ICPY473 Nuclear Physics
U. Robkob, Physics-MUSC
MUIC, Third Trimester, 2020-21
Monday 10, May 2021

5 Alpha Decays

5.1 Decay equation

α-particle is a nucleus of helium atom, i.e., ${}_2^4He_2^{+2}$. Alpha emission of ${}_Z^AX_N$ nucleus is

$$ {}_Z^AX_N \to {}_{Z-2}^{A-4}Y_{N-2} + \alpha + Q \tag{5.1} $$

The decay energy Q can be calculated by mass defect formula

$$ Q = [M_X - M_Y - M_\alpha]c^2 \tag{5.2} $$

In the rest from of X, we can evaluate

$$ p_Y = p_\alpha \to K_Y = \frac{M_\alpha}{M_Y} K_\alpha \tag{5.3} $$

$$ Q = K_Y + K_\alpha = \left(1 + \frac{M_\alpha}{M_Y}\right) K_\alpha \tag{5.4} $$

$$ \to K_\alpha = \frac{Q}{1 + M_\alpha/M_Y} \simeq \frac{A}{A+4} Q, \quad A \gg 4 \tag{5.5} $$

For example of α-emission of ${}_{84}^{214}Po$, we have $Q = 7.83MeV$ and $K_\alpha = 7.68MeV$.

5.2 Gamow theory of α-emission

Assume the α-particle already exist inside the nucleus at nuclear energy level $E_\alpha = K_\alpha \simeq Q$, the kinetic energy outside the nucleus. The emission occur by quantum tunneling though the Coulomb potenlai neck

$$ V_C(r) = 2Z_Y \frac{k_e e^2}{r^2}, r \geq a = r_0 A^{1/3} \tag{5.6} $$

See figure (5.1). We observe that $K_\alpha = V_C(b)$.

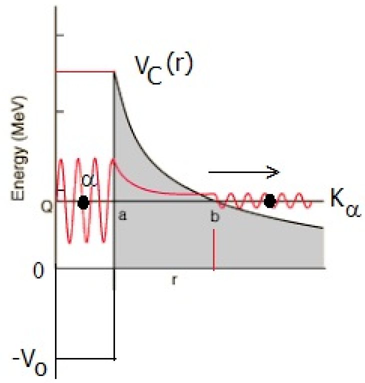

Figure 5.1:

From WKB approximation, the tunneling coefficient is

$$T \simeq e^{-G} \to \ln T = -G \tag{5.7}$$

$$\begin{aligned} G &= \frac{2}{\hbar}\int_a^b dr\sqrt{2M_\alpha(V_C(r)-E_\alpha)} \\ &= \frac{2}{\hbar}\sqrt{2M_\alpha E_\alpha}\int_a^b dr\sqrt{\frac{b}{r}-1} \end{aligned} \tag{5.8}$$

Let us define $r = b\cos^2\theta,\ \theta \in [0, \cos^{-1}\sqrt{a/b} = \theta_1]$,and $dr = -2bcos\theta\sin\theta d\theta$, then evaluate

$$\begin{aligned} I = \int_a^b dr\sqrt{\frac{b}{r}-1} = b\int_0^{\theta_1} d\theta(2\sin^2\theta) &= b\int_0^{\theta_1} d\theta(1-\cos 2\theta) \\ = b\left(\theta - \frac{1}{2}\sin(2\theta)\right)_0^{\theta_1} &= b\left(\theta - \sin\theta\cos\theta\right)_0^{\theta_1} \\ &= b\left\{\cos^{-1}\sqrt{\frac{a}{b}} - \sqrt{\frac{a}{b}\left(1-\frac{a}{b}\right)}\right\} \end{aligned} \tag{5.9}$$

Assume $b \gg a$, so that

$$\cos^{-1}\sqrt{\frac{a}{b}} \simeq \frac{\pi}{2} - \sqrt{\frac{a}{b}}$$

Then we have from (5.9)

$$I \simeq b\left\{\frac{\pi}{2} - 2\sqrt{\frac{a}{b}}\right\} \tag{5.10}$$

From (5.7), we have

$$\ln T = 2b\sqrt{\frac{2M_\alpha E_\alpha}{\hbar^2}}\left(2\sqrt{\frac{a}{b}} - \frac{\pi}{2}\right) \tag{5.11}$$

Rewrite b in term of E_α, we have

$$\ln T \simeq 8\sqrt{\frac{Z_Y k_e e^2 M_\alpha a}{\hbar^2}} - \frac{4\pi Z_Y k_e e^2}{\hbar}\sqrt{\frac{M_\alpha}{2E_\alpha}} \tag{5.12}$$

Since

$$\frac{8}{\hbar}\sqrt{k_e e^2 M_\alpha} = 8\sqrt{\frac{\alpha M_\alpha c^2}{\hbar c}} = 2.97\ fm^{-1/2}$$

$$\frac{4\pi k_e e^2}{\hbar}\sqrt{M_\alpha} = 4\pi\alpha\sqrt{M_\alpha c^2} = 3.95\ MeV^{1/2}$$

with $\alpha = \frac{k_e e^2}{\hbar c} = \frac{1}{137}$ is fine structure constant. From (5.12) we have

$$\ln T \simeq 2.97\sqrt{Z_y a} - 3.95\frac{Z_Y}{\sqrt{E_\alpha}} \tag{5.13}$$

with $[a] = fm,\ [E_\alpha] = MeV$.

To evaluate the emission rate, we have to find the frequency f that the alpha particle hit the Coulomb neck barrier. For a nucleus of radius a, it is

$$f = \frac{v_\alpha}{2a} \tag{5.14}$$

The emission rate is

$$\lambda = f \times T \to \ln\lambda = \ln f + \ln T \simeq A - B\frac{Z_Y}{\sqrt{E_\alpha}} \tag{5.15}$$

$$A = \ln(v_\alpha/2a) + 2.97\sqrt{Z_Y a},\ B = 3.95 \tag{5.16}$$

Note that (.15) is known as *Geiger-Nuttall law*, See figure (5.2). Note that $[\lambda] = s,\ [Q_\alpha] = MeV$ and $Z = Z_Y$.

For example of alpha emission out of ${}^{238}_{92}U$ into ${}^{234}_{90}Th$, with $Q_\alpha = 4.27 MeV$, we observe that

$$\frac{Z}{\sqrt{Q_\alpha}} = 43.65 \to \ln\lambda \sim -40 \to \frac{1}{\lambda} \sim 10^{17.37} = 9.42\times 10^{18}s \tag{5.17}$$

$$T_{1/2} \sim 2\times 10^{11}yr\ \ (4.47\times 10^9 yr) \tag{5.18}$$

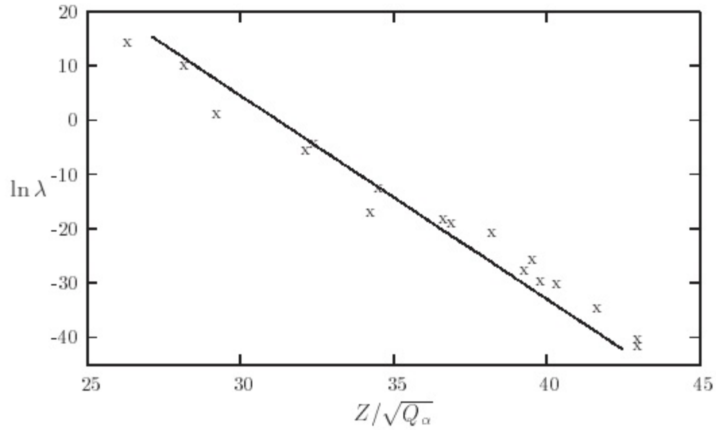

Figure 5.2:

Problems 5

5.1. Evaluate Q and K_α from the $^{228}_{90}Th$ α-emission.

5.2. Estimate the half-life time of the α-decay of ^{231}Pa, protactinium-231, using Geiger-Nuttall law.