Lecture 17 Quark Models of Hadrons

ICPY473 Nuclear Physics, MUIC, 3-Trimester, 2021

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Today Topics

- Symmetry group algebra
- su(2) models
- su(3) models
- Quark colors

Symmetry group algebra

- We first determine symmetry group algebra. Using example of SO(3) group, a rotational group of generators $\{L_x, L_y, L_z\}$. This group underline rotational symmetry in three dimensions. The so(3) algebra is

$$[L_x, l_y] = i\hbar L_z,\ [L_y, L_z] = i\hbar L_x,\ [L_z, L_x] = i\hbar L_y$$

$$\to L^2 = L_x^2 + L_y^2 + L_z^2,\ L^\pm = L_x \pm iL_y$$

$$[L^2, L_i] = 0, (i = x, y, z)\ [L_z, L^\pm] = \pm L^\pm,\ [L^+, L^-] = 2\hbar L_z$$

$$L^2|l, m\rangle = l(l+1)\hbar^2|l, m\rangle,\ L_z|l, m\rangle = m\hbar|l, m\rangle$$

$$L^\pm|l, m\rangle = \hbar\sqrt{l(l+1) - (m \pm 1)}|l, m \pm 1\rangle$$

  where $l = 0, 1, 2, ...$ and $m = -l, -(l-1), ..., (l-1)$, with multiplicity of $2l + 1$.
- For example of $l = 1, m = -1, 0, 1$, and matrix representations

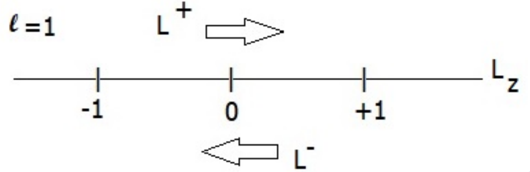
$$L^2 = 2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

- $j = 1$ matrix representations (cont.)

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; L_y = \frac{\hbar}{\sqrt{2}i} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\to L^+ = \sqrt{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \; L^- = \sqrt{2}\hbar \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

- Note that L^2 is called *Casimir operator:* C, since it commutes to all other operators, and L_z has diagonal matrix representation, it represents set of available states for each l. $L^\pm$ are called *ladder operators*.
- Weight diagram, to represent L_z

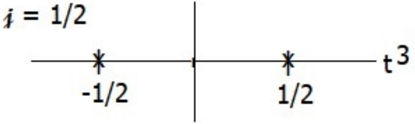


- su(2) algebra

$$t^a = \frac{1}{2}\sigma^a, \;\; a = 1, 2, 3 \qquad (1)$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (2)$$

$$\rightarrow C = t^2 = \frac{3}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; t^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (3)$$



- su(3) algebra

$$t^a = \frac{1}{2}\lambda^a, \;\; a = 1, 2, ..., 8 \qquad (4)$$

$$\lambda^{(1)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^{(2)} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^{(4)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^{(5)} = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^{(6)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^{(7)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^{(8)} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

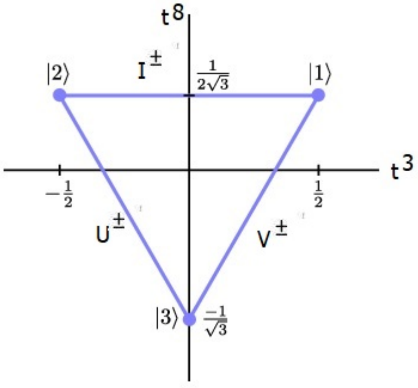
- We observe that su(3) algebra has two diagonal metrices t^3 and t^8. Their diagonal elements compose to be two dimension weight diagram coordinates as

$$(\frac{1}{2}, \frac{1}{2\sqrt{3}}), (-\frac{1}{2}, \frac{1}{2\sqrt{3}}), (0, -\frac{1}{\sqrt{3}})$$

- It has two casimir operators (hard to determine) and has three sets of ladder operators

$$I^{\pm} = t^1 \pm it^2, \ V^{\pm} = t^4 \pm it^5, \ U^{\pm} = t^6 \pm it^7$$

- su(3) weight diagram



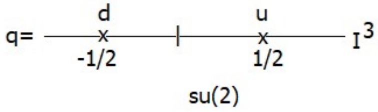
su(2) Quark Models

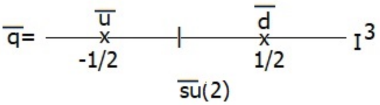
- su(2) representations for the quark model

$$I^3 = t^3 \rightarrow quark\ \ isospin\ doublet\ (u, d)$$

Quarks	M(MeV/c^2)	Q(e)	S	I, I_3	B	S
u	2.32	$+2/3$	1/2	$1/2, 1/2$	1/3	0
d	4.71	-1/3	1/2	$1/2, -1/2$	1/3	0
$\bar{u}$	2.32	-2/3	1/2	$1/, -1/2$	-1/3	0
$\bar{d}$	4.71	$+1/3$	1/2	$1/2, 1/2$	-1/3	0

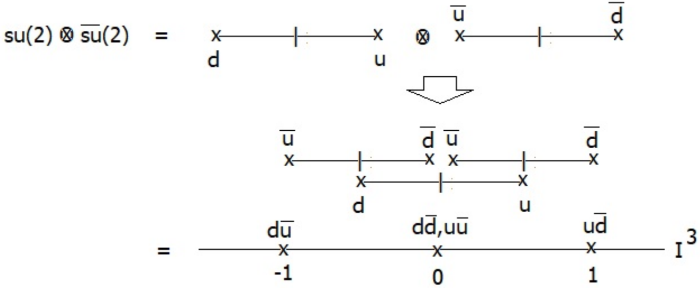
- Weight diagrams





- Meson = quark-antiquark bound state

$$|meson\rangle = q\bar{q} \rightarrow su(2) \otimes \bar{su}(2) = 2 \otimes \bar{2} = 3 \oplus 1$$



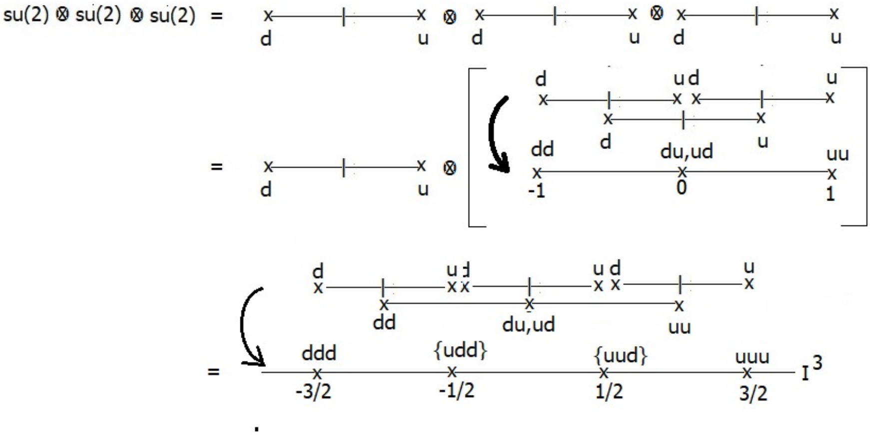
Match to two lightest mesons = pions (isospin = 1)

$$\pi^-(-1) = d\bar{u},\ \pi^0(0) = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),\ \pi^+(+1) = u\bar{d}$$

Pion masses $M(\pi^{\pm} = 139.57 MeV/c^2, M(\pi^0) = 134.98 MeV/c^2$.

- Baryons = three quarks bound state

$$|baryon\rangle = qqq \to su(2) \otimes su(2) \otimes su(2) = 2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$



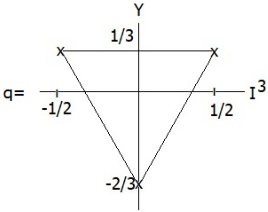
Match to two lightest baryons $p(1/2, +1/2) = uud$, $n(1/2, -1/2) = udd$

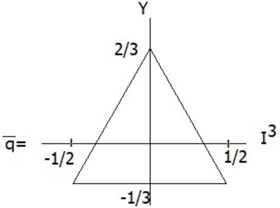
su(3) Quark Models

- su(2) representation

$$(I_3, Y) = (t^3, \frac{2}{\sqrt{3}} t^8) \to \{u, d, s\}$$

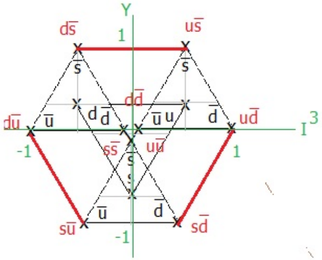
Quark	M (MeV/c^2)	Q(c)	S	Isospin	B	S
s	101	-1/3	1/2	0	1/3	-1
$\bar{s}$	101	+1/3	1/2	0	-1/3	+1

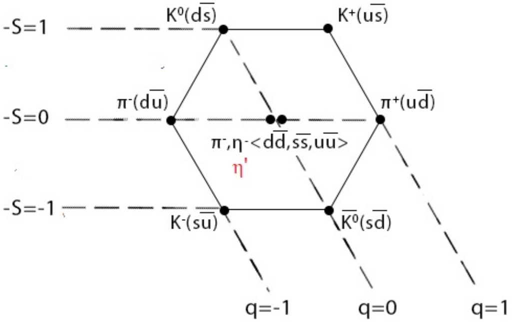




- su(3) mesons

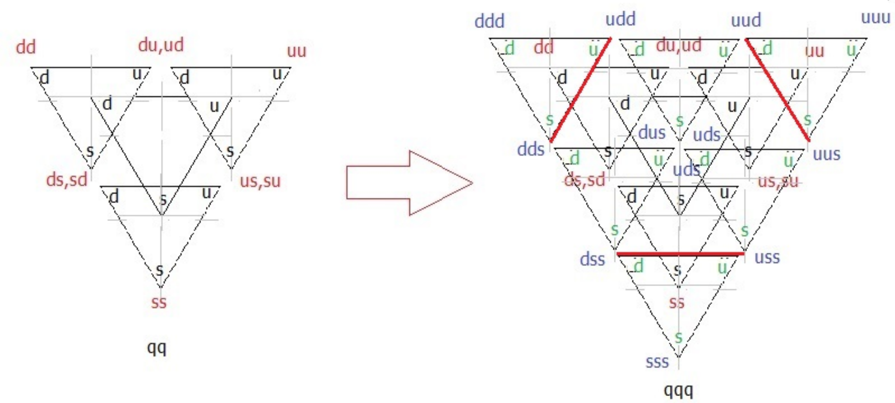
$$meson = q\bar{q} \rightarrow su(3) \otimes \bar{su}(3) = 3 \otimes \bar{3} = 8 \oplus 1$$



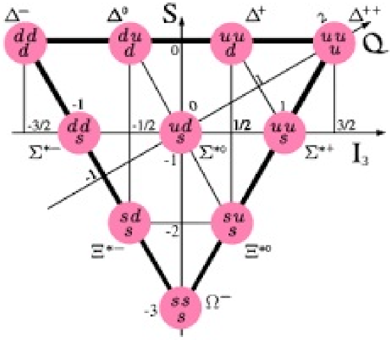


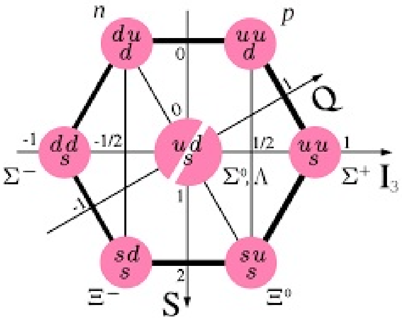
- su(3) baryons

$$baryon = qqq \to su(3) \otimes su(3) \otimes su(3) = 10 \oplus 8 \oplus 8 \oplus 1$$



- su(3) baryons (cont.)





Quark colors

- Color degrees of freedom (R, G, B) of quark was introduced to overcome the Pauli's exclusion of three quark state.
- It was introduced to follow the *color rule* which state "only colorless quark can be observe in nature" (Matter only appear in white.)