Lecture 18 The Standard Model and Fundamental Interactions

ICPY473 Nuclear Physics, MUIC, 3-Trimester, 2021

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Today Topics

- Standard model particles
- Fundamental interactions
- Toy model of particle interaction
- Electromagnetic and weak interactions
- Strong interaction

Standard model particles

Standard model particles = quarks, leptons and gauge bosons
Quarks and leptons appear in three generations and six flavors

Generation	Quarks		Leptons		Anti-Quarks		Anti-Leptons	
	Charge $+2/3$	Charge $-1/3$			Charge $-2/3$	Charge $+1/3$		
1st	up	down	electron	electron neutrino	Anti-up	Anti-down	positron	Anti-electron neutrino
	u	d	e	ν	$\bar{u}$	$\bar{d}$	e^+	$\bar{\nu}$
2nd	charm	strange	muon	muon neutrino	Anti-charm	Anti-strange	Anti-muon	Anti-muon neutrino
	c	s	μ	ν_μ	$\bar{c}$	$\bar{s}$	$\bar{\mu}$	$\bar{\nu}_\mu$
3rd	top	bottom	tau	tau neutrino	Anti-top	Anti-bottom	Anti-tau	Anti-tau neutrino
	t	b	τ	ν_τ	$\bar{t}$	$\bar{b}$	$\bar{\tau}$	$\bar{\nu}_\tau$

Quarks:

Quark	Symbol	Spin	Charge	Baryon Number	S	C	B	T	Mass*
Up	U	1/2	+2/3	1/3	0	0	0	0	1.7-3.3 MeV
Down	D	1/2	-1/3	1/3	0	0	0	0	4.1-5.8 MeV
Charm	C	1/2	+2/3	1/3	0	+1	0	0	1270 MeV
Strange	S	1/2	-1/3	1/3	-1	0	0	0	101 MeV
Top	T	1/2	+2/3	1/3	0	0	0	+1	172 GeV
Bottom	B	1/2	-1/3	1/3	0	0	-1	0	4.19 GeV(MS) 4.67 GeV(1S)

Leptons:

Lepton name		Mass $[MeV/c^2]$	Electric charge $[e]$	Spin (J)	Lepton number (L)	Electron lepton number (L_e)	Muon lepton number (L_μ)	Tau lepton number (L_τ)
electron	(e^-)	0.511	- 1	1/2	+1	+1	0	0
electron neutrino	(ν_e)	$<2\times10^{-6}$	0	1/2	+1	+1	0	0
muon	(μ^-)	105.66	- 1	1/2	+1	0	+1	0
muon neutrino	(ν_μ)	$<2\times10^{-6}$	0	1/2	+1	0	+1	0
tau	(τ^-)	1776.86	- 1	1/2	+1	0	0	+1
tau neutrino	(ν_τ)	$<2\times10^{-6}$	0	1/2	+1	0	0	+1

Gauge bosons

Gauge boson		Spin	Charge	Mass	Force
photon	γ	1	0	0	electromagnetic
W-boson	$W^{\pm}$	1	± 1	$80.4\ GeV$	weak
Z-boson	Z^0	1	0	$91.2\ GeV$	weak
gluons	g	1	0	0	strong
graviton	G	2	0	0	gravitation

Fundamental Interactions

Fundamental forces of nature

Name	Relative Strength	Range	Operates among
Gravitational Force	10^{-39}	Infinite	All objects in universe
Weak nuclear force	10^{-13}	Very short, sub nuclear size (approx. 10^{-16})	Some elementary particle, particularly electrons and neutrino)
Electromagnetic force	10^{-2}	Infinite	Charged particles
Strong nuclear force	1	Short nuclear size (approx. 10^{-5})	Nucleons, heavier elementary particles

Toy model of Yukawa-type interaction

We have learned about KG equation of $\phi(x)$,where $x \equiv x^\mu = (t, \vec{x})$ is spacetime coordinate, of the form

$$(\partial^2 + m^2)\phi(x) = 0, \quad \underbrace{\partial^2}_{d'Alembertian} = \partial_\mu \partial^\mu = \partial_t^2 - \underbrace{\nabla^2}_{Laplacian}$$

With generalized Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \phi^*} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^*} = 0$$

where $\mathcal{L}$ is known as Lagrangian density

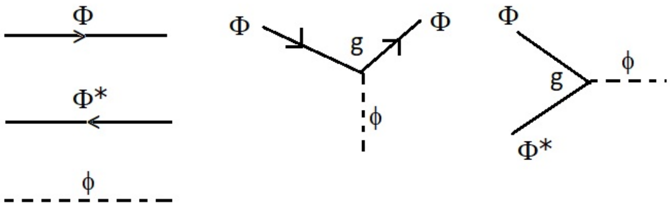
$$L = \int d^3x \mathcal{L}, \ \mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

Note that $\mathcal{L}$ appear in quadratic form, it contains only kinetic term and mass term.

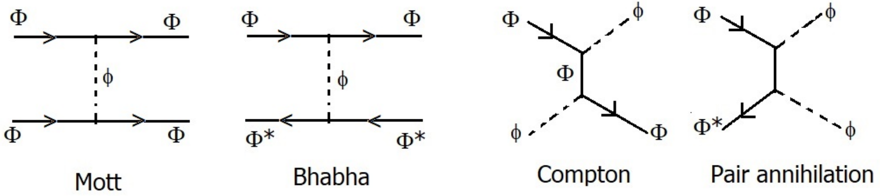
Yukawa-type interaction between massive complex scalar Φ and massless real scalar ϕ, and g is coupling constant (charge)

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - M^2 \Phi^* \Phi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \underbrace{g \phi \Phi^* \Phi}_{Yukawa\ term}$$

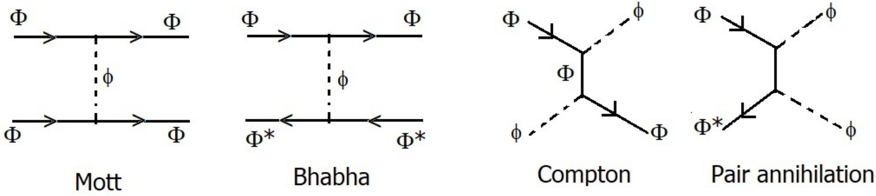
Feynman diagrams



Scattering diagrams



Scattering diagrams



Gauge field = vector field + gauge symmetry
In classical EM we have 4-potential $A^\mu = (\phi, \vec{A})$

$$\vec{B} = \nabla \times \vec{A},\ \vec{E} = -\nabla\phi - \partial_t \vec{A}$$

Its covariant form (field strength tensor) $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$$F^{00} = F^{11} = F^{22} = F^{33}, F^{0i} = \partial^0 A^i - \partial^i \phi = -E^i$$

$$F^{ij} = \partial^i A^j - \partial^j A^i = -\epsilon^{ijk} B^k$$

Gauge symmetry

$$A^{\mu} \to A^{\mu} + \partial^{\mu}\chi, \;\; F^{\mu\nu} \to F^{\mu\nu}$$

Gauge field dynamics

$$\mathcal{L}_{gauge\ \ field} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \xrightarrow[EL\ \ eqn.]{} \partial_{\mu}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) = 0$$

$$\partial_{\mu}A^{\mu} = 0 \;|_{\text{Lorentz gauge}} \to \partial^2 A^{\nu} = 0 \;|_{\text{Maxwell eqn.}}$$

Photon is a gauge boson. Matter coupling gauge field theory

$$\text{Matter; } \mathcal{L}_{matter} = \partial_{\mu}\phi^*\partial^{\mu}\phi - m^2\phi^*\phi$$

$$\text{Unitary trans. } u = e^{i\alpha(x)}, u^{\dagger} = e^{-i\alpha(x)}, \;\; uu^{\dagger} = u^{\dagger}u = 1$$

$$\phi \to \phi' = u\phi, \phi'^* = \phi^* u^{\dagger} \to \phi'^*\phi' = \phi^*\phi$$

$$\partial_{\mu}\phi \to \partial_{\mu}\phi' = \partial_{\mu}(u\phi) = u(\partial_{\mu} + i\partial_{\mu}\alpha)\phi$$

$$\partial_{\mu}\phi^* \to \partial_{\mu}\phi'^* = \left\{(\partial_{\mu} - i\partial_{\mu}\alpha)\phi^*\right\} u^{\dagger}$$

$$\to \partial_{\mu}\phi'^*\partial^{\mu}\phi' \neq \partial_{\mu}\phi^*\partial^{\mu}\phi$$

With the help of gauge field

$$A^\mu \to A'^\mu = A^\mu - \frac{1}{q}\partial_\mu \alpha = u(A_\mu - \frac{i}{q}\partial_\mu)u^\dagger$$
$$\partial_\mu \to D_\mu = \partial_\mu - iqA_\mu$$
$$D_\mu \phi \to D'_\mu \phi' = u(D_\mu \phi)$$
$$= uD_\mu u^\dagger u\phi = (\partial_\mu - i\partial_\mu \alpha - iqA_\mu + i\partial_\mu \alpha)\,\phi'$$
$$= (\partial_\mu - iqA_\mu)\phi' = D_\mu \phi'$$

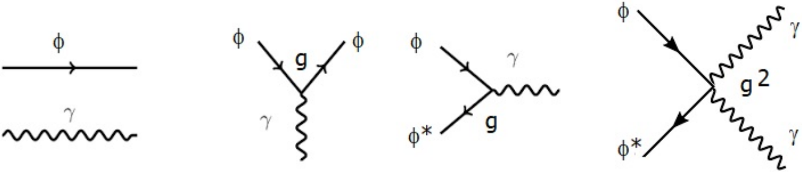
So that

$$(D'_\mu \phi')^*(D'^\mu \phi') = D_\mu \phi)^*(D^\mu \phi)$$

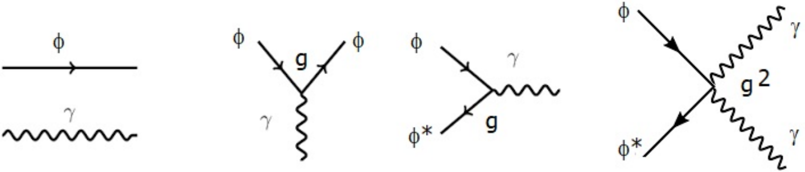
Model Lagrangian (scalar QED)

$$\mathcal{L} = (D_\mu \phi)^*(D^\mu \phi) - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$= \partial_\mu \phi^* \partial^\mu - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - qA_\mu J^\mu + q^2\phi^*\phi A_\mu A^\mu$$
$$J^\mu = i(\phi^*\partial^\mu \phi - \phi\partial^\mu \phi^*)$$

Scalar QED diagrams



Scalar QED diagrams



Quarks and leptons are Dirac particles, represented by Dirac spinors (solution of Dirac equation)

Derivation of Dirac equation (RQM, $c = 1 = \hbar$), we start form linearized relativistic energy-momentum relation

$$E = \vec{\alpha} \cdot \vec{p} + \beta m \xrightarrow[constraint]{} E^2 = p^2 + m^2$$

$$E^2 = \frac{1}{2}(\alpha_i \alpha_j + \alpha_j \alpha_i) p_i p_j + (\alpha_i \beta + \beta \alpha_i) p_i + \beta^2 m$$

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 2\delta_{ij},\ \alpha_i \beta + \beta \alpha_i = 0,\ \beta^2 = 1$$

We observe that $\{\alpha_i\}, \beta$ are not numbers but square matrices. Dirac observe that

$$\alpha_i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}_{4x4}, \ \beta = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}_{4x4}$$

where $\{\sigma^i\}$ is a set of Pauli matrices and 1_2 is 2x2 identity matrix. Quantum equation of Dirac energy is

$$E - \vec{\alpha} \cdot \vec{p} - \beta m = 0 \rightarrow (i\partial_t + i\vec{\alpha} \cdot \nabla - \beta m)\Psi(x) = 0$$

$$\gamma^\mu = (\gamma^0, \vec{\gamma}) \rightarrow \gamma^0 = \beta, \gamma^i = \beta\alpha^i, \ \beta^2 = 1_4$$

$$(i\gamma^\mu \partial_\mu - m 1_4)\Psi = 0, \quad \Psi = \begin{pmatrix} \chi_R \\ \bar{\psi}_L \end{pmatrix}$$

Note that Ψ is bi-spinor, and χ, ψ are spinors. Dirac Lagrangian is

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$$

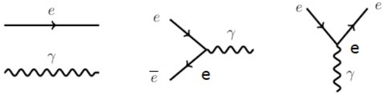
Electromagnetic interactions

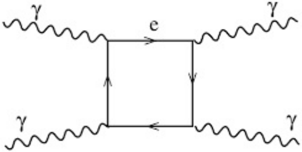
Quantum electrodynamics of QED

$$\mathcal{L} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \mathcal{L}_0 + \mathcal{L}_{int}$$

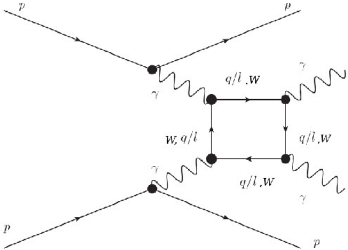
$$\mathcal{L}_{int} = -iq\bar{\psi}\gamma^{\mu}\Psi A_{\mu}$$

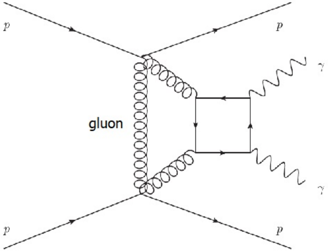
QED diagrams, QED = quantum of electromagnetic interactions, i.e., $\gamma\gamma \to \gamma\gamma$ scattering

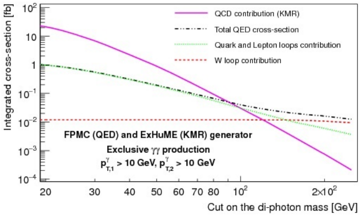




Light-by-light scattering with intact protons at the LHC: from Standard Model to New Physics - Fichet, Sylvain et al - arXiv:1411.6629

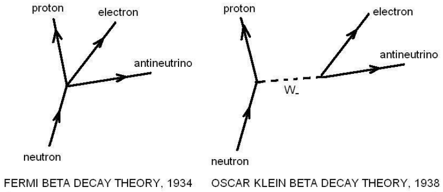




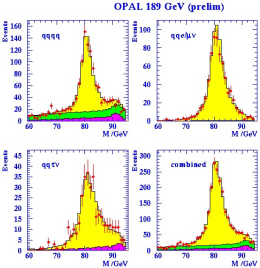


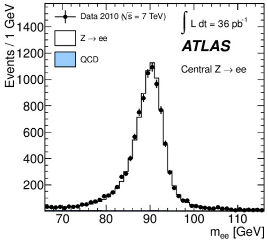
Weak interaction

Fermi's theory of weak interaction



Weak gauge bosons masses at LEP and LHC at CERN

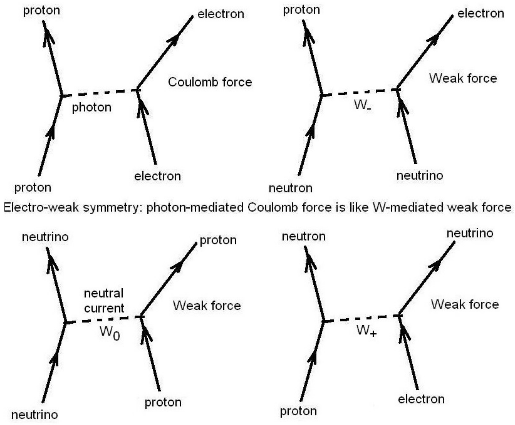




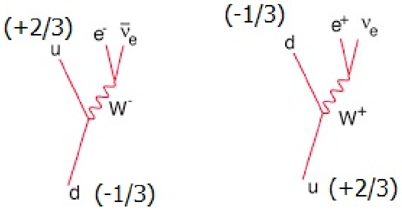
Weak gauage bosons B^μ=Proca particle, with Lagrangian

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}M^2 B_\mu B^\mu,\ G^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

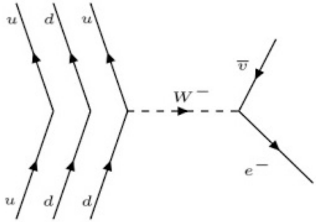
Weak interaction with $W^\pm$ and Z^0 weak gauge bosons



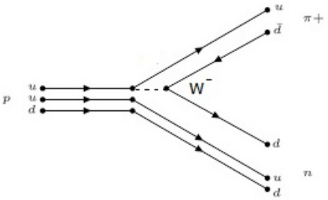
Weak interaction of quarks



Weak decays



n-decay (15 min)



p-decay (10 ^{34} yr.)

Next lecture:

- strong interaction
- meson decay with flavors oscillation