Lecture 2 Wave Optics, Interference and Diffraction

SCPY152, Second Semester 2021-22

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Topics

- Light wave and its phasor diagram
- Interference principle
- Double slit interference
- Triple slit interference
- Thin film interference
- Interferometers
- Huygens principle and diffraction phenomena
- Single slit diffraction
- Double slit diffraction
- X-rays diffraction and Bragg's law

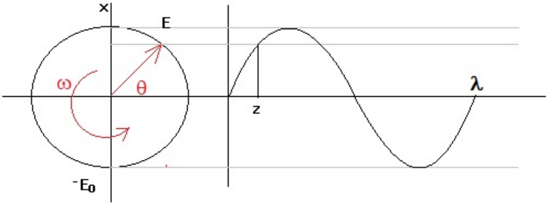
Light Wave and Its Phasor diagram

Light is a part of electromagnetic wave and is characterized by electric field $\vec{E}(z,t)$ in the wave. For monochromatic wave (λ) the electric field is a sinusoidal wave

$$\vec{E}(z,t) = \hat{x} E_0 \sin(kz - \omega t)$$

This can be though of x-projection of rotating vector, a phasor diagram,

$$\vec{E}(z,t) = \hat{\theta} E_0 e^{i(kz-\omega t)}$$

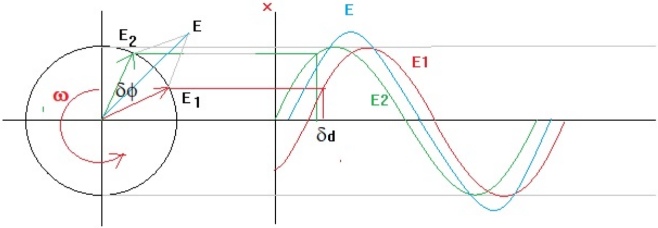


Interference Principle

Interference of two monochromatic waves can be determined from a vector sum of two phasors $\vec{E} = \vec{E}_1 + \vec{E}_2$, then we can capture sum of amplitudes and phases which relate the phase different $\delta\phi$ to the path different δ in the form

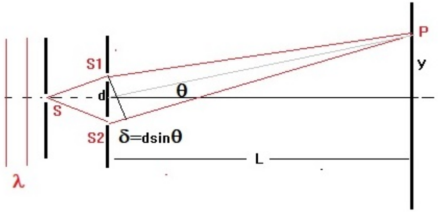
$$\frac{\delta\phi}{2\pi} = \frac{\delta}{\lambda}, \; |E| = 2E_0 |\cos \frac{\delta\phi}{2}|$$

When $\delta\phi = 2n\pi$ the interference is said to be *constructive*, while when $\delta\phi = (2n+1)\pi$ the interference is said to be *destructive*. This is called *interference principle*.



Double slit (Young) interference

Double slit is used to produce to *coherence light sources* (λ), with a separation d of the two slits, the interference is determined on a screen at a distance L behind the slits

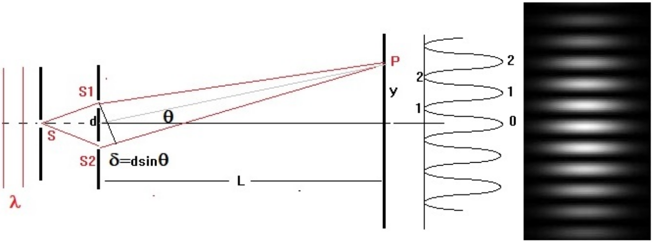


From interference principle we have

$$\text{Constrictive}: \quad \frac{2n\pi}{2\pi} = \frac{d\sin\theta}{\lambda} \mapsto d\sin\theta = n\lambda, \quad n = 0, 1, 2, \ldots \tag{1}$$

$$\text{Destructive}: \quad \frac{(2n-1)\pi}{2\pi} = \frac{d\sin\theta}{\lambda} \mapsto d\sin\theta = (n-1/2)\lambda, \quad n = 1, 2, 3, \ldots \tag{2}$$

Interference pattern of the double slit



Relative light intensity inside interference frings at y on the screen

$$\theta \sim \sin\theta \sim \tan\theta = \frac{y}{L} = \frac{\delta\phi(y)}{2\pi}\frac{\lambda}{d}$$

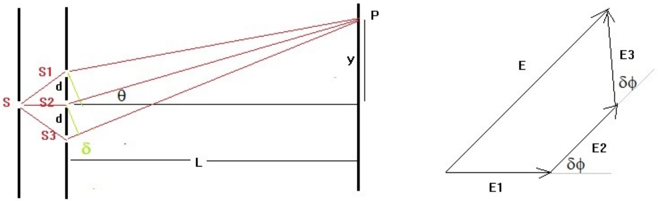
$$\frac{I(y)}{I_0} = \frac{|E(y)|^2}{4|E_0|^2} = \cos^2\left(\frac{\delta\phi(y)}{2}\right) = \cos^2\left(2\pi\frac{yd}{\lambda L}\right)$$

The lateral intensity are reduced by inverse distance square law, i.e. with a factor of $1/(1+y^2/L^2)$

Triple Slit Interference

From the three coherence light sources we will have three light phasors sum on the screen

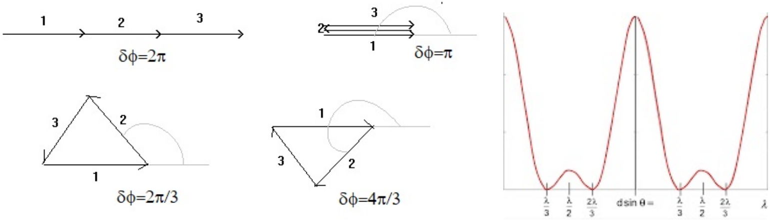
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



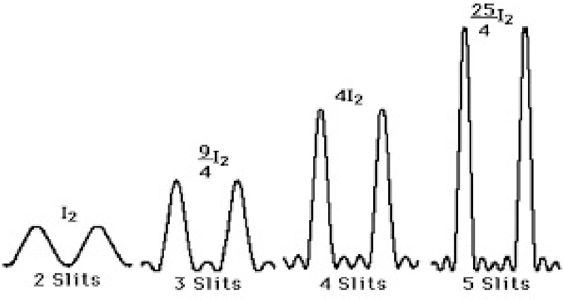
We observe that

$$\text{Big constructive :} \quad \delta\phi = 2n\pi, \;\; n = 0, 1, 2, \ldots$$
$$d \sin\theta = n\lambda \qquad (3)$$
$$\text{Small constructive :} \quad \delta\phi = (2n-1)\pi, \;\; n = 1, 2, \ldots$$
$$d \sin\theta = (n - 1/2)\lambda \qquad (4)$$

$$
\begin{aligned}
\text{First destructive}: \quad & \delta\phi = \frac{2n\pi}{3}, \;\; n = 1, 2, ... \\
& d \sin\theta = \frac{n\lambda}{3} \qquad (5) \\
\text{Second destructive}: \quad & \delta\phi = \frac{4n\phi}{3}, \;\; n = 1, 2, ... \\
& d \sin\theta = \frac{2n\lambda}{3} \qquad (6)
\end{aligned}
$$

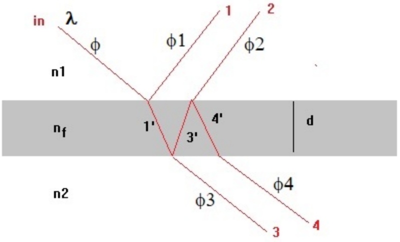


Multiple slits interference



Thin Films Interference

Uniform thin film interference



For a simple case $n_1, n_2 < n_f \mapsto \phi_1 = \phi + \pi$, $\phi_2 = \phi_3 = \phi_4 = \phi$ and $\delta\phi = \phi_1 + \phi_2 = \phi_3 + \phi_4$

$$\text{Front destructive :} \quad \delta\phi = 2n\pi \mapsto 2d = n\frac{\lambda}{n_f} \tag{7}$$

$$\text{Front constructive :} \quad \delta\phi = (2n+1)\pi \mapsto 2d = (n+1/2)\frac{\lambda}{n_f} \tag{8}$$

$$\text{Rare destructive :} \quad \delta\phi = (2n+1)\pi \mapsto 2d = (n+1/2)\frac{\lambda}{n_f} \tag{9}$$

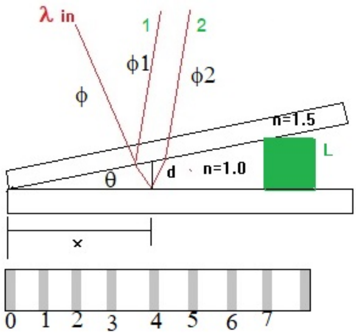
Example: Minimum thickness of reflective traffic film $n_f = 1.3$ at $\lambda = 500nm$

$$d_{min} = \frac{\lambda}{n_f} = \frac{500}{1.3} = 385nm = 0.385\mu m$$

Example: Minimum thickness of destructive screening film $(n_f = 1.3)$ at $\lambda = 650nm$

$$d_{min}\frac{\lambda}{n_f} = \frac{650}{1.3} = 500nm = 0.5\mu m$$

Non-uniform film interference-wedge film



$$\phi_1 = \phi, \phi_2 = \phi + \pi$$

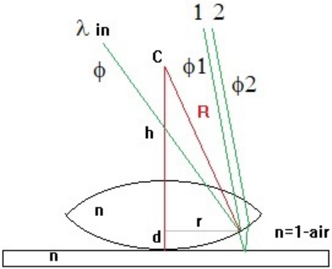
Destructive interference (dark frings)

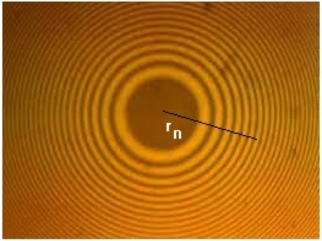
$$2d = n\lambda, \ \ n = 0, 1, 2...$$

Example What is the thickness L measured at 7^{th} frings of red light $\lambda = 640nm$?

$$2L = 7\lambda \mapsto L = \frac{7}{2}(640nm) = 2.24\mu m$$

Non-uniform film interference - Newton's rings





Destructive interference, with $r^2 = R^2 - (R-d)^2 \simeq 2Rd,\ d \ll R,$

$$\phi_1 = \phi,\ \phi_2 = \phi + \pi \mapsto 2d_n \simeq \frac{r_n^2}{R} = n\lambda,\ n = 1, 2, ...$$

Example What is R when $r_1 = 0.8mm$ is measured with $\lambda = 580nm$?

$$R = \frac{r_1^2}{\lambda} = \frac{6.4 \times 10^7 m^2}{5.8 \times 10^7 m} = 110.3cm$$

Interferometers

Michelson interferometer, destructive interference

$$2L = (m + 1/2)\lambda \mapsto 2\delta L = \delta m \lambda$$

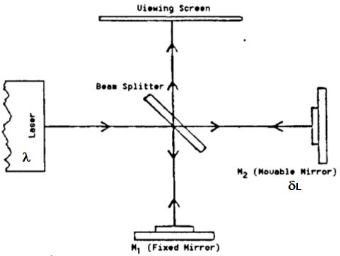
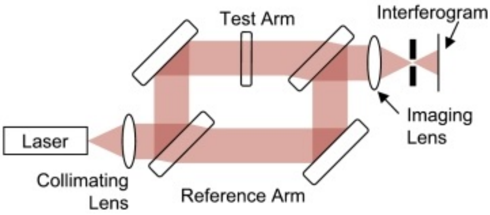


Figure 1: Michelson Interferometer

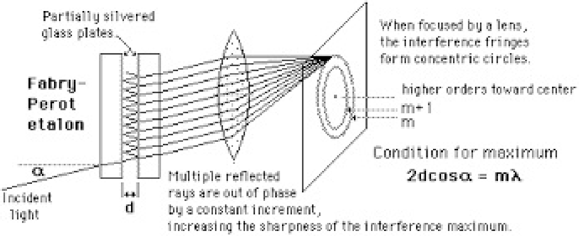
Note: the actual interference pattern will most probably be more irregular and show less fringes.

Figure 2: Interference Pattern
Note: the actual interference pattern will most probably be more irregular and show less fringes.

Mach-Zehnder Interferometer

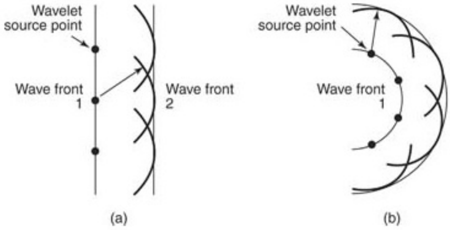


Fabry-Perot interferometer

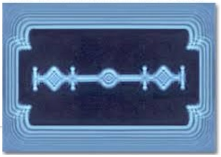


Huygens Principle

The principle say that "*all points at the wave front will become new source of wave*".

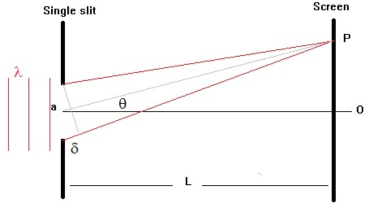


Light diffraction phenomena

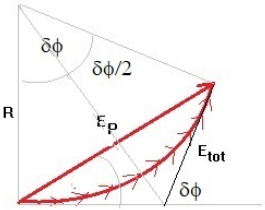


Single Slit Diffraction

Diffraction of monochromatic light λ from a slit of width a on screen at a distance L



Light phasor of superposition at P



$$E_{tot} = R\delta\phi, \ \sin\frac{\delta\phi}{2} = \frac{E_P/2}{R}$$

$$E_P = E_{tot}\frac{\sin(\delta\phi/2)}{\delta\phi/2} \qquad (10)$$

Destructive superposition

$$\delta\phi = 2n\pi, \ \delta = a\sin\theta \mapsto a\sin\theta = n\lambda, \ n = 1, 2, ... \tag{11}$$

$$\sin\theta \simeq \tan\theta = \frac{y}{L} \mapsto y_n = n\frac{L\lambda}{a} \tag{12}$$

Constructive superposition

$$\delta\phi = (2n+1)\pi \mapsto a\sin\theta = (n+1/2)\lambda, \ n = 1, 2, ... \tag{13}$$

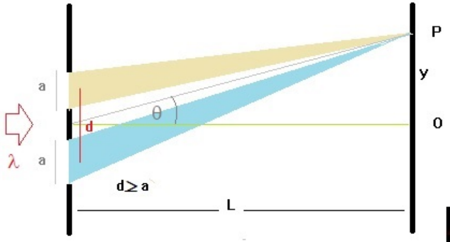
Relative light intensity of the constructive (bright) diffraction frings

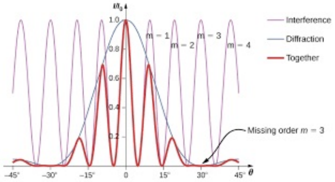
$$\frac{I_n}{I_0} = \frac{\sin^2((n+1/2)\pi)}{(n+1/2)^2\pi^2}, \ n = 1, 2, ... \tag{14}$$

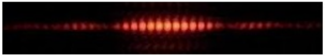
Question: What is the width on screen of the central bright fring?

Double Slit Diffraction

Diffraction from two slit of width *a* and separation *d* on screen at a distance *L*



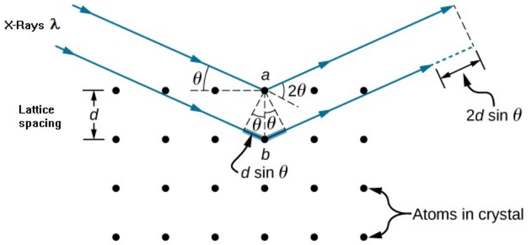




Question: What is the number of interference bright frings inside the central bright fring of diffraction?

X-Rays Diffraction

X-rays diffraction from crystalline structure



Constructive diffraction, Bragg's law,

$$2d \sin\theta = n\lambda \mapsto 2\theta \simeq \frac{n\lambda}{d}$$