Lecture 3 Particle-Wave Duality

SCPY152, Second Semester 2021-22

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Topics

- Particle and wave
- Wave particle
- Photoelectric effect
- Compton scattering

Particle and Wave

Nature is observed with particle and wave phenomenon

- Particle has path of motion described with position $x(t)$ and momentum $p(t) = m\dot{x}$. Its dynamics is described with Newton's equation of motion

$$\frac{dp}{dt} = F_x = m\ddot{x}$$

  Fundamentally particle carry total energy of $E = \frac{p^2}{2m} + U(x)$, when U is potential energy of the force $F_x = -dU/dx$ (According to Newton)

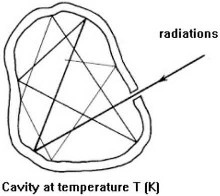
- Wave is determined from its wave function $f(x \mp vt)$ satisfy the wave equation

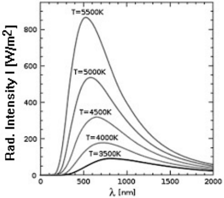
$$f'' - \frac{1}{v^2}\ddot{f} = 0$$

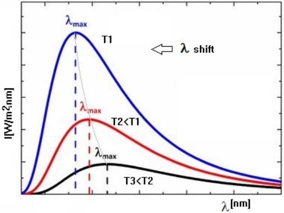
  The energy current density is carried along with the wave. Four characteristic properties of the wave, not for particle, are i) reflection, ii) refraction, iii) interference and iv) diffraction (according to Huygens)

Wave-Particle

We always observe radiations from hot body, it is explained by cavity absorption/emission of radiations







Wien's displacement law

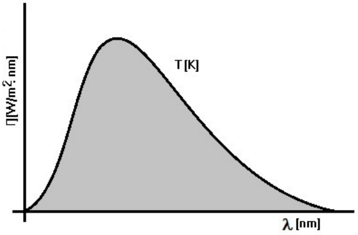
$$\lambda_{max} T = 2.898 \times 10^{-3} \; [m \cdot K]$$

$$T = 5800K \mapsto \lambda_{max} = 500nm$$

Stefan-Boltzmann's law

$$P = e\sigma A(T_S^4 - T_E^4)[W]$$

efficiency $e \in [0, 1]$, surface area-A, Stefan constant $\sigma = 5.67 \times 10^{-8}\ [W/m^2 \cdot K^4]$.



Example: The λ_{max} of our Sun light is $500nm$ (green), what is the T_S of our Sun?

$$T_S = \frac{2.898 \times 10^{-3}[m.K]}{500 \times 10^{-9}[m]} \simeq 5800[K]$$

Example: Power radiated from our Sun, with $T_S = 5800\ K$, $T_E = 0\ K$ (empty space) and assume $e = 1$

$$\frac{P}{A} = (1)(5.67 \times 10^{-8}[W/m^2 \cdot K^4)(5800[K])^4 = 6.42 \times 10^7[W/m^2]$$

EM theory of radiations emission from EM cavity:

- In cavity of volume $V = L^3$, the radiations are standing waves with possible wavelengths of

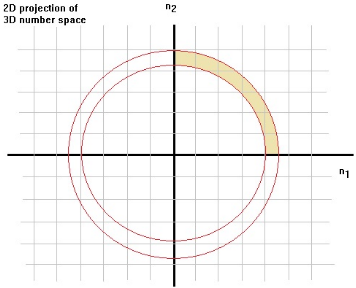
$$\lambda = \frac{2L}{n_a} \mapsto n_a = \frac{2L}{\lambda}, \;\; a = 1,2,3, \;\; n_a = 1,2,...$$

$$n_1^2 + n_2^2 + n_3^2 = \frac{4L^2}{\lambda^2} = \frac{4L\nu}{c} > 0$$

This represents number sphere of radius $2L\nu/\lambda$. The number of mode $n's$ between the frequency from $0 - \nu$ is determined from $1/8$ of this spherical volume (for positive n only) as

$$N(\nu) = (2)\frac{1}{8}\frac{4\pi}{3}\left(\frac{2L\nu}{c}\right)^3 = \frac{4\pi}{3}\frac{L^2\nu^3}{c^3}$$

with a factor 2 of two polarization.



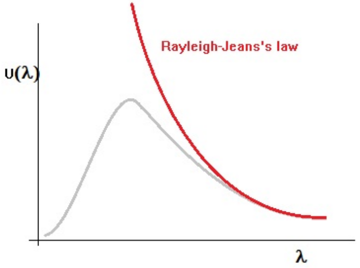
- Let $L = 1$, the number of modes $dN(\nu)$ of frequency between ν to $\nu + d\nu$ will be

$$dN(\nu) = 8\pi \frac{\nu^2 d\nu}{c^3} \mapsto dN(\lambda) = \frac{8\pi}{\lambda^4} d\lambda$$

According to equi-partition theorem, the thermal energy of this radiations will be

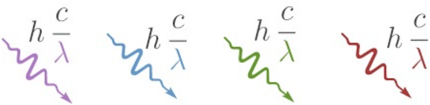
$$\begin{aligned} U(\lambda, T)d\lambda &= k_B T dN(\lambda) \\ &= \frac{8\pi k_B T}{\lambda^4} d\lambda \\ \mapsto U(\lambda, T) &= \frac{8\pi k_B T}{\lambda^4} \end{aligned} \qquad (1)$$

This is called *Rayleigh-Jeans's law*.



Plank's hypothesis of *photon* of EM radiations is *"EM radiations(λ) travel with not continuous energy but lump of energy $\epsilon(\lambda)$ with*

$$\epsilon(\lambda) = \frac{hc}{\lambda} = h\nu \mapsto E_n(\nu) = nh\nu, \;\; n = 0, 1, 2, \ldots$$
$$h = 6.626 \times 10^{-34}[J \cdot s] \text{ Plank's constant}$$



Thermal distribution of photon number will be in the form

$$N(n, T) = e^{-E_n(\nu)/k_B T} = e^{-nh\nu/k_B T}$$

The average total photon energy is then equal to

$$\langle E \rangle = \frac{\sum_n nh\nu e^{-nh\nu/k_B T}}{\sum_n e^{-nh\nu/k_B T}}$$

Calculation

$$x = \frac{h\nu}{k_B T} \mapsto \langle E \rangle = x k_B T \frac{\sum_n n e^{-nx}}{\sum_n e^{-nx}} = -x k_B T \frac{\frac{d}{dx}\sum_n e^{-nx}}{\sum_n e^{-nx}}$$

$$\sum_n e^{-nx} = \frac{1}{1-e^{-x}} \mapsto \frac{d}{dx}\frac{1}{1-e^{-x}} = \frac{-e^{-x}}{(1-e^{-x})^2}$$

$$\mapsto \langle E \rangle = \frac{x k_B T}{e^x - 1} = \frac{h\nu}{e^{h\nu/k_B T} - 1} \qquad (2)$$

Then we have radiations energy

$$U(\lambda, T)d\lambda = \langle E \rangle dN(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1} d\lambda$$

$$\mapsto U(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1}$$

This is called *Plank's distribution law*, it gives exact thermal radiations curve.

Derivation of Wien's displacement law

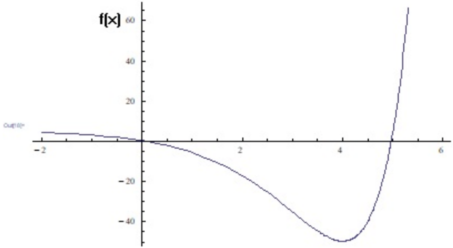
$$\left.\frac{dU(\lambda, T)}{d\lambda}\right|_{\lambda=\lambda_{max}} = 0$$

$$= \frac{8\pi hc}{\lambda^6}\frac{1}{e^{hc/\lambda k_B T}-1}\left(-5+\frac{1}{(e^{hc/\lambda k_B T}-1)}\frac{hc}{\lambda k_B T}e^{hc/\lambda k_B T}\right)$$

$$\alpha = \frac{hc}{k_B} \mapsto 0 = 5 - 5e^{\alpha/\lambda T} + \frac{\alpha}{\lambda T}e^{\alpha/\lambda T}$$

$$x = \frac{\alpha}{\lambda T} \mapsto f(x) = 5 - 5e^x + xe^x$$

Graphical solution of $f(x) = 0$, we observe that its solutions are $x = 0, 5$



At $x = 5(4.97)$, we have

$$\lambda T = \frac{\alpha}{5} = \frac{hc}{5k_B} = \frac{6.626 \times 10^{-34} \cdot 3 \times 10^{8}}{5 \cdot 1.38 \times 10^{-23}}$$
$$= 2.88 \times 10^{-3} [m \cdot K]$$

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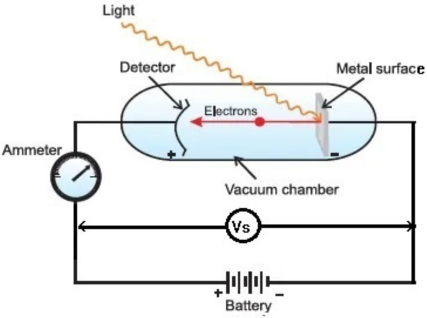
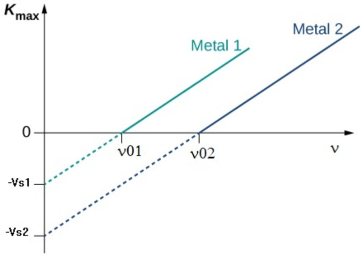
Derivation of Stefan-Boltzmann law

$$U(\lambda, T)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} d\lambda$$
$$\frac{P}{A} = \int_0^\infty d\nu U(\nu, T) = 8\pi \int_0^\infty d\lambda \frac{hc/\lambda^5}{e^{hc/\lambda k_B T} - 1}$$
$$x = \frac{hc}{\lambda k_B T} \mapsto dx = -\frac{hc}{\lambda^2 k_B T} d\lambda$$
$$\frac{P}{A} = 8\pi \frac{(k_B T)^4}{(hc)^3} \int_0^\infty dx \frac{x^3}{e^x - 1} = 8\pi \frac{(k_B T)^4}{(hc)^3} \frac{\pi^4}{15} \equiv \sigma T^4 \quad (3)$$
$$\mapsto \sigma = \frac{8\pi^5 k_B^4}{h^3 c^3} \quad (4)$$

Photoelectric Effect (PE)

PE experiment

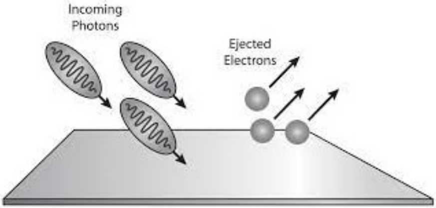




Einstein's explanation by a photon hits an electron on metal surface, ans it is released with kinetic energy K which is observed from stopping voltage V_s as

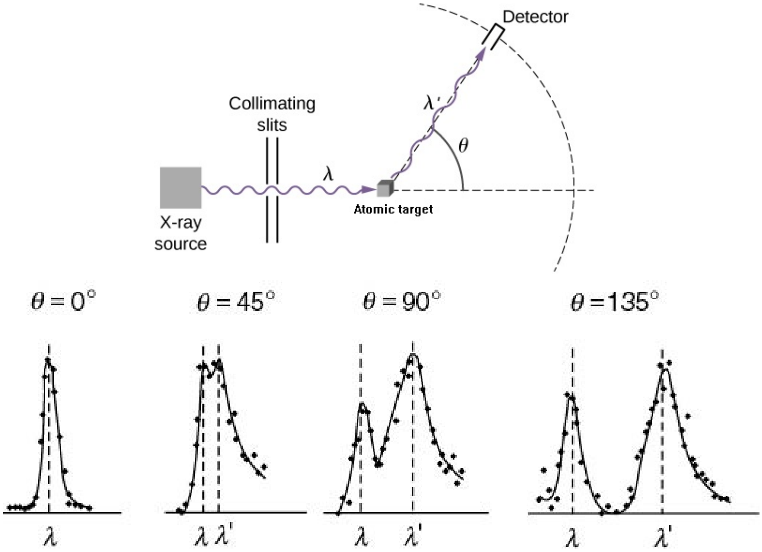
$$K = \frac{1}{2}m_e v^2 = h\nu - \phi \mapsto \frac{1}{2}m_e v_{max}^2 = eV_s$$

The work function ϕ of metal surface adhesion can be determined from ν_0.

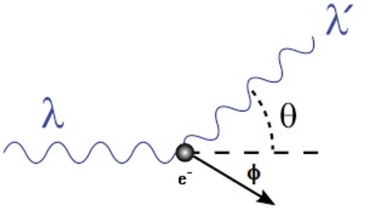


Compton Scattering

Compton scattering experiment



Compton explanation of collision of X-rays photon with atomic electron, from energy-momentum conservation law we have



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\begin{aligned}
\textit{Energy}: &\quad \frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2} m_e v^2 \\
\textit{Momentum}: &\quad \frac{h}{\lambda} = \frac{h}{\lambda'} \cos\theta + m_e v \cos\phi \\
&\quad 0 = \frac{h}{\lambda'} \sin\theta - m_e v \sin\phi \\
\textit{Compton shift}: &\quad \Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \quad (5) \\
\textit{Compton wavelength}(e^-): &\quad \lambda_C = \frac{h}{m_e c} = 2.43 \times 10^{-12} [m] \quad (6)
\end{aligned}
$$

λ_C is characteristic length (size) of an electron