Lecture 4 Nuclear Models II

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Today topics

- Fermi gas model
- Liquid drop model
- Collective model

Nuclear Fermi gas model

- Nucleon is a fermion
- Let us think about the nucleus of volume V contains $\tilde{N}$ nucleons in free particle state
- The number of nucleon states is determined from Heisenberg uncertainty principle to be

$$\tilde{N} = \frac{\int d^3\vec{r} \int d^3\vec{p}}{(2\pi\hbar)^3} = \frac{4\pi V \int p^2 dp}{(2\pi\hbar)^3}$$

- According to Pauli's exclusion principle, the highest occupied momentum state will be denoted with Fermi momentum p_F, and it is determined to be

$$\tilde{N} = \frac{4\pi V \int_0^{p_F} p^2 dp}{(2\pi\hbar)^3} = \frac{V p_F^3}{6\pi^2\hbar^3}$$

- We may think of two species of nucleons, the proton and neutron, then we will have two Fermi momenta

$$N = \frac{V(p_F^n)^3}{6\pi^2\hbar^3}, \;\; Z = \frac{V(p_F^p)^3}{6\pi^2\hbar^3}$$

- From $r = r_0 A^{1/3} \mapsto V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi A r_0^3 V$
- The number of nucleons occupied in a volume V of the nucleus will be

$$\bar{N} = 2 \cdot \tilde{N} = 2 \cdot \frac{V p_F^3}{6\pi^2 \hbar^3} = 2 \cdot \frac{4}{3}\pi A r_0^3 \cdot \frac{p_F^3}{6\pi^2 \hbar^3} = \frac{4 A r_0^3 p_F^3}{9\pi \hbar^3}$$

$$\mapsto p_F = \left(\frac{9\pi \bar{N}}{4A}\right)^{1/3} \frac{\hbar}{r_0}$$

- For the nucleus with $N = Z = A/2 = \bar{N}$, we will have

$$p_F = \left(\frac{9\pi}{8}\right)^{1/3} \frac{\hbar}{r_0} \simeq 250 MeV/c$$

$$\mapsto E_F = \frac{p_F^2}{2M} \simeq 33 MeV \quad (M = 938 MeV/c^2)$$

- Nuclear potential well depth can be determined to be

$$V_0 = E_F + E_B \simeq 33 + 8 = 41 MeV$$

- The averaged nucleon kinetic energy

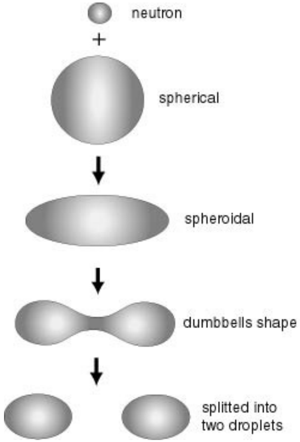
$$< E >= \frac{\int_0^{E_F} E \frac{dN}{dE} dE}{\int_0^{E_F} \frac{dN}{dE} dE} = \frac{\int_0^{p_F} E \frac{dN}{dEp} dp}{\int_0^{p_F} \frac{dN}{dp} dp}$$

with $dE = \frac{p}{E} dp$ and $\frac{dN}{dp} = Const.p^2$. Then we have

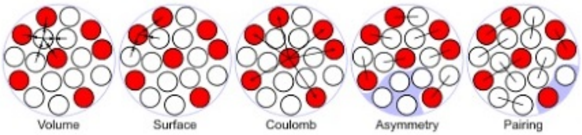
$$< E_{kin} >== \frac{\int_0^{p_F} \frac{p^2}{2M} p^2 dp}{\int_0^{p_F} p^2 dp} = \frac{3}{5} \frac{p_F^2}{2M} \simeq 20 MeV$$

Liquid drop model

- The model was introduced by G. Gamow, in order to explain nuclear fission



- Let us think about the nucleus as a drop of nucleon liquid, its nuclear binding energy will come from the energy of the drop as



- The empirical formula of this energy is known as *Bethe–Weizsäcker formula* and it appears in the form

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

$$a_V = 1585 MeV, a_S = 18.34 MeV, a_C = 0.714 MeV, a_A = 23.21$$

$$\delta(N, Z) = \begin{cases} +\delta_0, & A, Z - even \\ 0, & A - odd \\ -\delta_0, & A, Z - odd \end{cases}, \delta_0 = \frac{12.00 MeV}{A^{1/2}}$$

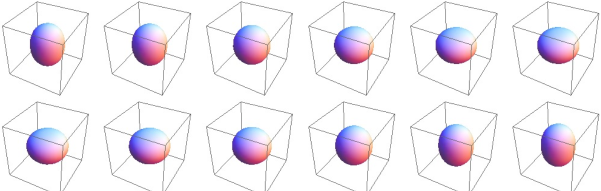
This is known as *pairing energy*.

Nuclear collective model

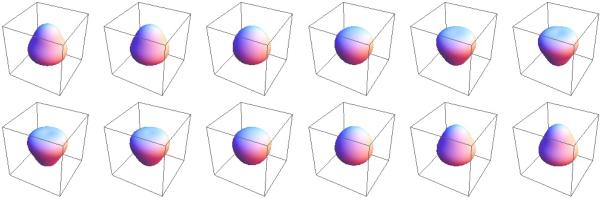
- The nuclear collective model is determined from the deformed nucleus

$$R(\theta,\phi) = R_0\left[1+\sum_{l,m}\alpha_{l,m}Y_{lm}(\theta,\phi)\right]$$

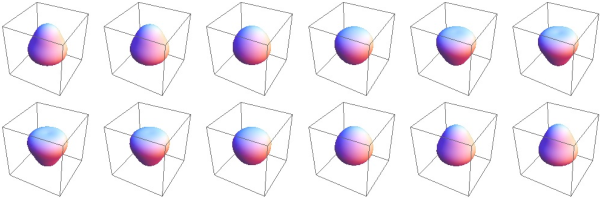
- Its dynamics can be *vibration* and *rotation*
- The vibration modes are
  - $l = 0$ (monopole) breathing mode
  - $l = 1$ (dipole) shifting center of mass
  - $l = 2$ (quadrupole)
  - $l = 3$ (octupole)
  - $l = 4$ (hexadecupole)
- Quadrupole vibration



- Octupole vibration



- Octupole vibration



- Rotation mode is determined from rotation of deformed sphere