ICPY473 Nuclear Physics
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6 Beta Emission

6.1 Beta decay equations

Beta particles are energetic electron β^- ans position β^+. The beta emission of any nucleus X into a daughter in the same isobar, always associated with *neutrinos* which was predicted by W. Pauli from the observed beta spectrum. Electron neutrino ν_e and its anti-particle positron neutrino $\bar{\nu}_e$ are charge neutral and assumed to be massless particles [1].

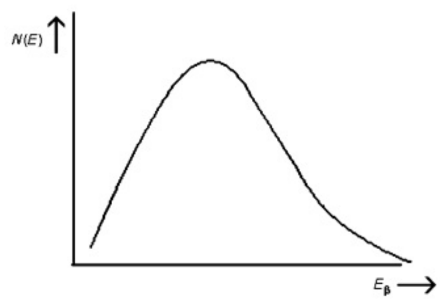

Figure 6.1: β-spectrum.

At fundamental level beta emission results from the proton and neutron decay as

$$p^+ \to n^0 + \beta^+ + \nu_e \tag{6.1}$$
$$n^0 \to p^+ + \beta^- + \bar{\nu}_e \tag{6.2}$$

One associate beta particle process is *electron capture*

$$p^+ + \beta^- \to n^0 + \nu_e \tag{6.3}$$

[1]Nowadays we have observed three generations of neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and also observed their flavor oscillation. This requires the existence of neutrino masses, even tiny masses.

Let us determine the beta emission and electron capture equations of any nucleus X, and their associated energy.

$$\beta^- : \quad {}_Z^AX_N \rightarrow {}_{Z+1}^{A}Y_{N-1} + \beta^- + \bar{\nu}_e + Q \tag{6.4}$$
$$Q = (M_X - M_Y)c^2 \tag{6.5}$$
$$\beta^+ : \quad {}_Z^AX_N \rightarrow {}_{Z_1}^{A}Y_{N+1} + \beta^+ + \nu_e + Q \tag{6.6}$$
$$Q = (M_X - M_Y - 2m_e)c^2 \tag{6.7}$$
$$e - capture : \quad {}_Z^AX_N + \beta^- \rightarrow {}_{Z-1}^{A}Y_{N+1} + \nu_2 + Q \tag{6.8}$$
$$Q = (M_X - M_Y - 2m_e)c^2 \tag{6.9}$$

For example

$$^{214}_{83}Bi \rightarrow {}^{214}_{84}Po + \beta^- + \bar{\nu}_e + 3.26MeV$$

From Weiszacker binding energy formula, we can have parabolic curve(s) of isobaric nuclei of the beta emission chain. We can observe the double beta decay only from the A-even nuclei.

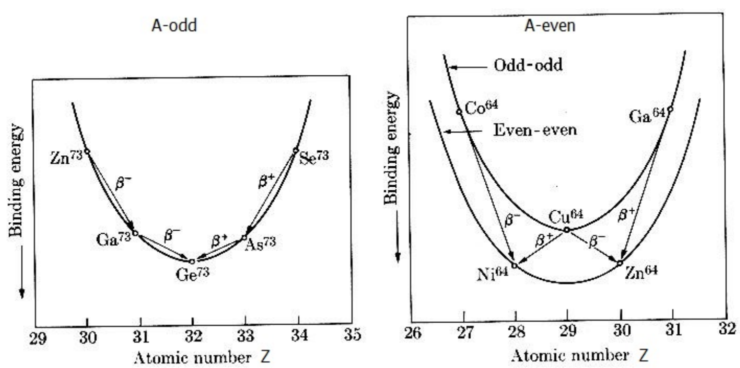

FIG. 17–9. Binding energies and decay properties of nuclides of odd mass number ($A = 73$).

FIG. 17–10. Binding energies and decay properties of nuclides of even mass number ($A = 64$).

Figure 6.2: A-odd and A-even isobaric beta emission chains.

6.2 Fermi theory of beta emission

We can determine the beta emission, using quantum description, as a transition of proton state into (neutron + beta+ neutrino) state from some interaction with strength g:

$$|\psi_p\rangle_i \rightarrow |\psi_n\psi_\beta\psi_\nu\rangle_f$$

From Fermi's golden rule we can calculate the transition rate to be

$$W_{fi} \equiv d\lambda = \frac{2\pi}{\hbar} g^2 |V_{fi}|^2 \frac{dn}{dE_0} \tag{6.10}$$

Since

$$\langle r|\psi_a\rangle = \frac{1}{\sqrt{V}} e^{ip_a \cdot r/\hbar}, \quad a = p, n, \beta, \nu \tag{6.11}$$

At $r = 0$ we have

$$d\lambda = \frac{2\pi}{\hbar} \frac{g^2}{V^2} |M_{fi}|^2 \frac{dn}{dE_0} \tag{6.12}$$

In order to evaluate dn/dE_e we use that fact from Bohr-Sommerfeld quantization rule

$$\begin{aligned}
dN_a &= \frac{V d^3 p_a}{h^3} = \frac{4\pi^2 V p_a^2 dp_a}{h^3}, \quad a = e, \nu && (6.13)\\
dn = dN_e dN_\nu &= \frac{16\pi^2 V^2}{h^6} p_e^2 p_\nu^2 dp_e dp_\nu && (6.14)\\
p_\nu c = E_\nu = E_0 - E_e &\rightarrow c dp_\nu = dE_0 \ (E_e - constant) && (6.15)\\
\rightarrow dn &= \frac{16\pi^2 V^2}{h^6 c^3} (E_0 - E_e)^2 p_e^2 dp_e dE_0 && (6.16)\\
\rightarrow \frac{dn}{dE_0} &= \frac{16\pi^2 V^2}{h^6 c^3} (E_0 - E_e)^2 p_e^2 dp_e && (6.17)\\
\rightarrow d\lambda(p_e) &= \frac{1}{2\pi^3 \hbar^7 c^3} g^2 |V_{fi}|^2 (E_0 - E_e)^2 p_e^2 dp_e && (6.18)
\end{aligned}$$

See figure (6.3).

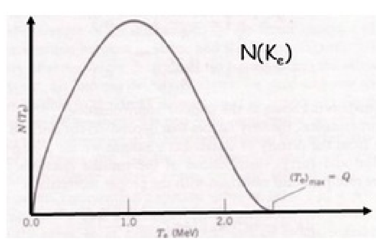

Figure 6.3: β-spectrum.

For correction we add a factor of Fermi function $F(Z, E_e)$, see figure (6.4), we have

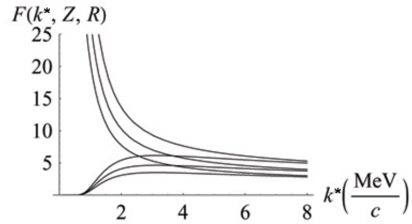

FIG. 3: The Fermi function $F(k^*, Z, R)$ for beta-decay to a final nucleus of charge $Z = 83$ as a function of the electron (decreasing curves) or positron momentum k^* and the nucleus radius $R = 2, 4, 8$ fm (in decreasing order).

Figure 6.4: Beta decay Fermi function.

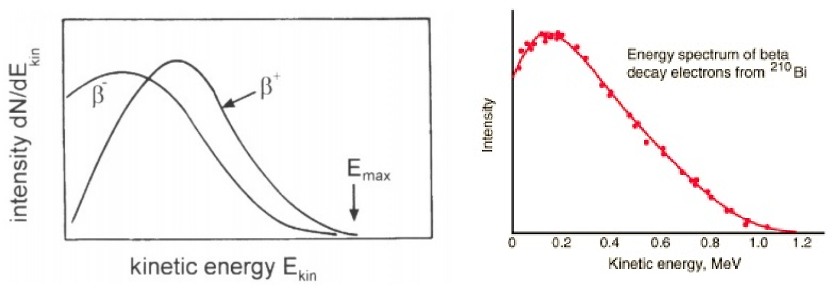

Figure 6.5: Fermi beta spectrum.

$$d\lambda(p_e) = \frac{1}{2\pi^3\hbar^7c^3} g^2 |V_{fi}|^2 F(Z, E_e)(E_0 - E_e)^2 p_e^2 dp_e \tag{6.19}$$

From this expression (6.19), we can have Kurie plot

$$\left\{ \frac{\lambda)p_e)}{p_e^2 F(Z, E_e)} \right\}^{1/2} \propto (E_0 - E_e)$$

which yield a straight line, see figure (6.6).

6.3 Neutrino mass

From (6.19), we can determine the effect of neutrino mass by including with a factor of neutrino mass as

$$d\lambda(p_e) = \frac{1}{2\pi^3\hbar^7c^3} g^2 |V_{fi}|^2 F(Z, E_e)(E_0 - E_e)^2 p_e^2 \sqrt{1 - \frac{m_\nu^2 c^4}{(E_0 - E_e)^2}} dp_e \tag{6.20}$$

Beta spectrum with neutrino mass appear in figure (6.7).

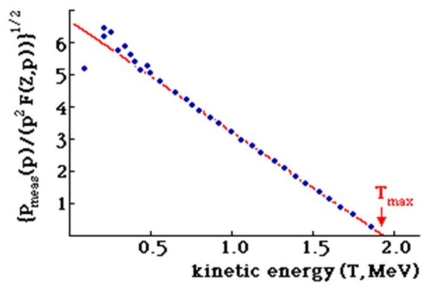

Figure 6.6: Kurie plot.

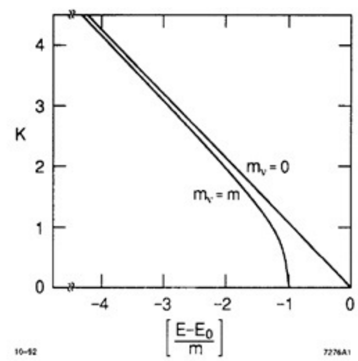

Figure 6.7: Effect of neutrino mass.

6.4 Beta decay classification

From (6.19), let us evaluate the total decay rate as

$$\begin{aligned} \lambda = \frac{1}{2\pi^3\hbar^7 c^3} g^2 |V_{fi}|^2 \int_0^{p_{max}} F(Z, E_e)(E_0 - E_e)^2 p_e^2 dp_e \\ = \frac{1}{2\pi^3\hbar^7} g^2 |V_{fi}|^2 m_e^5 c^4 f(Z, E_0) \end{aligned} \quad (6.21)$$

where $f(Z, E_0)$ is known as *Fermi integral.* Now we can define a *comparative half-life time* of the beta decay as

$$ft_{1/2} = \ln 2 \frac{2\pi^3\hbar^7}{g^2|V_{fi}|^2 m_e^5 c^4} \propto \frac{1}{g^2|M|^2} \quad (6.22)$$

This can be used to classify the beta transition as in the following table.

TABLE 8.2 Classifications of β-Decay Transitions

Transition Type	log *ft*	L_β	$\Delta\pi$	Fermi ΔI	Gamow–Teller ΔI
Superallowed	2.9–3.7	0	No	0	0
Allowed	4.4–6.0	0	No	0	0, 1
First forbidden	6–10	1	Yes	0, 1	0, 1, 2
Second forbidden	10–13	2	No	1, 2	1, 2, 3
Third forbidden	>15	3	Yes	2, 3	2, 3, 4

Figure 6.8: Classified beta transition.

6.5 Beta decay modes

Under beta decay, the electron spin and neutrino spin can be parallel or anti-parallel, these are called *Gamow-Teller* and *Fermi* decay modes, respectively. In heavy nuclei, Gamow-Teller decay mode is dominated. In mirror nuclei, Fermi decay mode is only the possible decay.

6.6 Double beta decays

The idea of double beta decay was first proposed by Maria Goeppert-Mayer in 1935. In 1937, Ettore Majorana demonstrated that all results of beta decay theory remain unchanged if the neutrino were its own antiparticle, now known as a Majorana particle.

Double beta decay observations

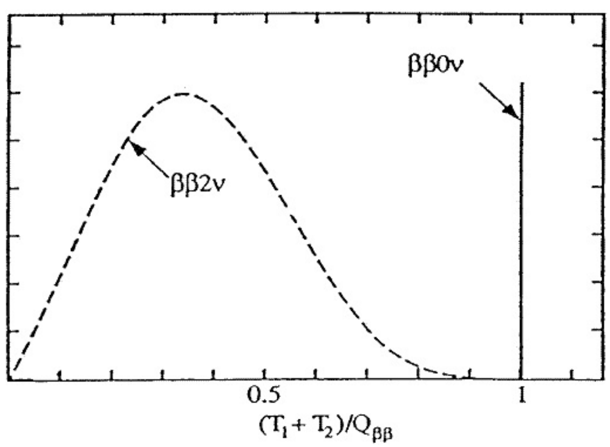

Figure 6.9: Possible double decay observations.

The observed $2\beta 2\nu$ isotopes

Table 1.1
The half-lives relatively to 2β decay of different isotopes measured in radiochemical and geochemical experiments.

Isotope	$Q_{2\beta}$, keV	Decay mode	$T_{1/2}$, years
^{82}Se	2995	$2\beta^-$	$= (1.2 \pm 0.1) \times 10^{20}$ [65]
^{96}Zr	3350	$2\beta^-$	$= (3.9 \pm 0.9) \times 10^{19}$ [66] $= (9.4 \pm 3.2) \times 10^{18}$ [67]
^{100}Mo	3034	$2\beta^-$	$= (2.1 \pm 0.3) \times 10^{18}$ [68]
^{128}Te	868	$2\beta^-$	$= (1.8 \pm 0.7) \times 10^{24}$ [65] $= (7.7 \pm 0.4) \times 10^{24}$ [69] $= (2.2 \pm 0.4) \times 10^{24}$ [70]
^{130}Te	2533	$2\beta^-$	$\sim 0.8 \times 10^{21}$ [70] $= (2.7 \pm 0.1) \times 10^{21}$ [71]
^{130}Ba	2611	$2\beta^+$, $\varepsilon\beta^+$, 2ε 2ε	$> 4.0 \times 10^{21}$ [72] $= (2.1^{+3.0}_{-0.8}) \times 10^{21}$ [72] $= (2.2 \pm 0.5) \times 10^{21}$ [73]
^{132}Ba	840	2ε	$> 3.0 \times 10^{20}$ [72] $> 2.2 \times 10^{21}$ [73]
^{238}U[1]	1150	$2\beta^-$	$= (2.0 \pm 0.6) \times 10^{21}$ [74]

Figure 6.10: Observed $2\beta2\nu$ isotopes.

Neutrinoless $2\beta2\nu$ decay and Majorana neutrinos

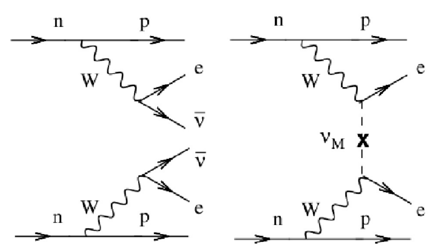

Figure 6.11: Majorana neutrino and neutrinoless $2\beta2\nu$ decay.

Problem 6