ICPY473 Nuclear Physics

Third Trimester, 2021

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10 Theory of Nuclear Fission

According to N. Bohr and J.A. Wheeler (1939).

10.1 Fission mechanism

The theory is based on liquid drop model of the nucleus.

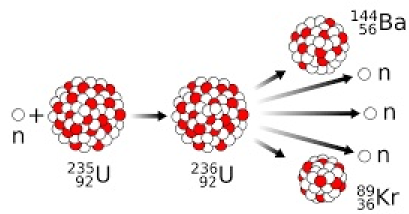

Figure 10.1: Liquid drop model of fission.

Mechanism: when a neutron hits a nucleus, the compound nucleus is formed with certain excited energy due to the extra neutron. The fission equation will be

$$n + {}^{A}_{Z}X_N \to {}^{A+1}_{Z}X^*_{N+1} \to {}^{A_1}_{Z_1}Y^1_{N_1} + {}^{A_2}_{Z_2}Y^2_{N_2} + fn \tag{10.1}$$

where f means a "few" neutrons.

The excited compound nucleus immediately falls into rapid oscillation. The vibration set up deformed nucleus and results to the exchange of liquid drop surface energy and Coulomb energy.

Set up the related liquid drop energy equation, we have

$$\begin{aligned} E_0 = (E_S + E_C)_0 &= a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} \\ &= 4\pi r_0^2 S A^{2/3} + \frac{3}{5}\frac{Z^2 e^2}{4\pi\epsilon_0 r_0 A^{1/3}} \end{aligned} \tag{10.2}$$

where S is the surface tension of the liquid drop.

Note that E_S tend to restore the spherical shape, while E_C effects to increasing the deformation.

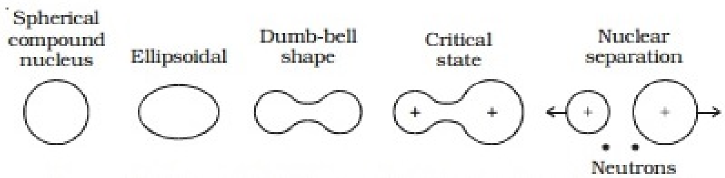

Figure 10.2: Deformed nucleus and fission.

10.2 Light nuclei ($E_C < E_S$)

Let there be two fission fragments of radius $R' = r_0(A/2)^{1/3}$.

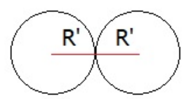

Figure 10.3: Lite nuclei fission fragments.

The energy just after fission is

$$\begin{aligned} E &= 2\left[a_2(A/2)^{2/3} + a_3\frac{(Z/2)^2}{(A/2)^{1/3}}\right] + \frac{(Ze/2)^2}{4\pi\epsilon_0 2R'} \\ &= 2\left[4\pi r_0^2 S(A/2)^{2/3} + \frac{3}{5}\frac{(Ze/2)^2}{4\pi\epsilon_0 r_0(A/2)^{1/3}}\right] + \frac{(Ze/2)^2}{4\pi\epsilon_0 2r_0(A/2)^{1/3}} \end{aligned} \tag{10.3}$$

So that the critical energy of the deformed nucleus is

$$\Delta E_{cr} = E - E_0 \tag{10.4}$$

Let

$$y = \frac{\Delta E_{cr}}{4\pi r_0^2 S A^{2/3}}, \quad x = (Z^2/A)\frac{3e^2}{4\pi\epsilon_0 \times 40\pi r_0^3 S} \tag{10.5}$$

$$(10.4) \to y = 0.26 - 0.215x \tag{10.6}$$

The critical energy of deformation causing fission is depend on Z^2/A for light nuclei.

10.3 Heavy nuclei ($E_C > E_S$)

Description of the deformed sphere, with axial symmetry, is

$$R(\theta) = R\sum_{l=0}^{\infty} \alpha_l P_l(\cos\theta) = R + \alpha_2 P_2(\cos\theta) + \alpha_3 P_3(\cos\theta) + ... \quad (10.7)$$

where $P_l(x)$ is Legendre polynomial. The zeroth order is a sphere, while the first order is displacement and is ignored from (10.7). For $l > 1$ they are multipole deformations.
The surface energy of the deformed nucleus is

$$\begin{aligned} E_{S0} &= 4\pi R^2 S = 4\pi r_0^2 S A^{2/3} && (10.8) \\ E_S &= 4\pi R^2(\theta) S \\ &= 4\pi r_0^2 S A^{2/3} \left[1 + \alpha_2 P_2(\cos\theta) + \alpha_3 P_3(\cos\theta) + ...\right]^2 \\ &= 4\pi r_0^2 S A^{2/3} \Big[1 + \frac{\alpha_2}{2}\left(3\cos^2\theta - 1\right) + \\ &\qquad + \frac{\alpha_3}{2}\left(5\cos^3\theta - 3\cos\theta\right) + ...\Big]^2 \\ &= 4\pi r_0^2 S A^{2/3}\left[1 + \frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + ...\right] && (10.9) \end{aligned}$$

Therefore

$$\begin{aligned} \Delta E_{S,cr} &= E_S - E_{S0} = 4\pi r_0^2 S A^{2/3}\left[\frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + ...\right] && (10.10) \\ &= E_{S0}\left[\frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + ...\right] && (10.11) \end{aligned}$$

The Coulomb energy of the deformed nucleus is

$$\begin{aligned} E_{C0} &= \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 R} = \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}} && (10.12) \\ E_C &= \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 R(\theta)} \\ &= \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}}\left[1 + \frac{\alpha_2}{2}\left(3\cos^2\theta - 1\right) + ...\right]^{-1} \\ &= \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}}\left[1 - \frac{1}{5}\alpha_2^2 - \frac{10}{49}\alpha_3^2 + ...\right] && (10.13) \end{aligned}$$

Therefore

$$\Delta E_{C,cr} = E_C - E_{C0} = \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}}\left[-\frac{1}{5}\alpha_2^2 - \frac{10}{49}\alpha_3^2 + ...\right] \quad (10.14)$$

$$= -E_{C0}\left[\frac{1}{5}\alpha_2^2 + \frac{10}{49}\alpha_3^2 + ...\right] \quad (10.15)$$

Finally we get

$$\Delta E_{cr} = \Delta_{S,cr} + \Delta E_{C,cr} = \frac{1}{5}\alpha_2^2(2E_{S0} - E_{C0}) \quad (10.16)$$

$$= \frac{\alpha_2^2}{5}\left(2a_2A^{2/3} - a_3\frac{Z^2}{A^{1/3}}\right) \quad (10.17)$$

$$< 0, \text{ for large } Z \quad (10.18)$$

Note that for $\Delta E_{cr} > 0$ the drop is stable, while for $\Delta E_{cr} < 0$ the drop is unstable and leads to *spontaneous fission*. This refers to a *critical parameter* of spontaneous fission

$$\frac{Z^2}{A} > \frac{2a_2}{a_3} \simeq 50 \quad (10.19)$$

10.4 Fission fragments

The distribution of yield of fission products of charge Z is *generally assumed* to be gaussian for each fragment mass A and is given by

$$P(Z) = \frac{1}{\sqrt{c\pi}}\exp\left[-\frac{(Z-Zp)^2}{c}\right] \quad (10.20)$$

where c is proportional constant and Z_p is fragment charge. It was found that $c = 0.86$ fits a goos deal to the observed data.

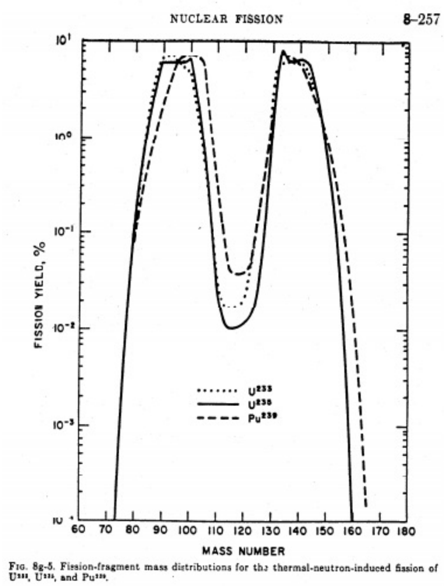

Figure 10.4: Mass distribution of fission fragments.