Lecture 9 Hydrogen Atom

SCPY152, Second Semester 2021-22

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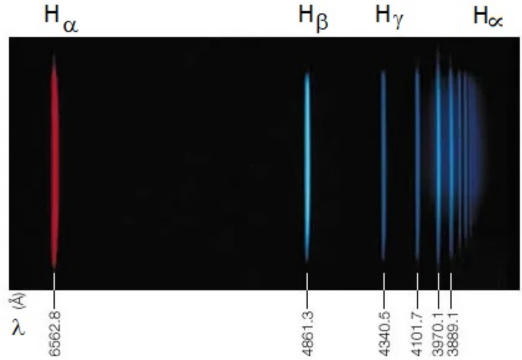
Topics

1. Hydrogen spectra
2. Bohr hypothesis
3. Atomic orbits and energies

Hydrogen spectra

At the early days of quantum physics, there appear observations of hydrogen spectra. The famous one is known as Balmer (visible) series



They are also appear in other series, i.e., Lyman (UV), Paschen (IR), Brackett (FIR) and Pfund (FIR) series

Rydberg formula Johannes Rydberg can immediately give an empirical formula for the spectral series in the form

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \quad R = 1.097 \times 10^7 m^{-1} \qquad (1)$$

$$\begin{aligned} &Layman\ series: && n_f = 1, n_i = 2, 3, ... && (2)\\ &Balmer\ series: && n_f = 2, n_i = 3, 4, ... && (3)\\ &Paschen\ series: && n_f = 3, n_i = 4, 5, ... && (4)\\ &Brackett\ series: && n_f = 4, n_i = 5, 6, ... && (5)\\ &Pfund\ series: && n_f = 5, n_i = 6, 7, ... && (6)\end{aligned}$$

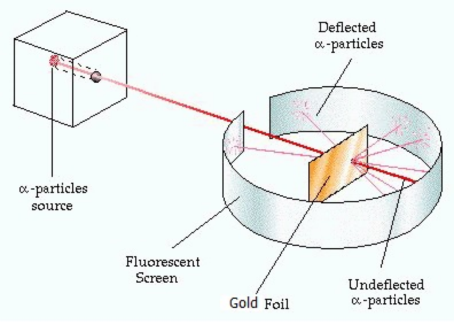
Calculation of H_α line:

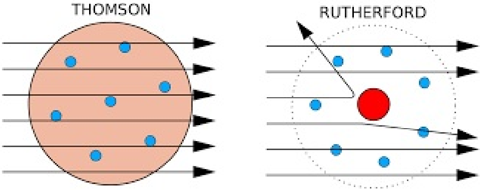
$$\frac{1}{\lambda} = 1.097\times 10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 0.152\times 10^7 m^{-1} \mapsto \lambda = 656.3\times 10^{-9} m$$

Bohr hypothesis of hydrogen atom

Rutherford experiment and atomic structure

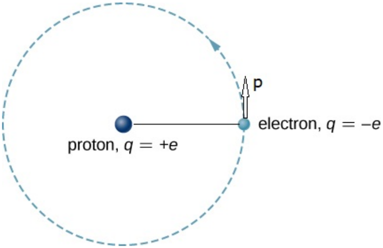
Rutherford experiment and atomic model





Bohr hypothesis

Hydrogen structure



Bohr hypothesis

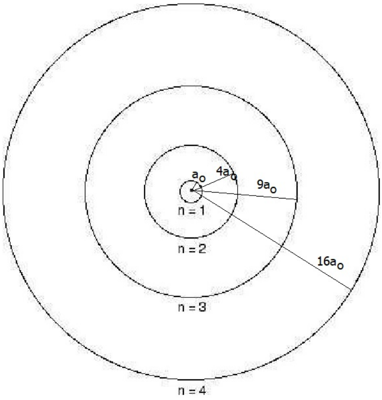
1. Newton's law of motion can be applied to atomic system
2. Electron orbits are discrete, with corresponding discrete angular momenta of $L = mvr = n\hbar, n = 1, 2, ...$
3. Change of electron orbit can occur by absorption/emission of photon with energy $hf = \frac{hc}{\lambda} = |E_f - E_i|$

Evaluation within Bohr hypothesis

$$F = \frac{Ke^2}{r^2} = \frac{mv^2}{r} \mapsto (mvr)^2 = mrKe^2 = (n\hbar)^2$$

$$\mapsto r \equiv r_n = \frac{\hbar^2 n^2}{mKe^2} = a_0 n^2 \quad (7)$$

$$Bohr\ radius:\ a_0 = \frac{\hbar^2}{mKe^2} = \frac{(\hbar c)^2}{mc^2 Ke^2} = 0.529 \times 10^{-10} m = 0.529^0 A$$

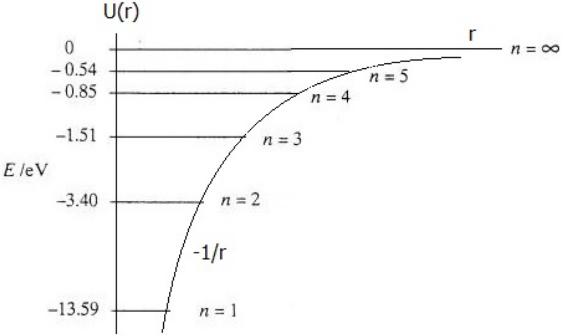


Electron energy

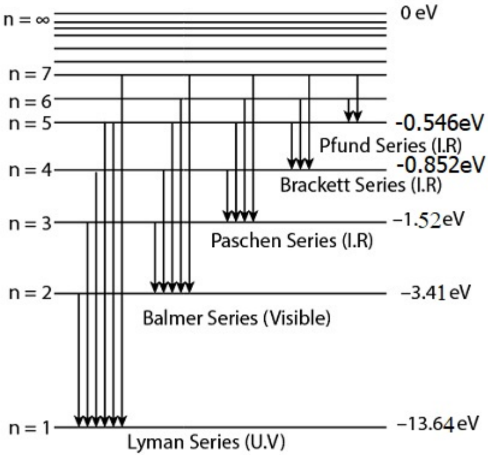
$$E = K + U = \frac{1}{2}mv^2 - \frac{Ke^2}{r} = -\frac{1}{2}\frac{Ke^2}{r}$$

$$\mapsto E \equiv E_n = -\frac{1}{2}\frac{Ke^2}{r_n} = -\frac{1}{2}\frac{Ke^2}{a_0 n^2} = -\frac{1}{2}\frac{m(Ke^2)^2}{\hbar^2 n^2} = -\frac{1}{2}mc^2\alpha^2\frac{1}{n^2} \quad (8)$$

$$\alpha = \frac{Ke^2}{\hbar c} = \frac{1}{137}, \;\; mc^2 = 0.512MeV \mapsto E_n = -\frac{13.6eV}{n^2} \quad (9)$$



Transitions for hydrogen (emission) spectral series



Calculation of H_α *line*:

$$|E_2 - E_3| = 1.89eV = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-6} eV \cdot m}{\lambda} \mapsto \lambda = 656.1 \times 10^{-9} m$$

Calculation of H_∞ line:

$$|E_2 - E_\infty| = 13.64 eV = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-6} eV \cdot m}{\lambda} \mapsto \lambda = 110.0 \times 10^{-9} m$$

Expression of Rydberg constant R

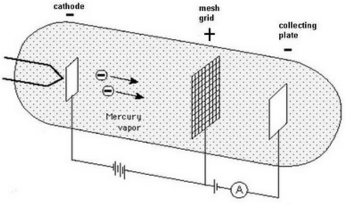
$$\frac{hc}{\lambda} = \frac{1}{2} m_e c^2 \alpha^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \; n_i > n_f$$

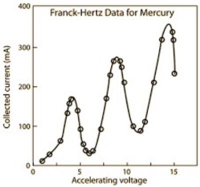
$$\frac{1}{\lambda} = \frac{m_e c^2 \alpha^2}{2hc} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\mapsto R = \frac{m_e c^2 \alpha^2}{2hc} = \frac{(0.512 MeV)(1/137)^2}{2(1.24 \times 10^{-6} eV \cdot m)} = 1.099 \times 10^7 m^{-1}$$

Does hydrogen has shell structure of electron?

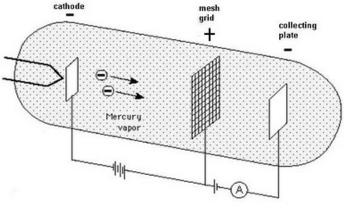
Frank-Hertz experiment:

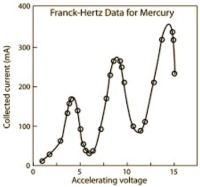




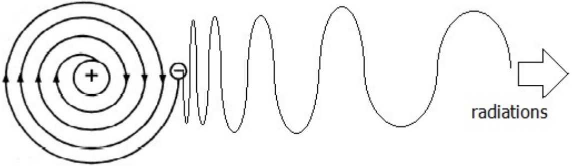
Does hydrogen has shell structure of electron?

Frank-Hertz experiment:



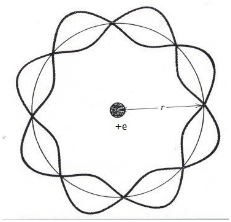


Instability of electron orbit due to EM theory



de Broglie hypothesis of electron wave

Standing electron wave for stable orbit



$$2\pi r_n = n\lambda = n\frac{h}{p} \mapsto r_n p \equiv L_n = n\frac{h}{2\pi} = n\hbar, \;\; n = 1, 2, ... \qquad (10)$$

It shows the origin of discrete electron orbits in Bohr hypothesis. This leads to think that electron behave like a wave inside hydrogen atom.

Next task, we will solve Schrodinger equation with Coulomb potential to looking for electron wave function inside hydrogen atom.