Lecture 2 Nuclear Force and Potential

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Today topics

- Nucleon and isospin symmetry
- Deuterium
- Nature of nuclear strong force
- Tensor force
- Spin-orbit interaction
- Yukawa theory of nucleon interaction

Nucleon and Isospin Symmetry

- Let us look at mass and charge of proton and neutron

	Mass (MeV/c^2)	Charge (e)
Proton (p)	938.272 088	$+1$
Neutron (n)	939.565 420	0

  They are nearly the same mass but differ in charge, which do not feel by nuclear strong force.

- Eugene Wigner (1938) was invent *isospin doublet* of nuclear particle called *nucleon*, similar to particle spin, with isospin quantum number $I = 1/2, I_3 = \pm 1/2$, and was assigned

$$|p>=|I=1/2, I_3=+1/2>, |n>=|I=1/2, I_3=-1/2>$$

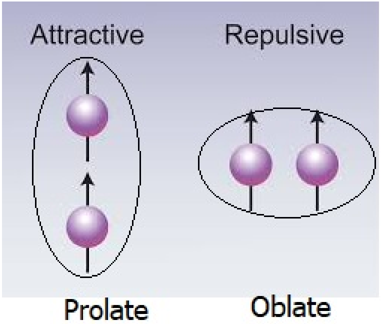
- In nuclear physics proton and neutron are identical, within isospin symmetry, and known as nucleon.

Deuterium

- Deuterium (d) is hydrogen isotope with one neutron (^2_1H)
- It is the ideal system to display the nature of nuclear strong force
- Deuterium physical properties

On Earth abundance	0.0156%
Isotope mass	2.014101 u
Spin	$+1$
Binding energy	2.224 MeV
Magnetic dipole moment μ	$0.857\mu_N$
Electric quadrupole moment Q	$+0.288\ e \cdot fm^2$

- From the value of d spin $=+1$, the spin triplet state, the possible configuration of proton and neutron inside d can be



And the nuclear shape will be prolate ellipsoid.

- Its magnetic moment

$$\mu = g_p \mu_N / + g_n \mu_n / = 2.793\mu_N - 1.913\mu_N = 0.88\mu_N$$

- Non-zero electric quadrupole moment means that the nucleus is not a spherical symmetric, i.e, the nuclear potential will not be a spherical symmetric potential.
- The electric quadrupole moment $Q$of d is determine only form proton, for prolate ellipsoid shape its expression will be

$$Q/e = r^2(3\cos^2\theta - 1) = \sqrt{\frac{16\pi}{5}} r^2 Y_{20}(\theta, \phi)$$

when r is a distance between $p - n$, so that in the COM a distance of p is $r/2$ and we have

$$Q/e = \sqrt{\frac{\pi}{5}} r^2 Y_{20}(\theta, \phi)$$

- Its quantum expectation value is

$$|\psi_d\rangle = \sum_{l=0,2} |l, S=1, J=1, M=1\rangle \frac{u_l(r)}{r} \tag{1}$$

$$\begin{aligned}\langle Q/e\rangle = \sqrt{\frac{\pi}{5}}\,\{&\langle 0,1,1,1|Y_{20}|2,1,1,1\rangle\langle r^2\rangle_{sd} \\ &+\langle 2,1,1,1|Y_{20}|0,1,1,1\rangle\langle r^2\rangle_{ds} \\ &\langle 2,1,1,1|Y_{20}|2,1,1,1\rangle\langle r^2\rangle_{dd}\}\end{aligned} \tag{2}$$

with$\langle r^2\rangle_{sd} = \int_0^\infty dr u_0(r) r^2 u_d(r)$. From the fact that

$$\langle 0,1,1,1|Y_{20}|2,1,1,1\rangle = \frac{1}{2}\sqrt{\frac{1}{10\pi}}$$

$$\langle 2,1,1,1|Y_{20}|2,1,1,1\rangle = -\frac{1}{2}\sqrt{\frac{1}{20\pi}}$$

$$\mapsto \langle Q/e\rangle = \sqrt{\frac{1}{50}}\langle r^2\rangle_{sd} - \frac{1}{20}\langle r^2\rangle_{dd} \tag{3}$$

- Let us assume

$$u_s(r) = N_s e^{-\alpha r}, \; u_d(r) = N_s n_d e^{-\alpha r} \tag{4}$$

where α is the inverse size of the deuterium, i.e., determined of uncertainty principle as

$$\alpha = \sqrt{\frac{Mc^2 \cdot E.B.}{\hbar^2 c^2}} \simeq 0.232 fm^{-1}$$

From above we observe that $N_s = \sqrt{2\alpha}$ and $|n_d|^2 = P_d$ is the radial distribution of d-state.

- From (3) we will get

$$\langle Q/e \rangle = \sqrt{\frac{1}{50}\frac{n_d}{2\alpha^2}} \Rightarrow 0.282 fm^2 \tag{5}$$

$$\mapsto n_d = 0.214 \rightarrow P_d = 0.046 \tag{6}$$

Nuclear Potential

- According to magnetic (dipole-dipole) interaction

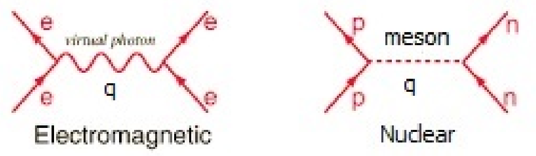
$$V_{dd}(r) = \frac{3(\vec{m}_1 \cdot \vec{r})(\vec{m}_2 \cdot \vec{r})}{r^5} - \frac{\vec{m}_1 \cdot \vec{m}_2}{r^3} \tag{7}$$

So that the asymmetric nuclear potential should be appear in the from

$$U(r) = \left\{ \frac{3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}}{r^5} - \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{r^3} \right\} V_C(r) \tag{8}$$

Yukawa theory of nucleon interaction

- Meson exchange of nucleon interaction



$$V_{Coulomb}(q) = \frac{4\pi e^2}{q^2} \mapsto V_{Coulomb}(r) = \frac{e^2}{r}$$

$$V_{Yukawa}(r) = \frac{4\pi Q^2}{q^2 + M^2} \mapsto V_{Yukawa}(r) = \frac{Q^2}{r} e^{-r/M}$$

where Q is nuclear hypercharge.

Yukawa's Meson

The Japanese physicist Hideki Yukawa had the idea of developing a quantum field theory that would describe the force between nucleons—analogously to the electromagnetic force.

To do this, he had to determine the carrier or mediator of the nuclear strong force analogous to the photon in the electromagnetic force which he called a ***meson*** (derived from the Greek word *meso* meaning "middle" due to its mass being between the electron and proton masses).

Hideki Yukawa (1907-1981)

Summary on the Nature of Nuclear Strong Force

- short range
- hard core repulsion (against the collapse of nucleus)
- asymmetric central potential (tensor interaction)