Lecture 2 Nuclear Strong Force

ICPY473 Nuclear Physics
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Today Topics

- Nuclear isospin symmetry
- Nuclear strong force
- Deuteron
- Yukawa theory of nucleon interaction

Nuclear isospin

- W. Heisenberg was noticed that proton p and neutron n have nearly the same mass but differ in electric charge

	p	n
Mass (MeV/c^2)	938.27	939.57
Charge (e)	$+1$	0
Spin (s)	$1/2$	$1/2$

- He was assigned the isospin state $|I, I_3\rangle$, with the isoispin quantum number $I = 1/2, I_3 = \pm 1/2$, in analogy to spin quantum number.
- A Nucleon N is *nuclear particle* with *isospin symmetry* and appear as it isospin doublet as

$$|N\rangle = |I = 1/2, I_3\rangle \rightarrow \begin{cases} |1/2, +1/2\rangle = |p\rangle \\ |1/2, -1/2\rangle = |n\rangle \end{cases} \quad (1)$$

Full description of nucleon state is $|N\rangle = |S = 1/2, S_3; I = 1/2, I_3\rangle$.

- The isoispin quantum number can be assigned to the three pions $(\pi^0, \pi^{\pm})$

	π^+	π^0	π^-
Mass (MeV/c^2)	139.570	134.977	139.570
Charge (e)	+1	0	-1
Spin (s)	0	0	0

- The *pion* isospin state is

$$|\pi\rangle = |I = 1, I_3\rangle \rightarrow \begin{cases} |1, +1\rangle = |\pi^+\rangle \\ |1, 0\rangle = |\pi^0\rangle \\ |1, -1\rangle = |\pi^-\rangle \end{cases} \tag{2}$$

Full description of pion state $|\pi\rangle = |S = 0, S_3 = 0; I = 1, I_3\rangle$, it is an isospin triplet.

- *Isopsin* becomes a good quantum number for nuclear and subnuclear particles. The multiplet formed from each isospin quantum number is said to arise from *isopsin symmetry* of the system with degenerate mass.

Nuclear strong force

- An atomic nucleus is a bound state of nucleon, under the action of nuclear strong force. The nature of the strong force is
  - short range
  - charge independent (blind from electric charge)
  - isotropic
  - isospin independent (blind from isospin charge?)
  - spin-dependent ?
  - hard core repulsion
- We can determine the properties of nuclear force from *deuteron* D, i.e., a system of two nucleons, and N-N scattering experiment.

Deuteron

- Basic (observed) properties of deuteron:
  - Mass =2.014735u
  - Binding energy $= 2.224 MeV$
  - Size (mean radius) $r = 2.1 fm$
  - Spin S=1
  - Isospin I=0
  - Magnetic dipole moment $\mu = 0.857\mu_N$
  - Electric quadrupole moment $Q = 0.00286 eb$
- Theoretical descriptions:
  - D is two nucleon system, each nucleon has spin $S_{1,2} = 1/2$, so that the total spin quantum numbers are $S = 1, 0$. The corresponding spin states $S, S_3\rangle$ are

$$\text{spin singlet}: \ |0,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \tag{3}$$

$$\text{spin triplet}: \left\{ \begin{array}{c} |\uparrow\uparrow\rangle \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ |\downarrow\downarrow\rangle \end{array} \right. \tag{4}$$

D is in the symmetric spin triplet.

- Theoretical descriptions (cont.):
  - From the aspect of isopsin quantum number, D has two isospin $I_{1,2} = 1/2$. The total isopin is $I = 1, 0$.

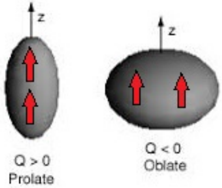
$$|0,0\rangle = \frac{1}{\sqrt{2}}(pn - np),\ |1,\pm 1\rangle = pp, nn, \frac{1}{\sqrt{2}}(pn + np) \tag{5}$$

    D is in anti-symmetric isospin singlet.
  - Nuclear magnetic dipole moment of D can be calculated as

$$\mu_D = \mu_p + \mu_n = \frac{1}{2}(g_p + g_n \mu_N) \tag{6}$$

$$g_p = 5.585,\ g_n = -3.825 \rightarrow \mu_D = 0.857\mu_N \tag{7}$$

    This comes to a problem of what is the shape of D?
  - The final decision comes from the value of electric quadrupole moment Q, which shows that D has a prolate shape.



$$Q_0 = \int \rho(3z^2 - r^2)dV$$

$$Q = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0$$

- In general, D is a system of two fermions, its state must be ani-symmetric under permutation $|N_1, N_2\rangle = -|N_2, N_1\rangle$, according to *spin-statistic theorem*.
- The state $D\rangle$ consists of

$$|D\rangle_A = |nlm_l; S, S_3; I, I_3\rangle_A = |nlm_l\rangle_S \otimes |S, S_3\rangle_S \otimes |I, I_3\rangle_A \qquad (8)$$

$$|nlm_l\rangle_A \to l = 0, 2, 4, 6, ... \qquad (9)$$

We can observe that only $l = 0$ state can be formed to be a bound state of D.

- Schrodinger equation of D

$$H_D = \frac{1}{2\mu}p^2 + V(r) \to (H_D - E_n)\psi_{nlm} = 0 \qquad (10)$$

where μ is a reduced mass, p is relative momentum, and $V(r)$ is finite spherical potential well of depth $-V_0$ and radius a. The space state function is $\psi_{nlm}(r, \theta, \phi)$.

- In its $l = 0$ state, the radial equation of (10) reads

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{n0}(r)}{dr}\right) + \frac{2\mu}{\hbar^2}(E_n - V(r))R_{n0}(r) = 0 \qquad (11)$$

$$U(r) = rR_{n0}(r) \rightarrow \frac{d^2U_n(r)}{dr} + \frac{2\mu}{\hbar^2}(E_n - V(r))U_n(r) = 0 \qquad (12)$$

$$V(r) = \begin{cases} -V_0, & 0 \leq r \leq a \\ 0, & r > a \end{cases} \qquad (13)$$

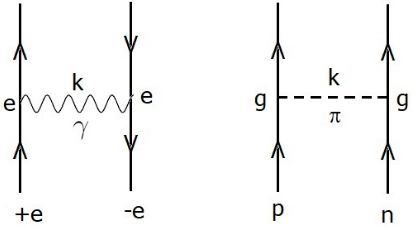
$$\rightarrow U(r) = \begin{cases} A\sin(kr), & 0 \leq r \leq a \\ Be^{-\kappa r}, & r > a \end{cases} \qquad (14)$$

where $k^2 = \frac{2\mu}{\hbar^2}(E + V_0)$, $\kappa^2 = \frac{2\mu E}{\hbar^2}$, with $U(0) = 0$.

- Connect graphically the two solutions at the boundary $a = 2.1fm$, and constraints with the ground state energy of $-2.224MeV$, we find that the depth of the well is $V_0 = 23.1MeV$.
- Details calculation will be left for exercise.

Yakawa theory of nucleon interaction

- H. Yukawa (1934) was suggested that the nuclear strong interaction may be mediated by massive bosonic particle, the *meson*, instead of the massless bosonic particle like photon.

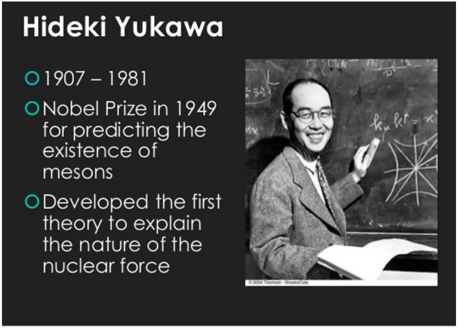


In the rest frame of particles, the interaction amplitudes are

$$\mathcal{M}_{ee} = -\frac{e^2}{k^2} \equiv V_{ee}(k) \qquad \mathcal{M}_{pn} = -\frac{g^2}{k^2 + m_\pi^2} \equiv V_{pn}(k) \quad (15)$$

$$V_{ee}(r) = -\frac{e^2}{4\pi}\frac{1}{r} \qquad V_{pn}(r) = -\frac{g^2}{4\pi}\frac{e^{-rm_\pi}}{r} \quad (16)$$

- $V_{pn}(r)$ in (16) in known as Yukawa-type potential, and the $r_0 = m_\pi^{-1}$ is called the *range* of the interaction force.
- For $r_0 = 2.1 fm \rightarrow m_\pi \sim 200 MeV$, which match to the mass of pion (140 MeV) that was found a few year later.



Yukawa's Prediction of the Meson

by

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1. Introduction

In October 1934, at a meeting of the Osaka Branch of the Physico-Mathematical Society of Japan, Hideki Yukawa proposed a new theory of nuclear forces involving the exchange between neutron and proton of an electrically charged "heavy quantum".[1] Yukawa's theory is known today as the *meson theory of nuclear forces* and his "heavy quantum" is called the meson.[2] The theory provided a fundamental explanation for the charge-exchange nuclear force that Werner Heisenberg had proposed in 1932 and had associated with the exchange of an *electron* between a neutron and a proton.[3] In the same paper Yukawa suggested an alternative form of Enrico Fermi's beta-decay theory and showed that the two forms were phenomenologically equivalent in the treatment of nuclear beta decay, although Yukawa used the meson as an intermediary in beta decay and clearly distinguished the "strong" nuclear binding force from the "weak" force responsible for radioactive beta decay.[4,5]