ICPY473 Nuclear Physics
MUIC, Third Trimester 2020-21
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7 Gamma Emissions

7.1 Decay equation

Gamma particle (γ) is energetic photon ($\geq MeV$).
Its emission results from the process of de-excitation of excited nucleus.
The decay equation reads

$$ {}^A_Z X^*_N \to {}^A_Z X_N + \gamma + Q \tag{7.1} $$

Decay energy is calculated as

$$ Q = (M_{X^*} - M_X)c^2 = E_\gamma + K_X \tag{7.2} $$

where K_X is the recoil kinetic energy of X nucleus. In the rest frame of X^* nucleus we will have

$$ p_X = p_\gamma = \frac{E_\gamma}{c} \to K_X = \frac{p_X^2}{2M_X} = \frac{E_\gamma^2}{2M_X c^2} \tag{7.3} $$

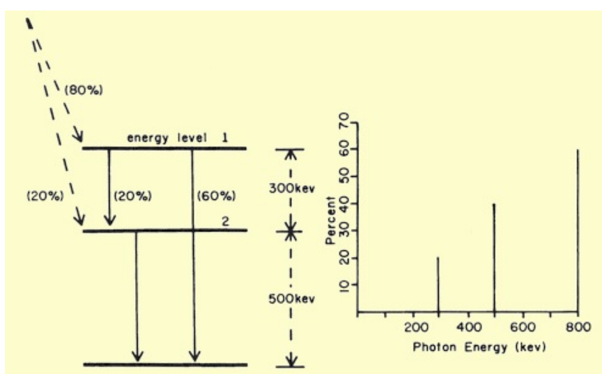

Figure 7.1: Schematic diagram of gamma spectrum.

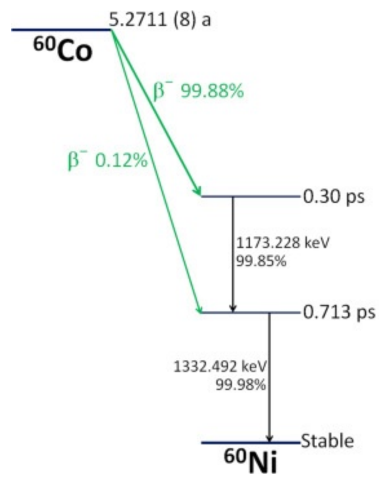

Figure 7.2: Gamma emissions of ${}^{60}_{28}Ni^{*}$.

7.2 Nuclear electromagnetic transitions

The transition rate is derived basically from Fermi's golden rule

$$W_{fi} = \lambda = \frac{2\pi}{\hbar} |\langle \psi_f | V | \psi_i \rangle|^2 \rho(E_f) \tag{7.4}$$

where $|\psi_i\rangle$ is initial state of excited nucleus and $|\psi_f\rangle$ is product of final nuclear and photon states. The last factor, $\rho(E_f)$ is the product of the density of nuclear and photon states that are available to the system after the transition.

The matrix element $\langle \psi_f | V | \psi_i \rangle$ deal with electromagnetic interactions of photon with proton in nuclear levels. According to V. Weisskopf calculation[1], with single proton approximation, we can derive expressions of transition rate in the form

$$\lambda(E(M), l) = \frac{8\pi(l+1)}{\hbar l[(2l+1)!!]^2} k^{2l+1} B(l, E(M)), \ k = \frac{E_\gamma}{\hbar c} \tag{7.5}$$

$$B(E, l) = \frac{k_e e^2}{4\pi} \left[\frac{3}{l+3}\right]^2 R^{2l}, \quad k_e = \frac{1}{4\pi\epsilon_0}, \ R = r_0 A^{1/3} \tag{7.6}$$

$$B(M, l) = \frac{10\mu_n^2}{\pi} \left[\frac{3}{l+3}\right]^2 R^{2l-2}, \ \mu_n = \frac{e\hbar}{2m_p c} \tag{7.7}$$

We can express (7.5) in terms of E_γ and A, with specified radiation modes with (l), as appear in the following figures.

TABLE 9.2 Weisskopf Single-Particle Transition Rates (E_γ is in MeV)

Multipole	E	M
l	$\lambda\ (s^{-1})$	$\lambda\ (s^{-1})$
1	$1.03 \times 10^{14}\ A^{2/3} E_\gamma^3$	$3.15 \times 10^{13}\ E_\gamma^3$
2	$7.28 \times 10^{7}\ A^{4/3} E_\gamma^5$	$2.24 \times 10^{7}\ A^{4/3} E_\gamma^5$
3	$3.39 \times 10^{1}\ A^{2} E_\gamma^7$	$1.04 \times 10^{1}\ A^{4/3} E_\gamma^7$
4	$1.07 \times 10^{-5}\ A^{8/3} E_\gamma^9$	$3.27 \times 10^{-6}\ A^{2} E_\gamma^9$
5	$2.40 \times 10^{-12}\ A^{10/3} E_\gamma^{11}$	$7.36 \times 10^{-13}\ A^{8/3} E_\gamma^{11}$

Figure 7.3: Gamma emissions modes, replace T with λ.

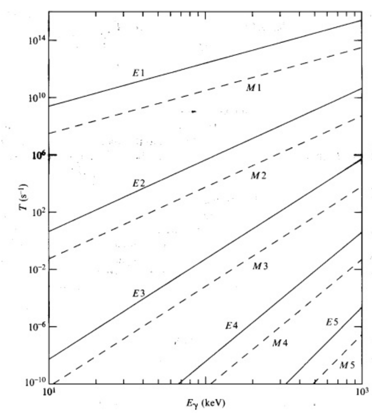

Figure 7.4: Gamma emissions modes.

Gamma emission selection rules: $\Delta l = \pm 1$, see figure (7.5).

$$L_i \to L_f, \qquad |L_i - L_f| \geq l \geq |L_i + L_f| \tag{7.8}$$

$$\pi_i \to \pi_f \qquad \pi_\gamma = \pi_i \pi_f \tag{7.9}$$

$$\pi_\gamma(E) = (-1)^l,\ \pi_\gamma(M) = (-1)^{l+1} \tag{7.10}$$

7.3 Isomeric transitions

Ordinary electromagnetic transitions take place within $10^{-15} - 10^{-12} s$. One may observe transitions with longer lifetime, i.e., in ns or longer, these are

[1] Blatt and Weisskopf, THeoretical Nuclear Physics (Springer, 1979)

TABLE 9.1 γ-Ray Selection Rules and Multipolarities

Radiation Type	Name	$l = \Delta I$	$\Delta\pi$
E1	Electric dipole	1	Yes
M1	Magnetic dipole	1	No
E2	Electric quadrupole	2	No
M2	Magnetic quadrupole	2	Yes
E3	Electric octupole	3	Yes
M3	Magnetic octupole	3	No
E4	Electric hexadecapole	4	No
M4	Magnetic hexadecapole	4	Yes

Figure 7.5: Gamma emissions modes.

called *isomeric transitions* of *isomeric nucleus*.

Nuclear isomers are atoms with the same mass number (A) and atomic number (Z), but with different states of excitation in the atomic nucleus. The higher or more excited state is called a *metastable state*, while the stable, unexcited state is called the ground state. The nuclear symbol for isomeric excited state will indicated my m, i.e., ${}^{Am}_{Z}X$ or ${}^{A}_{Z}X^{m}$.

Otto Hahn discovered the first nuclear isomer in 1921. This was Pa-234m, which decays in Pa-234.

$${}^{234m}_{91}Pa \xrightarrow[t_{1/2}=1.17min]{} {}^{234}_{91}Pa + \gamma$$

The reason metastable states form is because a larger nuclear spin change is needed in order for them to return to the ground state. High spin change makes the decays "forbidden transitions" and delays them. Decay half-life is also affected by how much decay energy is available.

For example of isomeric decay diagram

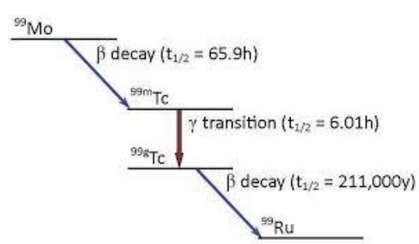

Figure 7.6: Isomeric transition of ${}^{99m}Tc$.

7.4 Internal conversion

The emitted gamma particle hit atomic electron to be ionized, we observe x-ray spectrum from atomic de-ionization instead to the gamma, see figure (7.7).

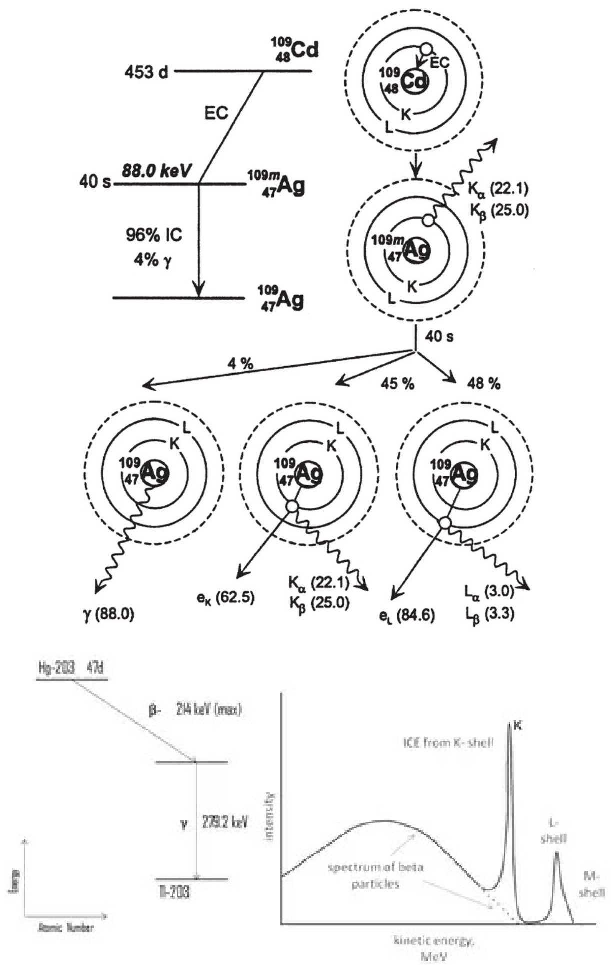

Figure 7.7: Internal conversion.

Internal conversion coefficient

$$\alpha = \frac{\#\ of\ e - emission}{\#\ of\ \gamma\ decays} = \frac{\lambda_e}{\lambda_\gamma} \tag{7.11}$$

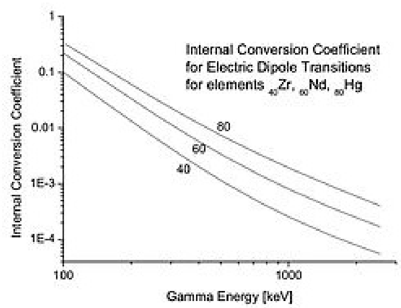

Figure 7.8: Internal conversion coefficient.

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