Lecture 6 Zeemann Effects

ICPY471 AMP, Second Trimester 2020-21

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Wednesday 27, 2021

The Zeemann effect.

- Pieter Zeeman in 1896 discovered that a magnetic field broadened the yellow D lines produced when sodium is placed in a flame. This is known as *normal Zeemann effect*.
- In 1897, Thomas Preston reported a similar effect to Zeeman but with more complicated results, it is known as *anomalous Zeemann effect*.
- The line splitting results from the effect of the magnetic field on the atomic energy levels. The magnetic interaction of atomic electron produces an amount of energy

$$U_B = -\vec{\mu}_e \cdot \vec{B}, \tag{1}$$

$$\vec{\mu}_e = \frac{-e}{2m_e}(\vec{L} + g\vec{S}), \quad g = 2 \tag{2}$$

- For a uniform magnetic in z-direction, $\vec{B} = B\hat{z}$, we will have

$$U_B = \frac{e}{2m_e}(L_z + 2S_z)B = \frac{\mu_B B}{\hbar}(L_z + 2S_z) \tag{3}$$

where $\mu_B = \frac{e\hbar}{2m_e} = 5.788 \times 10^{-5} eV/T$ is Bohr magneton.

Weak-field Zeemann effect

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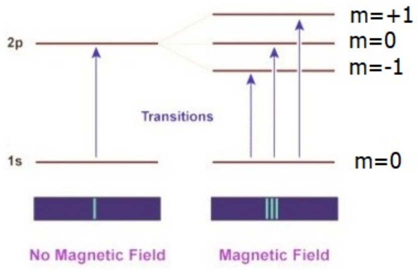
- *Normal Zeeman effect* is determined from the magnetic interaction energy

$$U_B = \frac{\mu_B B}{\hbar} L_z \tag{4}$$

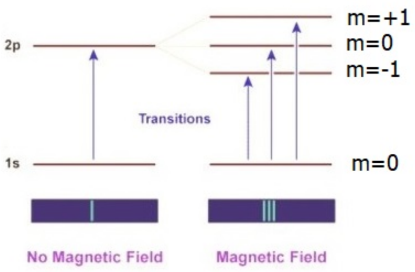
Takes this as the perturbation to the hydrogen energy level, when apply with perturbation theory using the unperturbed atomic state $|nlm>$, we will have perturbed atomic energy, at the first order, as

$$\Delta E^{(1)}_{nlm} = < nlm|U_B|nlm > = m\mu_B B \tag{5}$$

$$E_n \to E^{(1)}_{nlm} = E^{(0)}_n + m\mu_B B \tag{6}$$



m=+1
2p
m=0
m=-1
Transitions
1s
m=0
No Magnetic Field
Magnetic Field



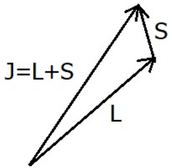
- *Anomalous Zeemann effect* is determined from the magnetic interaction energy

$$U_B = \frac{\mu_B B}{\hbar}(L_z + gS_z), \; g = 2 \tag{7}$$

Takes this as the perturbation of atomic energy level. In the weak field limit, the magnetic interaction energy is comparable to the spin-orbit coupling energy, so that we have to determine it from the total angular momentum state $|nljm_j>$.

- Let us determine the total angular momentum

$$\vec{J} = \vec{L} + \vec{S} \tag{8}$$



The projection of $\vec{S}$ onto the direction of $\vec{J}$ is written as

$$\vec{S}_J = \frac{\vec{S} \cdot \vec{J}}{J^2} \vec{J} \tag{9}$$

It will be used to evaluate the quantum expectation value from the state $|nljm_j>$.

- (Cont.) We can evaluate $\vec{S}\cdot\vec{J}$ from the fact that

$$\vec{L} = \vec{J} - \vec{S} \rightarrow L^2 = J^2 + S^2 - 2\vec{S}\cdot\vec{J} \tag{10}$$

$$\rightarrow \vec{S}\cdot\vec{J} = \frac{1}{2}(J^2 - L^2 - +S^2) \tag{11}$$

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- It follows that

$$\langle nljm_j|(L_z + gS_z)|nljm_j\rangle = \left\langle nljm_j \left| \left(1 + (g-1)\frac{\vec{S}\cdot\vec{J}}{J^2}\right) J_z \right| nljm_j \right\rangle$$
$$= \hbar m_j \left(1 + \frac{(g-1)}{2}\frac{j(j+1) - l(l+1) + s(s+1)}{j(j+1)}\right) = \hbar g_J m_j (12)$$

where we have defined *Lande g-factor*

$$g_J = 1 + \frac{g-1}{2}\frac{j(j+1) - l(l+1) + s(s+1)}{j(j+1)} \quad (13)$$

Lande g-factors for one electron atoms

L	Term	g_j	$g_j\,m_j$
0	$^2S_{1/2}$	2	± 1
1	$^2P_{1/2}$	2/3	$\pm 1/3$
	$^2P_{3/2}$	4/3	$\pm 2/3,\ \pm 6/3$
2	$^2D_{1/2}$	4/5	$\pm 2/5,\ \pm 6/5$
	$^2D_{3/2}$	6/5	$\pm 3/5,\ \pm 9/5,\ \pm 15/5$
3	$^2F_{1/2}$	6/7	$\pm 3/7,\ \pm 9/7,\ \pm 15/7$
	$^2F_{3/2}$	8/7	$\pm 4/7,\ \pm 12/7,\ \pm 20/7,\ \pm 28/7$

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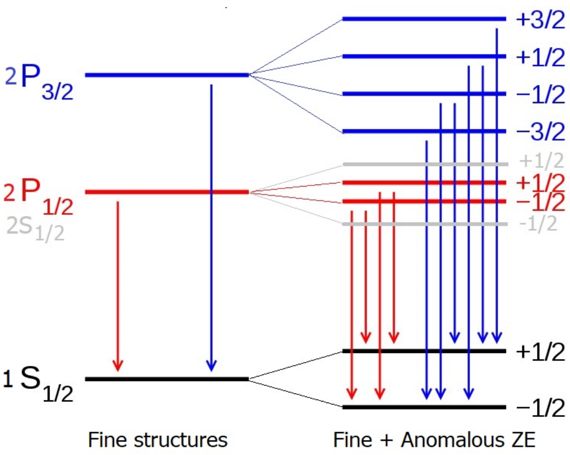
- From (7) we have the perturbed energy, at the first order level, equal to

$$\Delta E^{(1)}_{nljm_j} = g_J m_j \mu_B B \tag{14}$$



We have no transitions $2S \to 1S$, according to the *selection rules*.

Strong-field Zeemann effect

- In the strong field limit, the magnetic interaction energy become greater than the spin-orbit coupling energy, so the we may think of the magnetic interaction independently with orbital and spin magnetic moments.
- Takes this interaction as the perturbation of the atomic levels, and the first order perturbation energy will be determined form the atomic state $|nlm_l sm_s>$

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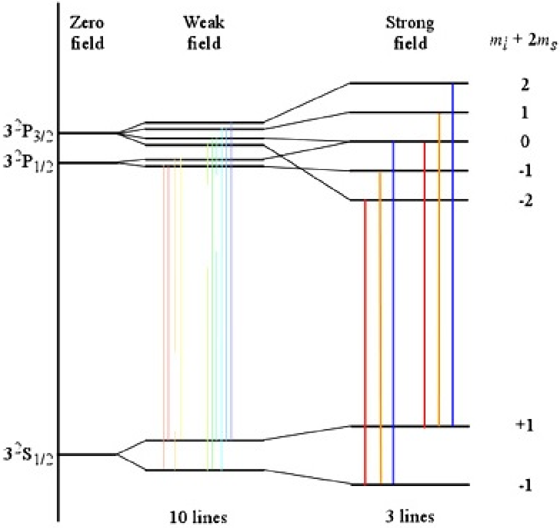
- Takes this interaction as the perturbation of the atomic levels, and the first order perturbation energy will be determined form the atomic state $|nlm_l sm_s>$
- Then we have

$$\Delta E^{(1)}_{nlm_l m_s} = \left\langle nlm_l sm_s \left| \frac{\mu_B B}{\hbar}(L_z + gS_z) \right| nlm_l sm_s \right\rangle = (m_l + gm_s)\mu_B B \tag{15}$$

It is known as the *Paschen-Back effect*.



** Magnetic interaction effects to the atomic energy levels splitting in the same order of atomic fine structures.

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- We choose $|ls; m_l m_s >$ as a good set of atomic state, and will do the expansion of the other set $|ls; jm_j >$ on this set as

$$|ls; jm_j >= \sum_{m_l, m_s} C^{lsj}_{m_l m_s m_j} |ls; m_l m_s > \qquad (16)$$

$$C^{lsj}_{m_l m_s m_j} =< ls; m_l m_s | ls; jm_j > \rightarrow CG - coefficient \qquad (17)$$

where we have ignored the independent n-quantum number.

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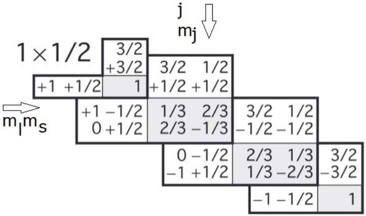
- For $l = 0$, we have $\psi_{l,i} = |jm_j >, i = 1, .., 2j + 1$ as

$$\psi_{0,1} = |1/2, +1/2 >= |0, 1/2; 0, +1/2 > \tag{18}$$

$$\psi_{0,2} = |1/2, -1/2 >= |0, 1/2; 0, = 1/2 > \tag{19}$$



- For $l = 1$, we will have

$$\psi_{1,1} = |\frac{3}{2}, +\frac{3}{2} >= |1, \frac{1}{2}; +1, +\frac{1}{2} > \tag{20}$$

$$\psi_{1,2} = |\frac{3}{2}, -\frac{3}{2} >= |1, \frac{1}{2}; -1, -\frac{1}{2} > \tag{21}$$

$$\psi_{1,3} = |\frac{3}{2}, +\frac{1}{2} >= \sqrt{\frac{1}{3}}|1, \frac{1}{2}; 1, -\frac{1}{2} > +\sqrt{\frac{2}{3}}|1, \frac{1}{2}; 0, +\frac{1}{2} > \tag{22}$$

$$\psi_{1,4} = |\frac{1}{2}, +\frac{1}{2} >= \sqrt{\frac{2}{3}}|1, \frac{1}{2}; +1, -\frac{1}{2} > -\sqrt{\frac{1}{3}}|1, \frac{1}{2}; 0, +\frac{1}{2} > \tag{23}$$

- For $l = 1$, cont.

$$\psi_{1,5} = |\frac{3}{2}, -\frac{1}{2} >= \sqrt{\frac{2}{3}}|1, \frac{1}{2}; 0, -\frac{1}{2} > +\sqrt{\frac{1}{3}}|1, \frac{1}{2}; -1, +\frac{1}{2} > \quad (24)$$

$$\psi_{1,6} = |\frac{1}{2}, -\frac{1}{2} >= \sqrt{\frac{1}{3}}|1, \frac{1}{2}; 0, -\frac{1}{2} > -\sqrt{\frac{2}{3}}|1, \frac{1}{2}; -1, +\frac{1}{2} > \quad (25)$$

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- The weak-field Zeemann effect will be calculated in term of the matrix elements as

$$W_{l,ij} = \mu_B B \langle \psi_i | (m_l + g m_s) | \psi_j \rangle \quad (26)$$

- For $l = 1$, cont.

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- For $l = 0$

$$W_{0,ij} = \mu_B B \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (27)$$

- For $l = 1$

$$W_{1,ij} = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & -\sqrt{2}/3 & 0 & 0 \\ 0 & 0 & -\sqrt{2}/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2/3 & -\sqrt{2}/3 \\ 0 & 0 & 0 & 0 & -\sqrt{2}/3 & -1/3 \end{pmatrix} \tag{28}$$

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- The following sub matrices need diagonalization

$$W_{1,(34)_{ij}} = \begin{pmatrix} 2/3 & -\sqrt{2}/3 \\ -\sqrt{2}/3 & 1/3 \end{pmatrix} \tag{29}$$

$$W_{1,(56)_{ij}} = \begin{pmatrix} -2/3 & -\sqrt{2}/3 \\ -\sqrt{2}/3 & -1/3 \end{pmatrix} \tag{30}$$

- Conclusions
  - In the weak field limit, $|nl; jm_j>$ is a good quantum state
  - In the strong field limit, $|nls; m_l m_s>$ is a good quantum state, and related to $|nl; jm_j>$, as appear in (18-25).
- The diagonalization process will be posted in Mathematica Code later!