Lecture 14 Two-Particle Scatterings, Particle Accelerators and Detectors

ICPY473 Nuclear Physics, MUIC, 3-Trimester, 2020-21

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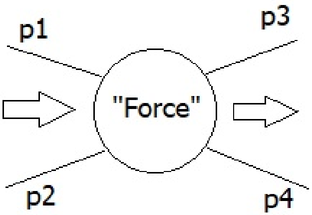
Today Topics

- Continue studying relativistic kinematics of two particle scattering
- Physics of particle accelerators and detectors

14.1 Two-Particle Scattering

- Diagram of 2-to-2 particle scattering
- Energy-momentum conservation

$$(p_1 + p_2)^\mu = (p_3 + p_4)^\mu$$



- Mandelstam variables

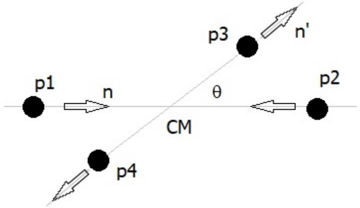
$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \quad (1)$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \quad (2)$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \quad (3)$$

Exercise: Shows that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

- In CM-frame, we have

$$p_1^\mu = (E_1, p\hat{n}),\ \ p_2^\mu = (E_2, -p\hat{n}) \tag{4}$$
$$p_3^\mu = (E_3, p'\hat{n}'),\ \ p_4^\mu = (E_4, -p'\hat{n}') \tag{5}$$

where $\hat{n}, \hat{n}'$ are directional unit vectors, i.e., $\hat{n} \cdot \hat{n}' = \cos\theta$.



- The Mandelstam variables become

$$s = (E_1 + E_2)^2 \rightarrow \ \textit{scattering energy} \tag{6}$$
$$t = m_1^2 + m_3^2 - 2E_1E_3 + pp'\cos\theta \tag{7}$$
$$u = m_1^2 + m_4^2 - 2E_1E_4 - pp'\cos\theta \tag{8}$$

- In case of elastic scattering, i.e., $p = p'$, $E_1 = E_3, m_1 = m_3$, we have

$$t = 2m_1^2 - 2E_1^2 + 2p^2 \cos\theta = -2p^2(1 - \cos\theta) \tag{9}$$

$$dt = 2p^2 d\cos\theta = \frac{1}{\pi} p^2 d\Omega \rightarrow scattering\ angle \tag{10}$$

- Scattering cross section

$$d\sigma = \frac{scattering\ rate\ into\ solid\ angle\ d\Omega}{incident\ particle\ flux} = \frac{W_{fi}(d\Omega)}{F} \tag{11}$$

The incident particle flux F is determined as

$$\begin{aligned} F &= |\psi_1|^2|\psi_2|^2 v_{12} = 2E_1 2E_2(v_1 + v_2) \\ &= 4E_1E_2\left(\frac{|p_1|}{E_1} + \frac{|p_2|}{E_2}\right) = 4(|p_1|E_2 + |p_2|E_1) \end{aligned} \tag{12}$$

In the CM-frame

$$|p_1| = |p_2| = p \rightarrow F = 4|p|(E_1 + E_2) = 4|p|\sqrt{s}$$

- From Lorentz covariant Fermi's golden rule of 2-to-2 particle transition, we have

$$W_{12\to34} = \int |M_{fi}|^2(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4) \times \underbrace{\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}}_{LIPS} \tag{13}$$

with the fact that $(2\pi)^4\delta^{(4)}(\{p\}) = \delta(\{E\})\delta^{(3)}(\{\vec{p}\})$ is Lorentz invariant 4-dimensional delta function.

- From (11), we have

$$d\sigma = \frac{1}{4|p|\sqrt{s}}\int |M_{fi}|^2 2(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4) \times \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4} \tag{14}$$

- In the CM frame, we have

$$d\sigma = \frac{1}{16\pi^2|p|\sqrt{s}} \int |M_{fi}|^2 \delta(\sqrt{s} - E_3 - E_4) \times \delta^{(3)}(\vec{p}_3 + \vec{p}_4) \frac{d^3\vec{p}_3}{2E_3} \frac{d^3\vec{p}_4}{2E_4} \quad (15)$$

$$= \frac{1}{64\pi^2|p|\sqrt{s}} \int |M_{fi}|^2 \delta\left(\sqrt{s} - \sqrt{p_3^2 + m_3^2} - \sqrt{p_3^2 + m_4^2}\right) \frac{d^3\vec{p}_3}{E_3 E_4} \quad (16)$$

$$d^3\vec{p}_3 = p_3^2 dp_3 d\Omega \quad (17)$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\sqrt{s}|p|} \int |M_{fi}|^2 \delta(f(p_3)) g(p_3) dp_3 \quad (18)$$

$$\text{where } f(p_3) = \sqrt{s} - \sqrt{p_3^2 + m_3^2} - \sqrt{p_3^2 + m_4^2} \quad (19)$$

$$g(p_3) = \frac{p_3^2}{E_3 E_4} = \frac{p_3^2}{\sqrt{p_3^2 + m_3^2}\sqrt{p_3^2 + m_4^2}} \quad (20)$$

- Let $f(p') = 0$, this leads to

$$\sqrt{s} = \sqrt{p'^2 + m_3^2} + \sqrt{p'^2 + m_4^2} \quad (21)$$

$$s = 2p'^2 + m_3 + m_4 + 2\sqrt{(p'^2 + m_3^2)(p'^2 + m_4^2)}$$

$$\rightarrow (s - m_3^2 - m_4^2)^2 + 4p'^4 - 4p'(s - m_3 - m_4)^2$$

$$= 4(p'^2 + m_3^2)(p'^2 + m_4^2)$$

$$\rightarrow (s - m_3^2 - m_4^2)^2 - 4p'^2(s - m_3 - m_4)^2$$

$$= 4p'^2(m_3^2 + m_4^2) + m_3^2 m_4^2 \quad (22)$$

$$\rightarrow |p'| = \frac{1}{2\sqrt{s}}\sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2} = \frac{\lambda^{1/2}(s, m_3^2, m_4^2)}{2\sqrt{s}} \quad (23)$$

- Using identity

$$\delta(f(p_3)) = \frac{\delta(p_3 - p')}{|f'(p')|}, \text{ with } f'(p') = |p'|\frac{E_3 + E_4}{E_3 E_4} = \frac{|p'|\sqrt{s}}{E_3 E_4} \quad (24)$$

- From (18), integrate dp_3 using delta function, we have

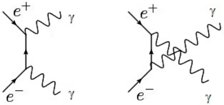
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|p'|}{|p|}|M_{fi}|^2 \tag{25}$$

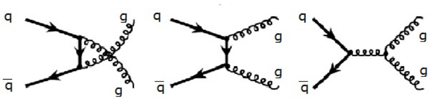
$$\sigma_{tot} = \frac{1}{64\pi^2 s}\frac{|p'|}{|p|}\int_0^{4\pi}|M_{fi}|^2 d\Omega \tag{26}$$

- From PDG ($\alpha = q^2/\hbar c$)

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\frac{u^2+t^2}{tu}$$

$$\frac{d\sigma}{d\Omega}(q\bar{q} \to gg) = \frac{8\alpha_s^2}{27s}(t^2+u^2)\left(\frac{1}{tu} - \frac{9}{4s^2}\right)$$





14.2 Particle Accelerators

- Equation of motion of particle of charge q, mass m, under the action of *Lorentz force* is

$$m\dot{\vec{v}} = q\vec{E} + q\vec{v} \times \vec{B} \tag{27}$$

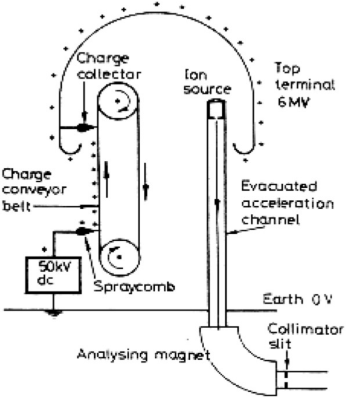
A particle is accelerated by electric field with power consumption rate of $q\vec{E} \cdot \vec{v}$. But it is changed direction by the magnetic field, with the rate of change of direction

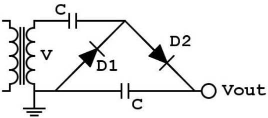
$$\omega_c = \frac{qB}{m} \ rad/s$$

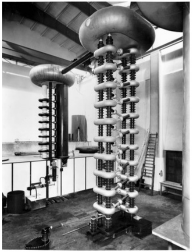
- Energy loss rate of charged particle under acceleration is

$$P = \frac{2}{3}\frac{q^2}{c^3}\gamma^4(a_\perp^2 + \gamma^2 a_{||}^2) \tag{28}$$

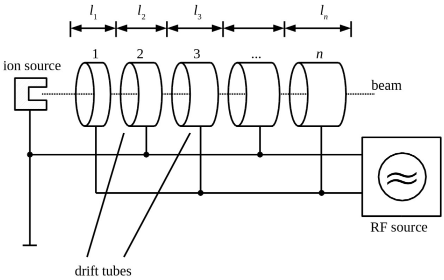
- Static linear accelerators
  - van de graaff
  - Walton-Cockroft





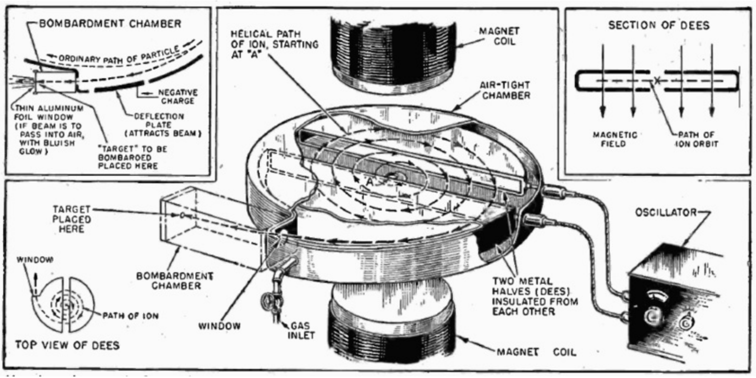


- Synchronous linear accelerators



SLAC at Standford Uni.

- Synchronous circular accelerators, betatron or cyclotron



- Synchrotron accelerators / radiations (X-rays) source

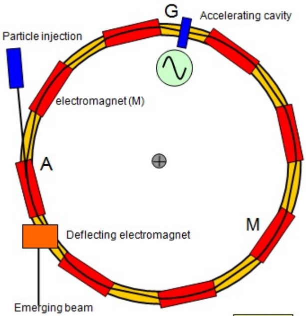
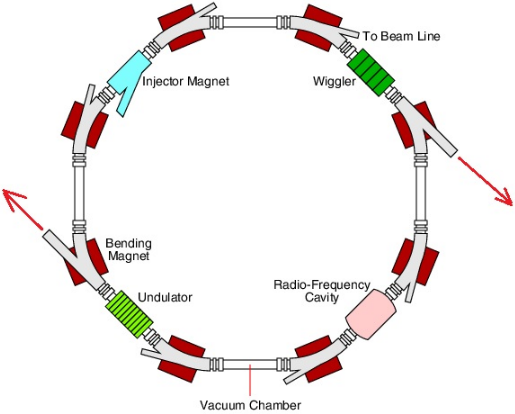
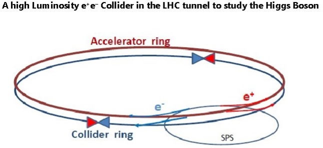
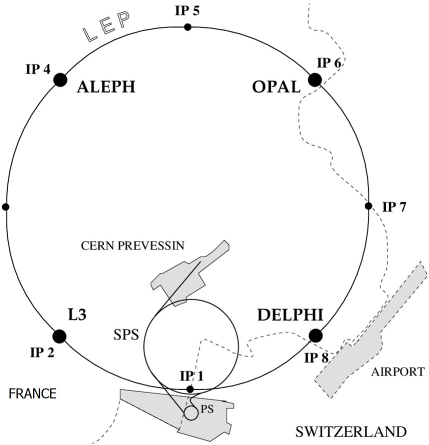


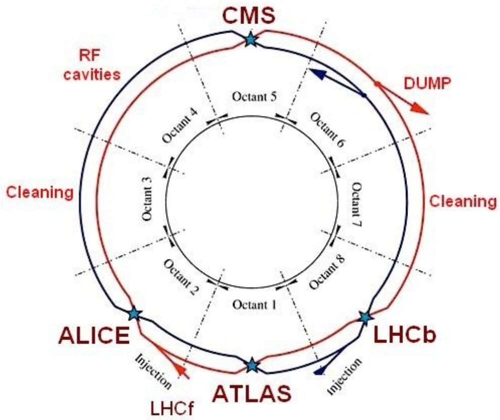
Figure 1

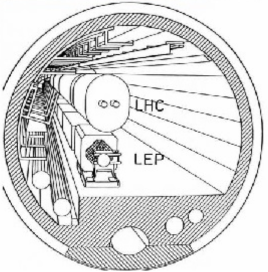


- Colliders, i.e., LEP and LHC at CERN



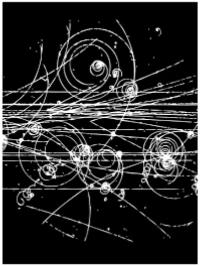


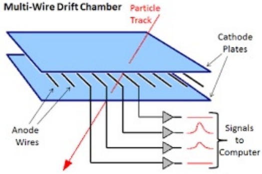

CMS
RF cavities
DUMP
Octant 5
Octant 4
Octant 6
Cleaning
Octant 3
Octant 7
Cleaning
Octant 2
Octant 8
Octant 1
ALICE
LHCb
Injection
Injection
ATLAS
LHCf


LHC
LEP

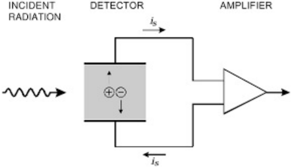
14.3 Particle Detectors

- Interactions of particles with matter, i.e., charged particle, photon, neutral particles
- Type of detectors
  - Tracking
  - Calorimeter
  - Scintillation
- Detector systems
  - Gaseous
  - Drift chamber
  - Solid state
  - Cerenkov

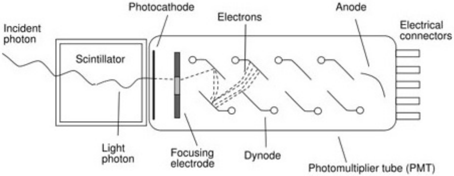




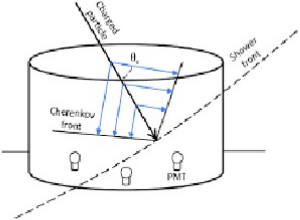
Multi-Wire Drift Chamber
Particle Track
Cathode Plates
Anode Wires
Signals to Computer



INCIDENT RADIATION
DETECTOR
AMPLIFIER
i_s
i_s



Incident photon
Photocathode
Electrons
Anode
Electrical connectors
Scintillator
Light photon
Focusing electrode
Dynode
Photomultiplier tube (PMT)



Charged particle
θ_c
Shower front
Cherenkov front
PMT

- Modern detector, i.e, CMS (Compact Muon Solenoid) detector