Lecture 1 Geometrical Optics

SCPY152, Second Semester 2021-22

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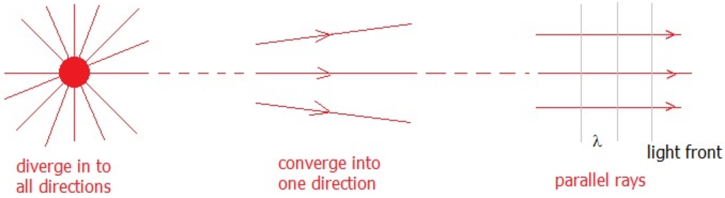
Topics

- Light rays
- Law of light reflection
- Law of light refraction
- Image forming
- Mirror images
- Refractive images
- Matrix method
- Lens
- Composite systems of lens and mirrors
- Optical instruments

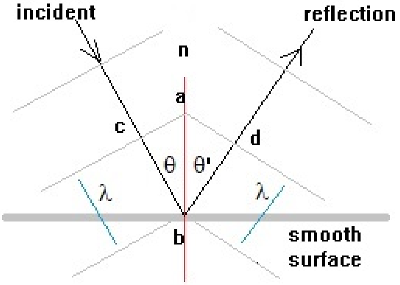
Light Rays

Let us determine rays from light source, at large distance the rays seem to be parallel and a few of them can be used to determine reflection and refraction laws of light



Law of Light Reflection

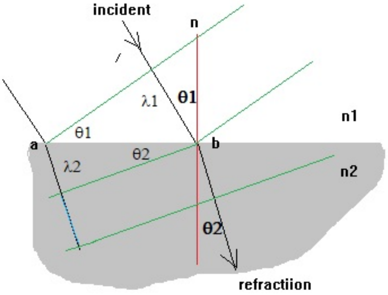
Let us determine light ray reflects from smooth surface, the reflection is characterize with incident angle θ and reflective angle θ', measured from normal direction $\hat{n}$



$$\Delta abc = \Delta abd \rightarrow \theta = \theta' \quad \text{"The law of light reflection"} \qquad (1)$$

Law of Light Refraction

Let us determine light ray refracts smooth surface between two media characterized with refractive index n_1 and n_2, where $n_a = c_a/c_0$ and $\lambda_a = \lambda_0/n_a$ $(a = 1, 2)$, with incident angle θ_1 and refractive angle θ_2

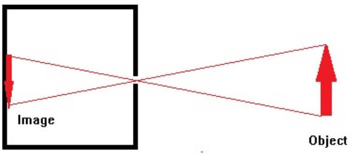


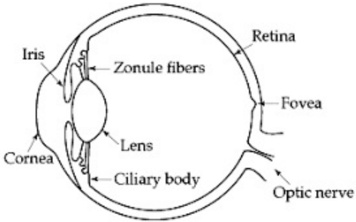
$$\tan\theta_1 = \frac{\lambda_1}{ab},\ \tan\theta_2 = \frac{\lambda_2}{ab} \to \frac{1}{ab} = \frac{\tan\theta_1}{\lambda_1} = \frac{\tan\theta_2}{\lambda_2}$$

$$\tan\theta \xrightarrow{\theta\to 0} \sin\theta \mapsto n_1 \sin\theta_1 = n_2 \sin\theta_2 \ \text{"Snell's law"} \tag{2}$$

Image Forming

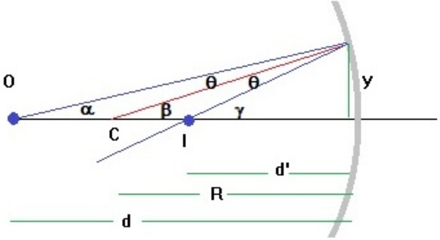
How image is form? Look at the image of pinhole camera, we observe that light from any point of the object will form point of the image.
Human eyes work light a pinhole camera, we have two eyes to see three dimensional object





Mirror Images

Let us determine image of the point object forming from light reflection from curved mirror (as a part of mirror sphere of radius R, i.e., $(+R)$ for concave mirror and $(-R)$ for convex mirror)



$$\alpha + \theta = \beta, \beta + \theta = \gamma \mapsto \alpha + \gamma = 2\beta$$

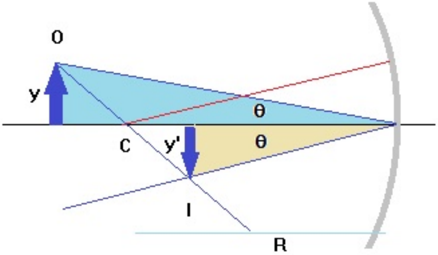
$$\alpha \simeq \tan\alpha = \frac{y}{d}, \gamma \simeq \tan\gamma = \frac{y}{d'}, \beta \simeq \tan\beta = \frac{y}{R}$$

$$\mapsto \frac{1}{d} + \frac{1}{d'} = \frac{2}{R} \equiv \frac{1}{f}, \; f = \frac{R}{2} \qquad (3)$$

Note that d is always positive, and $(+d')$ for real front image and $(-d')$ for virtual rear image
Image magnification, the two color shaded angles are equivalent, so that the image magnification is

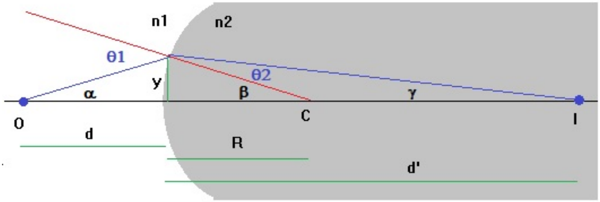
$$m = \frac{y'}{y} = \frac{-d'}{d} \tag{4}$$

Note that $(-m)$ for real downward image, and $(+m)$ for virtual upward image



Refractive Images

Let us determine image of point object from refractive light at smooth curved surface of radius R, i.e., ($+R$ for convex surface and ($-R$) for concave surface, between two media with refractive indices of n_1, n_2, using Snell's law of the form $n_1\theta_1 \simeq n_2\theta_2$

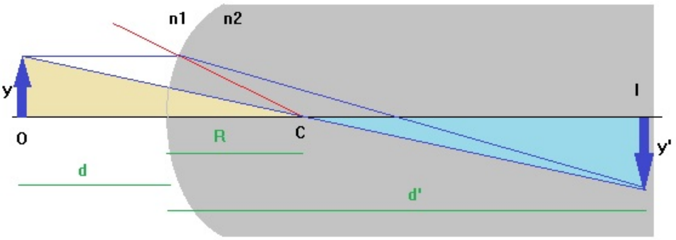


$$\theta_1 = \alpha + \beta,\ \theta_2 = \beta - \gamma \mapsto n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

$$\alpha \simeq \tan\alpha = \frac{y}{d}, \beta \simeq \tan\beta = \frac{y}{R}, \gamma \simeq \tan\gamma = \frac{y}{d'}$$

$$\mapsto \frac{n_1}{d} + \frac{n_2}{d'} = (n_2 - n_1)\frac{1}{R} \qquad (5)$$

Image magnification, the two color shaded triangles are equivalent so that



$$m = \frac{y'}{y} = -\frac{d' - R}{d + R} = -\frac{d'/n_2}{d/n_1} \tag{6}$$

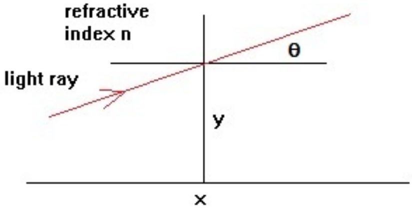
Exercise: Derive equation (6) from refractive image equation (5).

Matrix Method

To be more practical for complicate system, we can develop matrix method for image forming, base on the idea that

$$[image\ matrix] = [image\ forming\ matrix] \times\ [object\ matrix]$$

Object/Image matrix at any point x can be constructed from light ray as a 2x1 matrix as



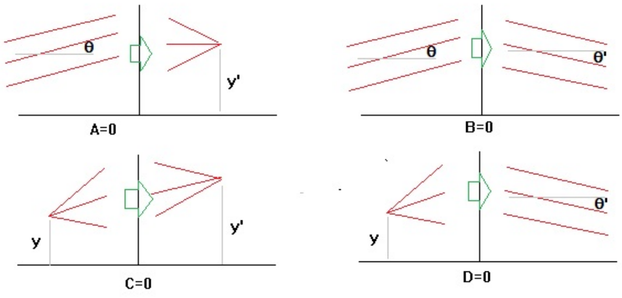
$$[object/image] = \begin{pmatrix} \theta \\ y \end{pmatrix} \quad (7)$$

Image forming must appear in matrix form as

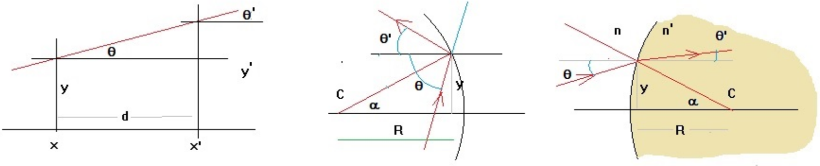
$$\begin{pmatrix} \theta' \\ y' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \theta \\ y \end{pmatrix}$$

with basic properties of image forming matrix

- $A = 0$, back focal point
- $B = 0$, parallel rays transparent
- $C = 0$, image forming
- $D = 0$, front focal point



There are translation, reflection, and refraction matrices in forming image of light rays



- translation

$$\theta' = \theta \simeq \frac{y'-y}{d} \mapsto trans[d] = \begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix}$$

- reflection

$$y' = y, \theta - \alpha = \theta' + \alpha, \theta' = \theta - 2\alpha \simeq \theta - \frac{2y}{R}$$

$$\mapsto reflect[R] = \begin{pmatrix} 1 & -2/R \\ 0 & 1 \end{pmatrix}$$

- refraction

$$y' = y, n(\theta + \alpha) = n'(\theta' + \alpha)$$

$$n'\theta' = n\theta + (n - n')\alpha \simeq n\theta = (n - n')\frac{y}{R}$$

$$\mapsto refract[R, n, n'] = \begin{pmatrix} n/n' & \frac{n-n'}{n'R} \\ 0 & 1 \end{pmatrix}$$

- reflective image $\mapsto trans[d2] \cdot [reflect[r] \cdot trans[d1]/C = 0$

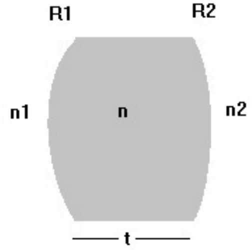
$$\begin{pmatrix} 1 - \frac{2\,d1}{r} & -\frac{2}{r} \\ d2 + d1\left(1 - \frac{2\,d2}{r}\right) & 1 - \frac{2\,d2}{r} \end{pmatrix}$$

- refractive image
  $\mapsto trans[d2] \cdot refrac[r, n1, n2] \cdot trans[d1]/C = 0$

$$\begin{pmatrix} \frac{n1}{n2} + \frac{d1\,(n1-n2)}{n2\,r} & \frac{n1-n2}{n2\,r} \\ \frac{d2\,n1}{n2} + d1\left(1 + \frac{d2\,(n1-n2)}{n2\,r}\right) & 1 + \frac{d2\,(n1-n2)}{n2\,r} \end{pmatrix}$$

Lens

Let us determine optical property of *thick lens*, i.e., consists of two curved refractive surfaces of radius R_1, R_2 of a medium with refractive index n and thickness t



The thick lens matrix can be constructed as

$$thick[r_1, r_2, t, n, n_1, n_2] = refrac[r_2, n, n_2] \cdot trans[t] \cdot refrac[r_1, n_1, n]$$

Thick lens matrix

```
In[7]:= thick[r1, r2, t, n, n1, n2] // MatrixForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} \frac{n1\left(\frac{n}{n2}+\frac{(n-n2)\,t}{n2\,r2}\right)}{n} & \frac{n-n2}{n2\,r2}+\frac{(-n+n1)\left(\frac{n}{n2}+\frac{(n-n2)\,t}{n2\,r2}\right)}{n\,r1} \\ \frac{n1\,t}{n} & 1+\frac{(-n+n1)\,t}{n\,r1} \end{pmatrix}$$

For example

```
In[8]:= thick[40, -40, 0.5, 1.5, 1, 1] //
  MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0.995833 & -0.0249479 \\ 0.333333 & 0.995833 \end{pmatrix}$$

Thick lens image can be determined from the image matrix

$$thickimage = trans[d_2] \cdot thick[r_1, r_2, t, n, 1, 1] \cdot trans[d_1]/C = 0$$

```
In[10]:= image3 = trans[d2].thick[r1, r2, t, n, 1, 1].trans[d1] // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} \frac{n+\frac{(-1+n)\,t}{r2}}{n} + d1\left(\frac{-1+n}{r2} + \frac{(1-n)\left(n+\frac{(-1+n)\,t}{r2}\right)}{n\,r1}\right) & \frac{-1+n}{r2} + \frac{(1-n)\left(n+\frac{(-1+n)\,t}{r2}\right)}{n\,r1} \\ \frac{t}{n} + \frac{d2\left(n+\frac{(-1+n)\,t}{r2}\right)}{n} + d1\left(1 + \frac{(1-n)\,t}{n\,r1} + d2\left(\frac{-1+n}{r2} + \frac{(1-n)\left(n+\frac{(-1+n)\,t}{r2}\right)}{n\,r1}\right)\right) & 1 + \frac{(1-n)\,t}{n\,r1} + d2\left(\frac{-1+n}{r2} + \frac{(1-n)\left(n+\frac{(-1+n)\,t}{r2}\right)}{n\,r1}\right) \end{pmatrix}$$

For example (all distances are in cm.)

```
In[29]:= image3 = trans [d2].thick [40, -40, 0.5, 1.5, 1, 1].trans [10] // MatrixForm
         c = image3 [[1, 2, 1]]
         Solve [c == 0, d2]

Out[29]//MatrixForm=
  (                          0.746354                                      -0.0249479            )
  ( 0.333333 + 10 (0.995833 - 0.0249479 d2) + 0.995833 d2   0.995833 - 0.0249479 d2 )

Out[30]= 0.333333 + 10 (0.995833 - 0.0249479 d2) + 0.995833 d2

Out[31]= {{d2 → -13.7893}}
```

For example (all distances are in cm.)

```
In[29]:= image3 = trans [d2].thick [40, -40, 0.5, 1.5, 1, 1].trans [10] // MatrixForm
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  (                         0.746354                                   -0.0249479           )
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Out[30]= 0.333333 + 10 (0.995833 - 0.0249479 d2) + 0.995833 d2

Out[31]= {{d2 → -13.7893}}
```

Thin lens is reduced from thick lens in the limit $t = 0$, in air we have

$$thin[r1, r2, n] = thick[r1, r2, 0, n, 1, 1]$$

```
In[32]:= thin[r1, r2, n] =
          thick[r1, r2, 0, n, 1, 1] // MatrixForm
```

Out[32]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{1-n}{r1} + \frac{-1+n}{r2} \\ 0 & 1 \end{pmatrix}$$

We have derived the *lens maker equation* in the form

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \mapsto thinf[f] = \begin{pmatrix} 1 & -1/f \\ 0 & 1 \end{pmatrix}$$

Thin lens image is determined from the image matrix

$$thinimage = trans[d2] \cdot thinf[f] \cdot trans[d1]/C = 0$$

```
In[33]:= thinf[f_] := {{1, -1 / f}, {0, 1}}
         thinimage =
          trans[d2].thinf[f].trans[d1] //
           MatrixForm
```

Out[34]//MatrixForm=

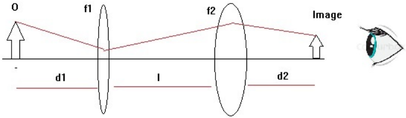
$$\begin{pmatrix} 1 - \frac{d1}{f} & -\frac{1}{f} \\ d2 + d1\left(1 - \frac{d2}{f}\right) & 1 - \frac{d2}{f} \end{pmatrix}$$

$$C = 0 \mapsto \frac{1}{d1} + \frac{1}{d2} = \frac{1}{f}$$

Composite Systems

The image matrix of a 2 lens system is

$$image = trans[d2] \cdot thinf[f2] \cdot trans[l] \cdot \cdot thinf[f1] \cdot trans[d1]$$



For example

```
In[47]:= image4 = trans[d3].thinf[10].trans[1.5].thinf[-30].trans[5] // MatrixForm
         c2 = image4[[1, 2, 1]]
         Solve[c2 == 0, d3]
```

Out[47]//MatrixForm=

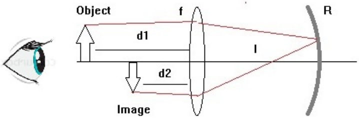
$$\begin{pmatrix} 0.491667 & -0.0716667 \\ 1.5\left(1-\frac{d3}{10}\right)+d3+5\left(1-\frac{d3}{10}+\frac{1}{30}\left(1.5\left(1-\frac{d3}{10}\right)+d3\right)\right) & 1-\frac{d3}{10}+\frac{1}{30}\left(1.5\left(1-\frac{d3}{10}\right)+d3\right) \end{pmatrix}$$

Out[48]= $1.5\left(1-\frac{d3}{10}\right)+d3+5\left(1-\frac{d3}{10}+\frac{1}{30}\left(1.5\left(1-\frac{d3}{10}\right)+d3\right)\right)$

Out[49]= {{d3 → -13.7288}}

The image matrix of lens and mirror system is

$$image = trans[d2]\cdot thinf[f]\cdot trans[l]\cdot reflec[r]\cdot trans[l]\cdot thinf[f]\cdot trans[d1]$$



For example

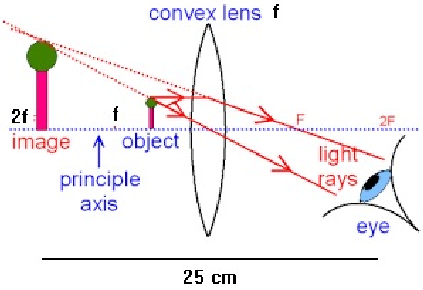
```
In[86]:= image5 = trans [d4].thinf [20].trans [2.5].reflect [20].trans [2.5].thinf [20].trans [5] // MAtrixForm

Out[86]= MAtrixForm[{{-0.289063, -0.164063}, {2.5 (1 - d4/20) + 2.5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) - d4/20) +
      5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) + 1/20 (-2.5 (1 - d4/20) - 2.5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) - d4/20) - d4) - d4/20) + d4,
      1 + 1/10 (-2.5 (1 - d4/20) - d4) + 1/20 (-2.5 (1 - d4/20) - 2.5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) - d4/20) - d4) - d4/20}}]

In[87]:= c3 = image5 [[1, 2, 1]]

Out[87]= 2.5 (1 - d4/20) + 2.5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) - d4/20) +
      5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) + 1/20 (-2.5 (1 - d4/20) - 2.5 (1 + 1/10 (-2.5 (1 - d4/20) - d4) - d4/20) - d4) - d4/20) + d4

In[88]:= Solve [c3 = 0, d4]

Out[88]= {{d4 → 24.3243}}
```

Optical Instruments

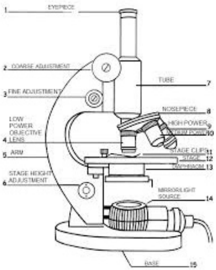
The most comfortable distance at which one can see an object with our eyes is $25cm$

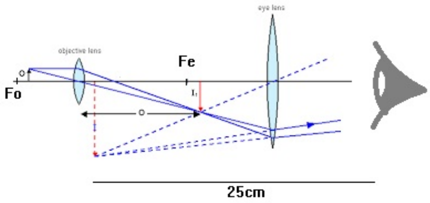
All optical instruments are designed or invented to improve our seeing the objects that are hard to have direct seeing, i.e, too small or too far

a) Magnifier

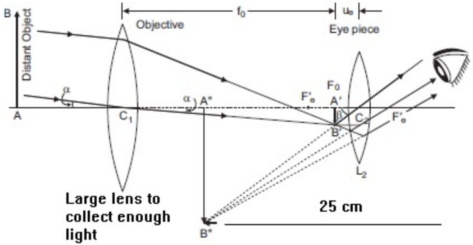


b) Microscope





c) Telescope



Parabolic mirror and lens/image aberration