Lecture 21 Discrete PCT Symmetries, CP-Violation and Axions

ICPY473 Nuclear Physics, MUIC, 3-Trimester, 2021

Udom Robkob, Physics-MUSC

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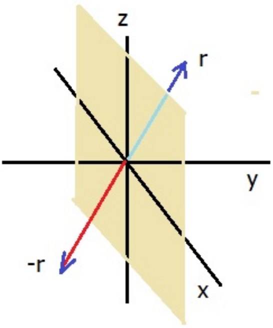
Today Topics

- Discrete PCT symmetries
- CP-violation
- CKM matrix
- Axions

Discrete PCT Symmetries

- Define a quantum mechanical operator O, if O describes a good symmetry *physics will looked the same before and after operation with* O. If not, we say that the symmetry is broken, i.e., the symmetry is not protected by the nature
- **Parity** P, is defined to be a reflection on a mirror plane followed by rotation of π around an axis of the mirror plane.



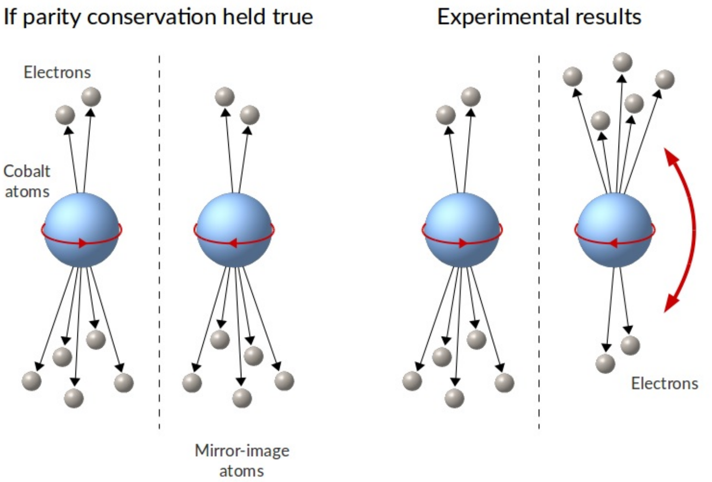
Note that

$$P[\vec{r}] = -\vec{r},\ P[\vec{p}] = -\vec{p},\ P[\vec{L}] = \vec{L}$$

Vector change sign under P, but pseudo-vector do not. Note also that

$$P^2 = 1,\ [P, H_W] \neq 0$$

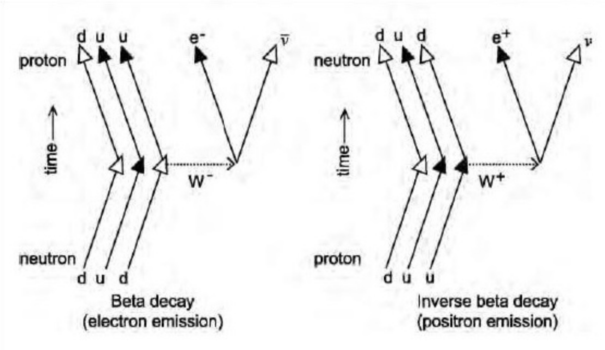
- P-violation in beta decay, C.N. WU (1956)



- **Charge conjugation** C is defined to complex conjugation of the quantum field $\phi \xrightarrow{C} \phi^*$, it changes matter into anti-matter and vice versa.. Note that

$$C[e^-] = e^+,\ C[\gamma] = \gamma,\ C^2 = 1,\ [C, H_W] \neq 0$$

This is matter and anti-matter have different weak interaction.



Note that CP is conserved in beta decay.

- **Time reversal** T is defined to be a reverse all time direction $t \xrightarrow{T} -t$. Note that

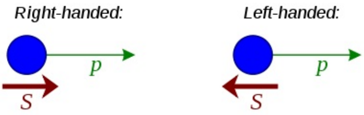
$$T[\vec{r}] = \vec{r}, \;\; T[\vec{p}] = -\vec{p}, \;\; T[\vec{L}] = -\vec{L}, \;\; T^2 = -1$$

  T is an anti-unitary operator.
- **PCT-theorem** Let $\phi(t, \vec{x})$ represent a quantum field particle, so that

$$PCT[\phi(t, \vec{x})] = \phi^*(-t, -\vec{x})$$

  This is PCT transform particle to its anti-particle moving backward in time ans has opposite helicity

$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|^2}$$

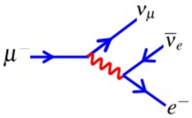


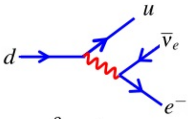
- PCT is conserved in standard model, but PC is violated in K and B mesons decay

Quarks Mixing

Different weak interactions between lepton and quark

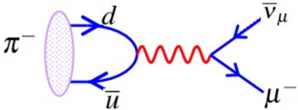
★ Slightly different values of G_F measured in μ decay and nuclear β decay:

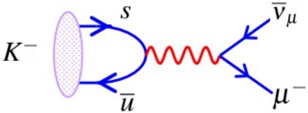




$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

★ In addition, certain hadronic decay modes are observed to be suppressed, e.g. compare $K^- \to \mu^- \overline{\nu}_\mu$ and $\pi^- \to \mu^- \overline{\nu}_\mu$. Kaon decay rate suppressed factor 20 compared to the expectation assuming a universal weak interaction for quarks.

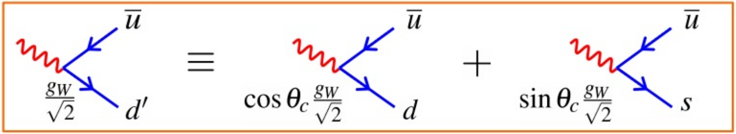




- Both observations explained by Cabibbo hypothesis (1963): weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

i.e. weak interaction couples different generations of quarks



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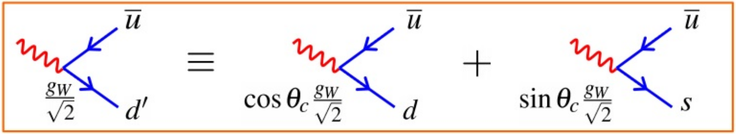
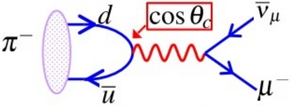
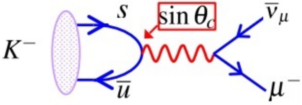


★ Can explain the observations on the previous pages with $\theta_c = 13.1^\circ$

- Kaon decay suppressed by a factor of $\tan^2\theta_c \approx 0.05$ relative to pion decay

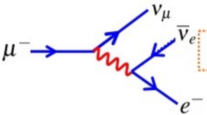




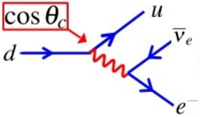
$\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c$

$\Gamma(K^- \to \mu^-\overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$

- Hence expect $G_F^\beta = G_F^\mu \cos\theta_c$



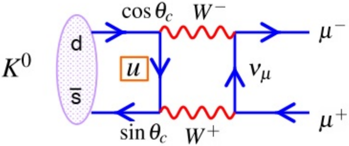
$\propto G_F.G_F$



$\propto G_F.(G_F\cos^2\theta_c)$

GIM mechanism of meson decay

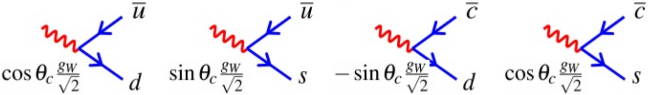
★ In the weak interaction have couplings between both ud and us which implies that neutral mesons can decay via box diagrams, e.g.



$$\boxed{M_1 \propto g_W^4 \cos\theta_c \sin\theta_c}$$

•Historically, the observed branching was much smaller than predicted

★ Led Glashow, Illiopoulos and Maiani to postulate existence of an extra quark - before discovery of charm quark in 1974. Weak interaction couplings become



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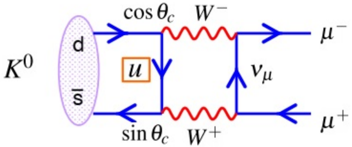

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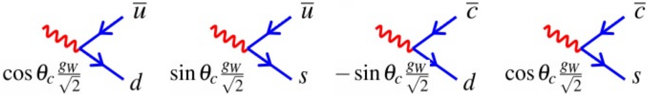
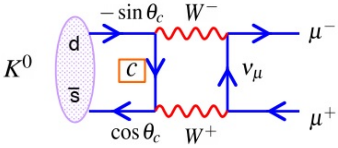


★ Gives another box diagram for $K^0 \to \mu^+\mu^-$



$$\boxed{M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c}$$

- Same final state so sum amplitudes

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

- Cancellation not exact because $m_u \neq m_c$

CKM matrix

★ Extend ideas to three quark flavours (analogue of three flavour neutrino treatment)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$

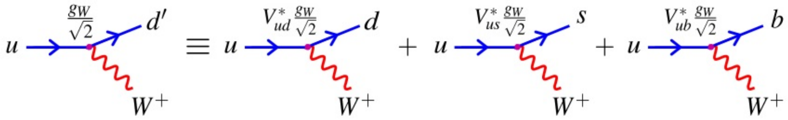
Weak eigenstates

CKM Matrix

Mass Eigenstates

(Cabibbo, Kobayashi, Maskawa)

★ e.g. Weak eigenstate d' is produced in weak decay of an up quark:



- The CKM matrix elements V_{ij} are complex constants
- The CKM matrix is unitary
- The V_{ij} are not predicted by the SM – have to determined from experiment

Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

★ NOTE: within the SM, the charged current, $W^{\pm}$, weak interaction:

① Provides the only way to change flavour !

② only way to change from one generation of quarks or leptons to another !

★ However, the off-diagonal elements of the CKM matrix are relatively small.

- Weak interaction largest between quarks of the same generation.
- Coupling between first and third generation quarks is very small !

★ The number of free parameters in the CKM matrix are three real parameters and one imaginary phase

★The presence of an imaginary phase is source of CP violation!

(From http://www.physik.uzh.ch/de/lehre/PHY213/FS2017.html)