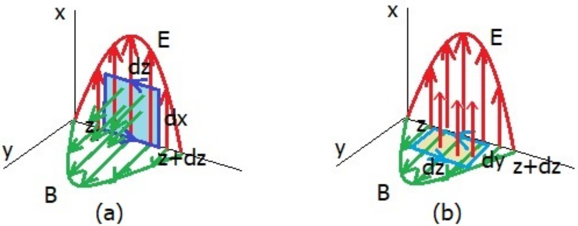
Plane electromagnetic wave



The wave functions are

$$\vec{E}(z,t) = \hat{x}E_0\cos(kz-\omega t) \quad (29)$$
$$\vec{B}(z,t) = \hat{y}B_0\cos(kz-\omega t) \quad (30)$$

To derive its wave equation, let us apply with Faraday's and Ampere-Maxwell's laws



We have

$$(a): \quad [E(z+dz,t)-E(z,t)]dx = -\frac{dB(z,t)}{dt}dxdz$$
$$\rightarrow \frac{dE(z,t)}{dz} = -\frac{dB(z,t)}{dt} \quad (31)$$
$$(b): \quad -[B(z+dz,t)-B(z,t)]dy = \mu_0\epsilon_0\frac{dE(z,t)}{dt}dydz$$
$$\rightarrow \frac{dB(z,t)}{dz} = -\mu_0\epsilon_0\frac{dE(z,t)}{dt} \quad (32)$$

So that

$$\frac{d^2E(z,t)}{dz^2} = -\frac{d}{dz}\frac{dB(z,t)}{dt} = -\frac{d}{dt}\frac{dB(z,t)}{dz} = \mu_0\epsilon_0\frac{d^2E(z,t)}{dt^2}$$
$$\rightarrow \frac{d^2E(z,t)}{dz^2} - \mu_0\epsilon_0\frac{d^2E(z,t)}{dt^2} = 0 \quad (33)$$

We derive wave equation for electric field, with wave velocity

$$c_0^2 = \frac{1}{\mu_0\epsilon_0} \rightarrow c_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8 m/s$$

Similar wave equation can be derived for magnetic field $B(z,t)$.

From (29,30), apply to the derived equations (31,32)

$$-kE_0 \sin(kz - \omega t) = -\omega B_0 \sin(kz - \omega t) \tag{34}$$

$$-kB_0 \sin(kz - \omega t) = -\frac{1}{c_0^2}\omega E_0 \sin(kz - \omega t) \tag{35}$$

$$\rightarrow c_0^2 \frac{B_0}{E_0} = \frac{E_0}{B_0} \rightarrow E_0 = c_0 B_0 \tag{36}$$

The amplitude of electric field is much larger than the amplitude of magnetic field inside the electromagnetic waves

Electromagnetic energy-momentum

Energy of electric and magnetic fields in a volume V is

$$U = \frac{1}{2}\int (\frac{1}{\mu_0}B^2 + \epsilon_0 E^2)dV = \epsilon_0 \int E^2 dV = \int u dV$$

$$\rightarrow u = \epsilon_0 E^2,\ S = uc = \epsilon_0 c E^2 = \frac{1}{\mu_0}EB \tag{37}$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0},\ \text{its average}: \langle \vec{S}\rangle = \frac{\vec{E}\times\vec{B}}{2\mu_0} \tag{38}$$

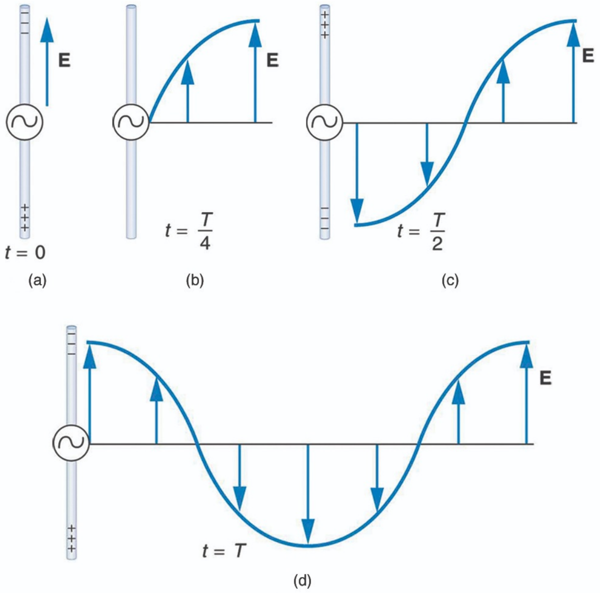
This is called Poynting vector, in unit of $[J/s\cdot m^2]$.
Radiation momentum and pressure are determined to be

$$p = \frac{U}{c} \rightarrow P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{Ac}\frac{dU}{dt} = \frac{S}{c} \tag{39}$$

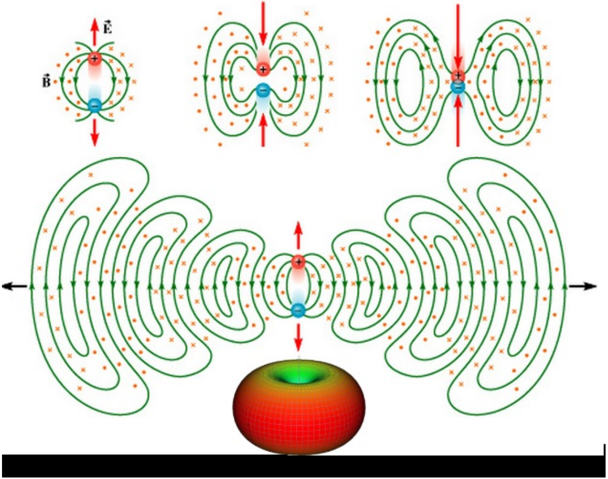
Momentum transfer for total absorption/reflection, will result to radiations pressure

$$dp_{absorp} = \frac{dU}{c} \rightarrow P = \frac{S}{c}, dp_{reflect} = \frac{2dU}{c} \rightarrow P = \frac{2S}{c} \tag{40}$$

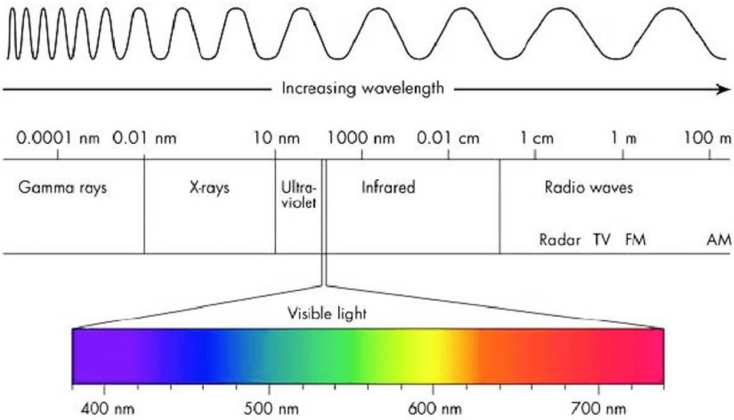
- Generation of EM waves - dipole antenna



- Dipole field in EM waves

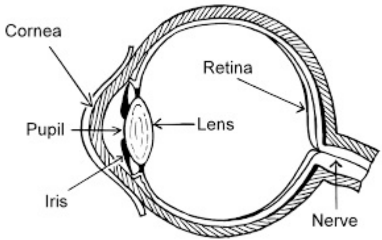


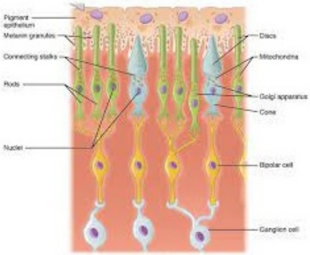
- Electromagnetic spectrum



Visible light

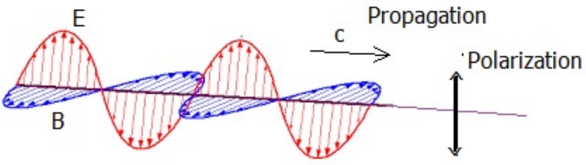
- Wavelength range of $400nm - 70nm$
- Human eyes



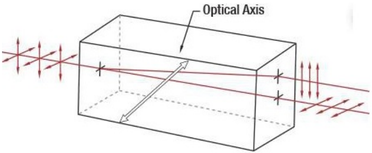


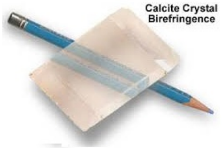
Rods	Cones
Coarse Detail	Fine Detail
Black and White	Colour
120 million cells	6 – 7 million cells
More sensitive to low light	Less sensitive to low light
Used for peripherial and night vision	Used for central and detailed vision

- Polarization - is defined in direction of electric field in EM waves

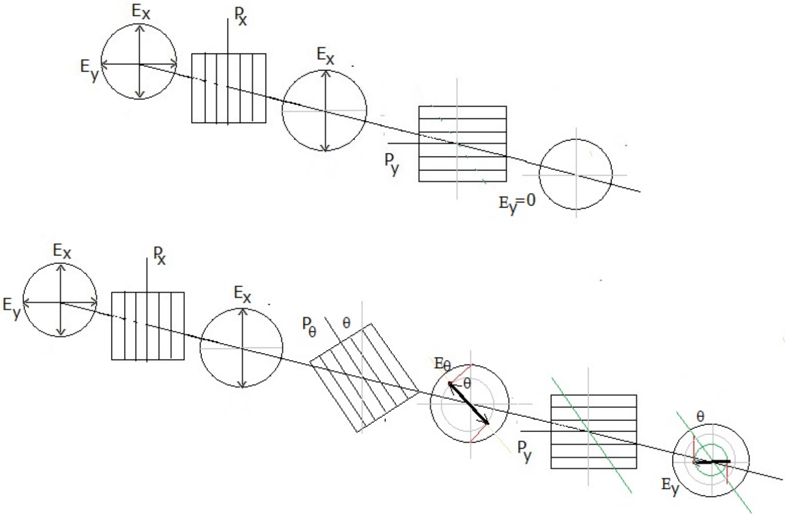


- Non-polarized light is light with polarization in all directions
- Production of polarized light
  - reflection at Brewster's angle
  - passing through birefringence crystal
  - passing through polaroid film

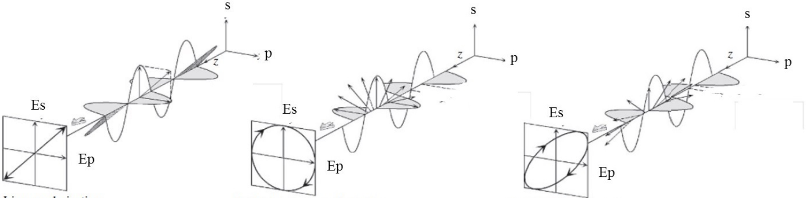




- Intrinsic polarization state of light



- Polarization states
  - linear (x,y) polarization
  - right/left circular polarization
  - elliptical polarization



(a) Linear polarization (b) Right-circular polarization (c) Elliptical polarization

- Polaroid sun glasses