Lecture 5 Alpha Emission

SCPY327 ANP, Physics-MUSC, 2021/2

Udom Robkob

March 24, 2022

Alpha emission equation

- $\alpha = He^{+2}$ and $M(\alpha) = 4.001506u = 3.7274MeV/c^2$
- Alpha emission equation

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha + Q $$

- Decay energy ($K = p^2/2m$)

$$ Q = (M_X - M_Y - M_\alpha)c^2 = K_Y + K_\alpha \quad (1) $$

$$ X - \text{rest frame}: \quad p_\alpha = p_Y \mapsto M_\alpha K_\alpha = M_Y K_Y \quad (2) $$

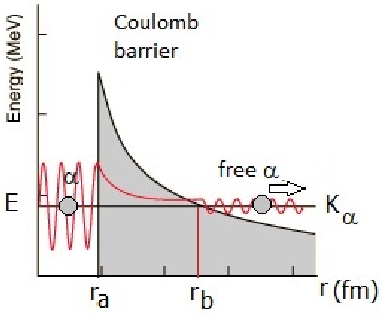
$$ \frac{M_\alpha}{M_Y} = \frac{4}{A-4} \mapsto K_\alpha = \frac{A-4}{A}Q \quad (3) $$

For example ${}^{214}_{84}Po \rightarrow {}^{210}_{82}Pb + \alpha$, with $Q = 7.83MeV$

$$ K_\alpha = \frac{210}{214} \times 7.83MeV = 7.68MeV $$

Gamow theory of alpha emission

- George Gamow (1928)
- Assumption: the α particle is already exist inside the nucleus. Its emission is quantum tunneling through Coulomb barrier



- Tunneling coefficient, with $Z = Z_Y$,

$$T(E) \simeq e^{-2\int_{r_a}^{r_b} k(r)dr}, \;\; k(r) = \sqrt{\frac{2M_\alpha}{\hbar^2}\left(\frac{2ZKe^2}{r} - E\right)}$$

- Calculation, using the fact that $E = K_\alpha = \frac{2ZKe^2}{r_b}$, then we can write

$$\ln T = -2\sqrt{\frac{2M_\alpha E}{\hbar^2}}\int_{r_a}^{r_b}\sqrt{\frac{r_b}{r}-1}dr \quad (4)$$

$$\text{Assign}: \; x = r_b\cos^2\theta, \; \theta = \cos^{-1}\sqrt{\frac{x}{r_b}}$$

$$dx = -2r_b\sin\theta\cos\theta d\theta$$

$$\mapsto \int\sqrt{\frac{r_b}{r}-1}dr = -2r_b\int\sin^2\theta d\theta = -2r_b\left[\frac{\theta}{2}-\frac{1}{4}\sin(2\theta)\right]$$

$$\mapsto \ln T = -2\sqrt{\frac{2M_\alpha E}{\hbar^2}}r_b\left[\cos^{-1}\sqrt{\frac{r_a}{r_b}}-\sqrt{\frac{r_a}{r_b}}\sqrt{1-\frac{r_a}{r_b}}\right] \quad (5)$$

In the limit of wide barrier, $r_b \gg r_a$, we will have

$$\cos(\pi/2-\theta) = \sin(\theta) \xrightarrow[\theta\to 0]{} \theta$$

$$\cos\left(\frac{\pi}{2}-\sqrt{\frac{r_a}{r_b}}\right) \simeq \sqrt{\frac{r_a}{r_b}} \mapsto \cos^{-1}\sqrt{\frac{r_a}{r_b}} \simeq \frac{\pi}{2}-\sqrt{\frac{r_a}{r_b}} \quad (6)$$

- Calculation (cont.), with $1 - r_a/r_b \simeq 1$, we have

$$\ln T \simeq -2\sqrt{\frac{2M_\alpha E}{\hbar^2}} r_b \left[\frac{\pi}{2} - \sqrt{\frac{r_a}{r_b}}\right] \tag{7}$$

  Replacing $r_b = \frac{2ZKe^2}{E}$, we will have

$$\ln T \simeq 8\sqrt{\frac{M_\alpha Ke^2}{\hbar^2}} Z^{1/2} r_a^{1/2} - 4\pi Ke^2 \sqrt{\frac{M_\alpha}{2\hbar^2}} ZE^{-1/2} \tag{8}$$

$$\mapsto 2.97 Z^{1/2} r_a^{1/2} - 3.95 ZE^{-1/2} \tag{9}$$

  with $[E] = MeV,\ [r_a] = fm$

- Using the fact that $\ln A = \frac{1}{\log_{10} e} \log_{10} A = 0.4343 \log_{10} A$, we will have from (9)

$$\log_{10} T \simeq 1.29 Z^{1/2} r_a^{1/2} - 1.72 ZE^{-1/2} \tag{10}$$

Decay constant

- Frequency of the alpha particle comes to hit the Coulomb barrier is determined to be

$$f = \frac{v_\alpha}{2r_a}, \ v_\alpha = \sqrt{\frac{2K_\alpha}{M_\alpha}}$$

- The decay constant becomes

$$\lambda = fT = \frac{v_\alpha}{2r_a} T \qquad (11)$$

$$\log_{10} \lambda = \simeq \log_{10} \frac{v_\alpha}{2r_a} + 1.29 Z^{1/2} r_a^{1/2} - 1.72 Z E^{-1/2} \qquad (12)$$

This is known as *Geiger–Nuttall law*:

$$\log_{10} \lambda \simeq -a_1 \frac{Z}{\sqrt{E}} + a_2, \ \log_{10} T_{1/2} \simeq a_1 \frac{Z}{\sqrt{E}} - a_2'$$

- Geiger-Nuttall plot