Lecture 15 Particles Zoo and Their Classification

ICPY473 Nuclear Physics, MUIC, 3-Trimester, 2021

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Today Topics

- Particle and anti-particle
- Chronological observations
- Particles classification
- Additional quantum numbers
- Unitary symmetry

15.1 Particle and Anti-particle

- Basic NRQM of stationary system

$$\{x, p\}_{Poisson} = 1,\ x \to \hat{x},\ p \to \hat{p} :\ [\hat{x}, \hat{p}] = i\hbar$$

$$H(x,p) = \frac{p^2}{2m} + V(x) = E \to \hat{H}(x,p) = H(\hat{x}, \hat{p}),\ \hat{H}|\psi\rangle = E|\psi\rangle$$

$$\langle x|x'\rangle = \delta(x - x'),\ \langle n|n'\rangle = \delta_{nn'} \to \langle x|n\rangle = \phi_n(x)$$

$$\hat{x}|x\rangle = x|x\rangle,\ \hat{p}|x\rangle = -i\hbar d_x|x\rangle$$

$$\left(-\frac{\hbar^2}{2m}d_x^2 + V(x)\right)\phi_n(x) = E_n\phi_n(x)$$

For non-stationary system $E \to \hat{E} = i\hbar d_t$ and $\phi_n(x) \to \phi(x,t)$

$$\left(-\frac{\hbar^2}{2m}\partial_x^2 + V(x)\right)\phi(x,t) = i\hbar\partial_t\phi(x,t)$$

$$\left(i\hbar\partial_t + \frac{\hbar^2}{2m}\partial_x^2 - V(x)\right)\phi(x,t) = 0 \to (\hat{E} - \frac{\hat{p}^2}{2m} - \hat{V})|\phi\rangle = 0$$

- Extension to RQM

$$E^2 = p^2c^2 + m^2c^4 \rightarrow (E^2 - p^2c^2 - m^2c^4) = 0$$

$$\rightarrow (\hat{E}^2 - \hat{P}^2c^2 - m^2c^4)|\phi\rangle = 0$$

$$-\hbar^2c^2\left(\frac{1}{c^2}\partial_t^2 - \partial_x^2 - \frac{m^2c^2}{\hbar^2}\right)\phi(x,t) = 0$$

We have derived what is called *Klein-Gordon equation*. In the unit in which $c = 1 = \hbar$, it appears in a simple form as

$$(\partial^2 - m^2)\phi(x,t) = 0 \qquad (15.1)$$

with $\partial^2 = \partial_\mu\partial^\mu = \partial_t^2 - \partial_x^2$, where $\partial_\mu = (\partial_t, \partial_x)$ is called 4-derivative , while $\partial^\mu = (\partial_t, -\partial_x)$

- Since ∂^2 is Lorentz scalar, so that $\phi(x,t)$ is also Lorentz scalar function or scalar function. It can be real or complex valued function.

- Let us determine the conserved Klein-Gordon current density

$$
\begin{aligned}
0 &= \phi^*(\partial^2 - m^2)\phi - \phi(\partial^2 - m^2)\phi^* \\
&= \phi^2\partial^2\phi - \phi\partial^2\phi^* \\
&= \partial_\mu\left(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*\right) = \partial_\mu j^\mu = \partial_t j_0 + \partial_x j_x \qquad (15.2) \\
j^\mu &= \phi^*\partial^\mu\phi - \phi\partial^\mu\phi^* = (j_0, j_x) \qquad (15.3) \\
&\to j_0 = \phi^*\partial_t\phi - \phi\partial_t\phi^* \qquad (15.4)
\end{aligned}
$$

We observe that j_0 is not positive definite. So that we cannot interpret ϕ as a usual quantum probability density amplitude. At first time KG equation is abandoned, but later it was re-interpreted in the meaning of particle field. Its quantum behavior need the method of *second quantization*, which is determined from its Fourier expansion

$$
\phi(x,t) = \int \frac{d^4k}{(2\pi)^4}\left(a(k)e^{-ik\cdot x} + b^\dagger(k)e^{ik\cdot x}\right) \qquad (15.5)
$$

- Second quantization

$$\phi(x,t) \to \hat{\phi}(x,t) \tag{15.6}$$

$$a(k) \to \hat{a}(k), \ [a(k), a^\dagger(k')] = \delta(k-k') \tag{15.7}$$

$$b(k) \to \hat{b}(k), \ [b(k), b^\dagger(k')] = \delta(k-k') \tag{15.8}$$

$$a(k) = b(k) \to \phi^\dagger = \phi \to \ \textit{real scalar} \tag{15.9}$$

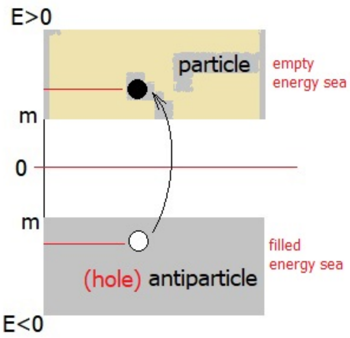
Now j_0 is interpreted to be charge density of scalar particle, with a total particle charge

$$Q = \int d^3x j_0 \to \pm|Q|$$

- Back to the expression of particle energy

$$E^2 = p^2 + m^2 \to E = \pm\sqrt{p^2+m^2}$$

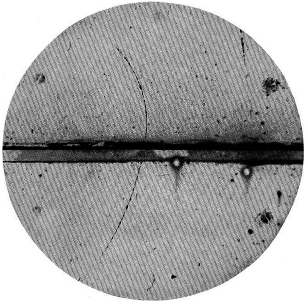
We face with a problem of negative energy. According to Dirac, the negative energy particle is interpreted as an *anti-particle*, which exits in negative energy sea.



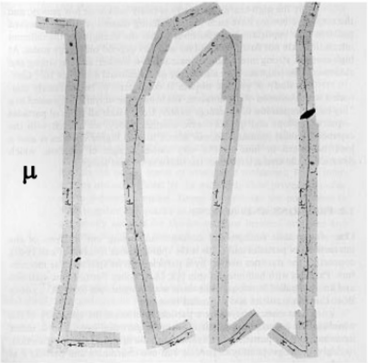
- Particle/antiparticle can exit in relativistic theory, with positive/negative energy.
- Note that particle/antiparticle always created in pair, but we have asymmetry of particle/antiparticle appearance in nature. This still be a big problem in particle physics theory.
- Particle/antiparticle also relate to positive/negative charge, depends on first assignment.

15.2 Chronological Particle Observations

- Electron was first known J.J. Thomson in studying structure of matter, proton was known by E. Rutherford in studying structure of atom and neutron was known by J. Chadwick to complete the constituent of atomic nucleus.
- The other are discovered in particle regime. These are major discovering
  - 1932, the positron, predicted by P. Dirac, was discovered by Carl Anderson

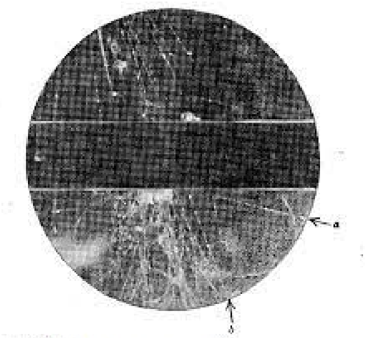


- Continue major discovering
  - 1937, the muon was discovered by S. Neddermeyer, C. Anderson, J. Street, and E. Stevenson
  - 1947, the pion, predicted by H. Yukawa, was discovered by C. Powell



$$\pi \rightarrow \mu\nu$$
$$\mu \rightarrow e\nu\nu$$

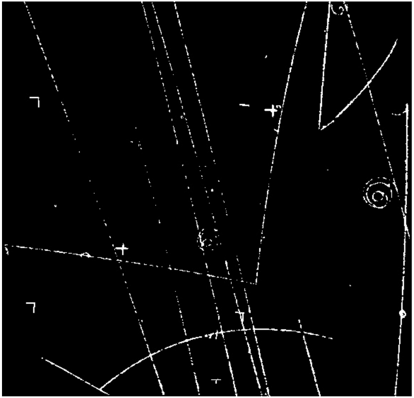
- Continue major discovering
  - 1947, the K-meson (kaon), the first strange particle, was discovered by G. Rochester and C. Butler

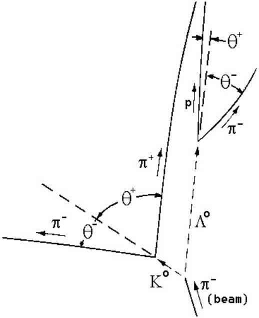


$K^0 \to \pi^+ + \pi^-$

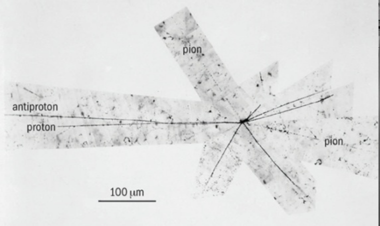
  - 1950, the Lambda baryon Λ was discovered by

- Continue major discovering
  - 1950, the Lambda baryon Λ was discovered at Berkeley Bevatron Facility



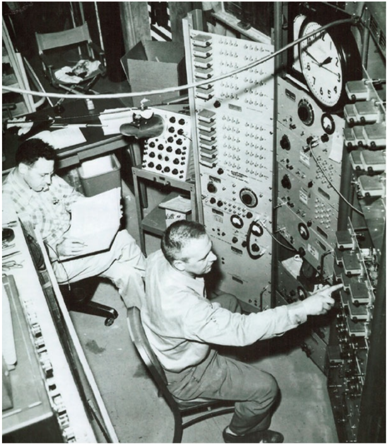


- Continue major discovering
    - 1955, the antiproton p^- was discovered by O. Chamberlain, E. Serge, C. Wiegand, and T, Ypsilantis at Berkeley Bevatron Facility

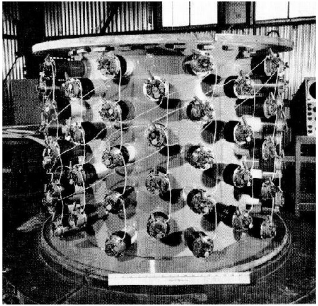


(Serge, Wiegand, (Lofgren), Chamberlain, Ypsilantis)

- Continue major discovering
  - 1956, the electron neutrino ν_e, proposed by W. Pauli in 1930, was discovered by F. Reines and C. Cowan

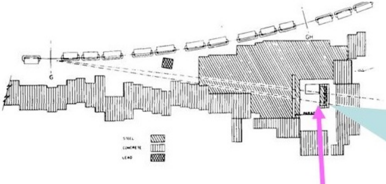


(Reines, Cowan)

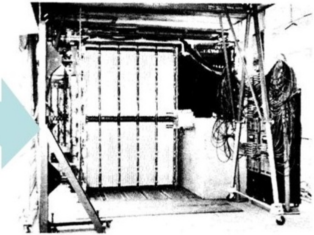


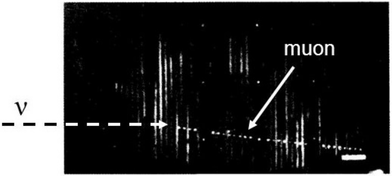
The Savannah River neutrinos detector of Reines and Cowan 1956

- Continue major discovering
  - 1962, the muon neutrino ν_μ, was discovered by L. Lederman, M. Schwartz, and J. Steinberger, done at Brookhaven NL



Neutrino detector



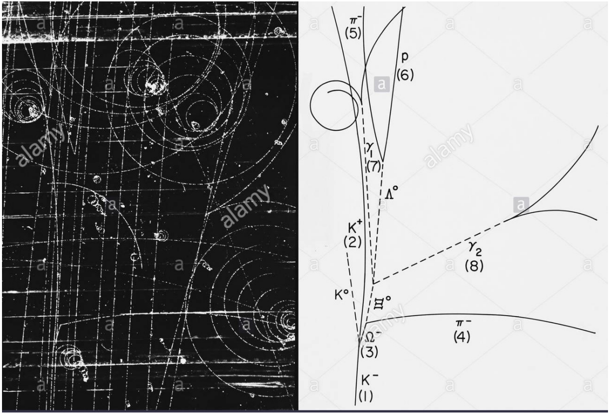


Observed event

G.Danby et al.,

Phys. Rev. Lett. 9 (1962) 36.

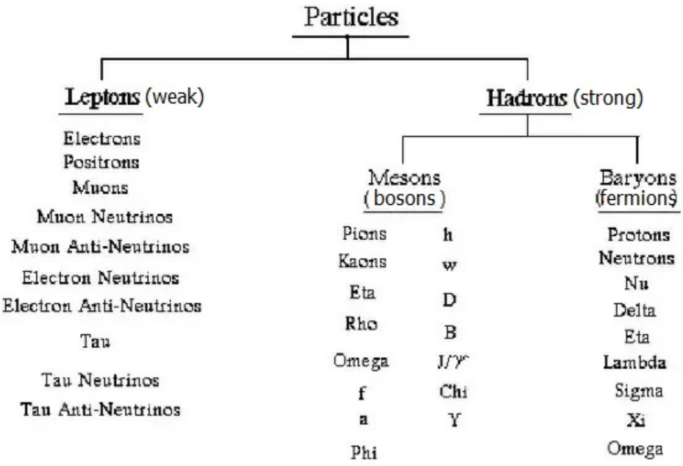
- Continue major discovering
    - 1964, the xi baryon Ξ, another strange baryon, was discovered at Brookhaven NL



- I will stop my list at this time, since after this it will begin the era of quark model. I will continue my list after stating the idea of quark model.

15.3 Particles Classification

- Particles are classified by their a) interactions, and b) spin. Additional quantum numbers are assigned in order to protect their elementary property.



15.4 Additional Quantum Numbers

- Additional quantum numbers are related to group of particles of degenerated *mass* and *spin*.
- A good example is *isospin* (I, I_3). This quantum number has the same structure as angular momentum quantum number, since particles are defined with state vectors from to be a basis set of symmetry space. This symmetry is determined as the invariant under rotation, which is generated by the angular momentum. So these state vectors are represented in terms of basis vector of the angular momentum operator $L^2|l,m\rangle = l(l+1)|l,m\rangle, L_3|l,m\rangle = m|l,m\rangle$, $(\hbar = 1)$.
- For example, $|N\rangle| = |I = \frac{1}{2}, I_3 = \pm\frac{1}{2}\rangle$, N=(p,n)

$$|p\rangle = |1/2, 1/2\rangle, \ |n\rangle = |1/2, -1/2\rangle$$

- For example $|Pion\rangle = |I = 1, I_3 = 0, \pm 1\rangle$, Pion $= (\pi^0, \pi^\pm)$

- Additional quantum numbers
  - Baryon quantum number $B = 1$ is assigned for all baryons, and $B = -1$ for their anti particles
  - Strangeness $S = -1$ is assigned for strange particles ($S = +1$ for their anti-particles), and $S = -2$ for more strange particles, and $S = -3$ for more and more strange particles

Particle	Symbol	Antiparticle	Baryon Number	Strangeness Number	Mass (MeV/C²)
Proton	p	$\bar{p}$	1	0	938.3
Neutron	n	$\bar{n}$	1	0	939.6
Sigma	Σ^+	Σ^-	1	-1	1189
	Σ^0	Σ^0	1	-1	1193
	Σ^-	Σ^+	1	-1	1197
Xi	Ξ^0	Ξ^0	1	-2	1315
	Ξ^-	Ξ^+	1	-2	1321
Lambda	Λ^0	$\overline{\Lambda^0}$	1	-1	1116
Omega *	Ω^-	Ω^+	1	-3	1672

- All additional quantum numbers of baryons are collected to be the *hypercharge* Y, since they are related to charge $Q(e)$ of baryons through *Gell-Mann-Nishijina formula*

$$Q = I_3 + \frac{1}{2}Y, \quad Y = B + S + \ldots \tag{15.10}$$

$Q(p) = +1, \; Q(n) = 0.$

- Charges of strange mesons (Δ) and baryons (Σ, Ξ, Ω)

	q	S		q	S
Δ^-	-1	0	Σ^{*0}	0	-1
Δ^0	0	0	Σ^{*+}	$+1$	-1
Δ^+	$+1$	0	Ξ^{*-}	-1	-2
Δ^{++}	$+2$	0	Ξ^{*0}	0	-2
Σ^{*-}	-1	-1	Ω^-	-1	-3

15.5 Unitary Symmetry

- Symmetry is determined from transformation, the transformation is a symmetry transformation when its generator T commute with the Hamiltonian H

$$H|E\rangle = E|E\rangle, \;\; T|E\rangle = |E'\rangle$$

$$\rightarrow T\{H|E\rangle\} = T\{E|E\rangle\} = E\{T|E\rangle\} = E|E'\rangle$$

$$TH = HT \rightarrow H\{T|E\rangle\} = H|E'\rangle = E|E'\rangle \rightarrow E' = E$$

$$TH = HT \rightarrow TH - HT = [T, H] = 0$$

$$\langle E'|E'\rangle = \langle E|T^\dagger T|E\rangle = \langle E|E\rangle \rightarrow T^\dagger T = 1, \;\; T = e^{i\epsilon t}$$

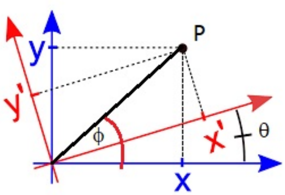
  T is said to be unitary operators, and t is hermitian, results to its unitary transformation and unitary symmetry.

- How can we determine the generator of any transformation?

- Let me see some examples of *infinitesimal transformation* ($\hbar = 1$)
  - **Translation** in x-direction

$$
\begin{aligned}
|x\rangle \to |x'\rangle = |x+a\rangle = |x\rangle + ad_x|x\rangle + \ldots \\
= |x\rangle - ia(-id_x)|x\rangle + \ldots = (1 - iaP_x + \ldots)|x\rangle \\
= w^{-iaP_x}|x\rangle \qquad (15.11)
\end{aligned}
$$

    The generator is P_x, i.e., $H = p^2/2m \to [P_x, H] = 0$, free particle is translation invariant.
  - **Rotation** on (x,y) plane



$x' = x\cos\theta + y\sin\theta \simeq x + \theta y, y' = y\cos\theta - x\sin\theta \simeq y - \theta x$

- Continue examples
  - Continue **rotation**

$$\begin{aligned} |x,y\rangle \to |x',y'\rangle &= |x+\theta y, y-\theta x\rangle \\ = |x,y\rangle + \theta y d_x|x,y\rangle - \theta x d_y|x,y\rangle + ... \\ = |x,y\rangle + i\theta[x(-id_y) - y(-id_x)]|x,y\rangle + ... \\ = (1 + i\theta(xP_y - yP_x) + ...)\,|x,y\rangle \\ = (1 + i\theta L_z + ...)|x,y\rangle = e^{i\theta L_z}|x,y\rangle \end{aligned} \qquad (15.12)$$

Note that L_z is a generator of rotation on xy-plane or around the z-axis.

  - **Unitary** transformation with $u = e^{i\alpha t}$, where α is a real parameter and t is hermitian generator.
    - U(1) transformation of $|\phi\rangle$, where ϕ is complex scalar quantity

$$t = 1 \to u = e^{i\alpha} \to |\phi'\rangle = e^{i\alpha}|\phi\rangle,\ \langle\phi'|\phi'\rangle = \langle\phi|\phi\rangle \quad (15.13)$$

    - SU(2) transformation of $|\psi\rangle$, where ψ is spinorial quantity

$$|\psi\rangle \equiv \begin{pmatrix} \chi \\ \eta \end{pmatrix} \to t = 2x2\ matrix \qquad (15.14)$$

- Continue example
  - Continue unitary
    - Continue SU(2) transformation

$$\alpha t = \alpha^a t^a, \; a = 1, 2, 3 = 2^2 - 1, \quad t^a = \frac{1}{2}\sigma^a \qquad (15.15)$$

$$\{\sigma^a\} - \mathit{Pauli's\ matrices}: \; \sigma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad (15.16)$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (15.17)$$

This means that SU(2) is just complex rotation on 2d-plane, i.e., $|nucleon\rangle$ is a spinor

$$\sigma^1|p\rangle = -|n\rangle, \; \sigma^1|n\rangle = -|p\rangle$$

$$\sigma^2|p\rangle = i|n\rangle, \sigma^2|n\rangle = -i|p\rangle$$

$$\sigma^3|p\rangle = |p\rangle, \; \sigma^3|n\rangle = -|n\rangle$$

So that nucleon isospin symmetry is SU(2) symmetry.

- Continue example
  - Continue unitary
    - SU(3) transformation of $|\Psi\rangle$

$$|\Psi\rangle = \begin{pmatrix} \lambda \\ \rho \\ \eta \end{pmatrix} \rightarrow t = 3x3 \ matrix \quad (15.18)$$

$$\alpha t = \alpha^{a} t^{a}, \ a = 1, 2, 3, ..., 8 = 3^{2} - 1, \ t^{1} = \frac{1}{2}\lambda^{a} \quad (15.19)$$

where $\{\lambda^{a}$ is a set of *Gell-Mann matrices*. The isospin symmetry of $|Pion\rangle$ is SU(3).

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$