Lecture 1
Nuclear Phenomenology
SCPY322 Nuclear and Particle Physics

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Today topics

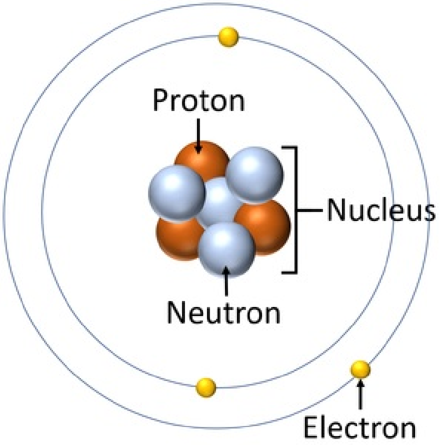
1. Nuclear history
2. Nuclear structure and symbol
3. Nuclear size
4. Nuclear stability
5. Nuclear decays
6. Nuclear activity

Nuclear histories

- 1896 Roentgen discovered X-rays
- 1897 Becquerel discovered nuclear radiations from Uranium
- 1898, Pierre and Marie Curie, first used the term radiations to describe the effects that they were observing from Uranium. The Curies' also discovered Radium.
- 1910, the existent of atomic nucleus was established by Rutherford experiment of alpha particle scattering.
- 1913 Niels Bohr published his model of the atom (Hydrogen Pictured) 1919, Rutherford was able to accomplish transmutation of nitrogen into oxygen by collision with alpha particle, at the University of Manchester
- 1932, Bohr's model was perfected James Chadwick who discovered the neutron and made Bohr's model work.

Nuclear structure and symbol

- A complete picture of atomic nucleus is a compact group of numbers of protons and neutron under the action of nuclear *strong force*.



- For a nucleus X with a number of Z protons and a number of N neutrons, its nuclear symbol is denoted in the form

$$ {}^{A}_{Z}X_N, \ {}^{A}_{Z}X, \ {}^{A}X \tag{1} $$

where $A = Z + N$ is called *atomic mass number*. For example of ${}^{4}_{2}He_2, \ {}^{4}_{2}He$, or ${}^{4}He$

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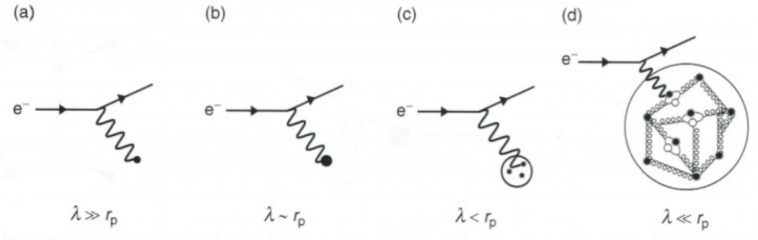
- A group of atomic nuclei is classified as in the following
  - **Isotope**: the same Z but differ in N and A
  - **Isotone**: the same N but differ in Z and A
  - **Isobar**: the same A but differ in Z and N

Name	*Definition*	*Examples*
Isotopes	Same Z, different N	O^{15}, O^{16}, O^{17}, O^{18}
Isotones	Same N, different Z	B^{12}, C^{13}, N^{14}, O^{15}
Isobars	Same A, different Z	Fe^{59}, Co^{59}, Ni^{59}, Cu^{59}
Isomers*	Same Z, same N, different energy	Kr^{81m}, Kr^{81}, Sr^{87m}, Sr^{87}, Tc^{99m}, Tc^{99}, In^{113m}, In^{113}

Nuclear size

- Robert Hofstadter (1961 Nobel prize in physics)
- Elastic electron-nucleus scattering



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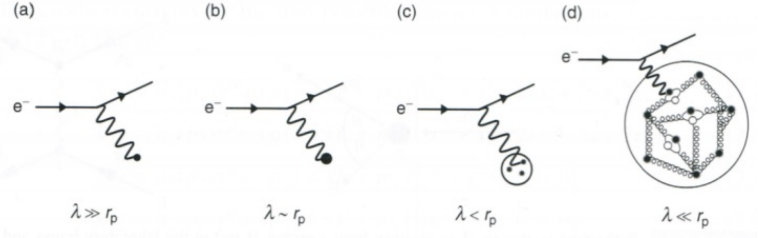

- Non-relativistic point-particle scattering (Rutherford) cross section, $E << m_e c^2$, is

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{\alpha^2}{16E^2 \sin^4(\theta/2)} \tag{2}$$

Note that $\alpha = \frac{k_e e^2}{\hbar c} \simeq \frac{1}{137}$ and $d\Omega = 2\pi d\cos\theta$.

- Relativistic point-particle scattering (Mott) cross section, $m_e c^2 << E << M_p c^2$, is

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{16E^2 \sin^4(\theta/2)} \left(1 - \beta^2 \sin^2(\theta/2)\right) \quad (3)$$

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- With a finite nuclear size, its from factor of the scattering cross section is introduced in the form

$$\left(\frac{d\sigma}{d\Omega}\right)_{Exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Point-like} |F(q^2)|^2 \tag{4}$$

where $F(q^2)$ is defined from the point-particle distribution inside a nuclear volume as

$$F(q^2) = \frac{1}{Ze} \int d^3 r \rho_e(r) e^{-iq\cdot r/\hbar} \tag{5}$$

- For a simple case of isotropic radial distribution, we will have

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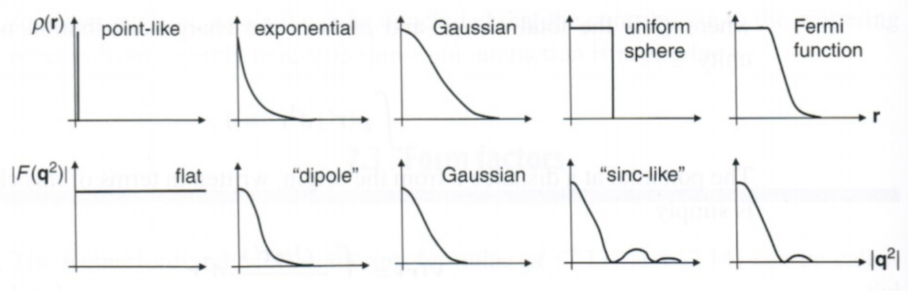
- Hard sphere distribution

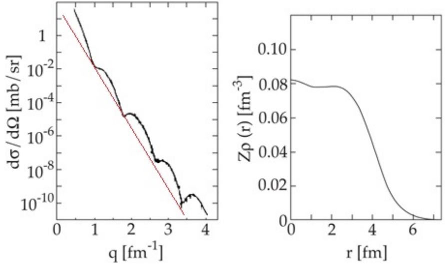
$$\rho_e(r) = \frac{3}{4\pi r^3} Ze, \ \ 0 \leq r \leq a \tag{8}$$

$$\rightarrow F(q^2) = 3\left(\frac{\hbar}{qa}\right)^3 \left[\frac{qa}{\hbar}\cos\left(\frac{qa}{\hbar}\right) - \sin\left(\frac{qa}{\hbar}\right)\right] \tag{9}$$

Other cases



From nuclear experiment

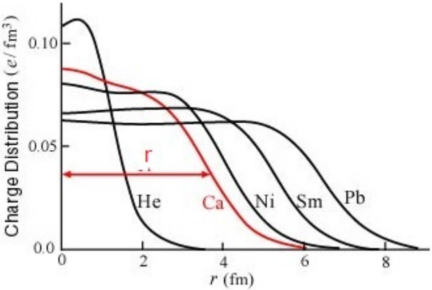


The mean nuclear radius is derived empirically to be

$$r = r_0 A^{1/3}, \quad r_0 \simeq 1.2 fm \tag{10}$$

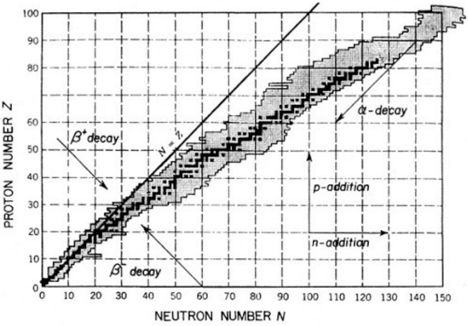
$$\rightarrow \rho_m = \frac{M_A}{\frac{4}{3}\pi r^3} = \frac{3u}{4\pi r_0^3} = \frac{3 \times 1.66 \times 10^{-27} kg}{4\pi(1.2 \times 10^{-15} m)^3} = 2.29 \times 10^{17} kg/m^3 \tag{11}$$

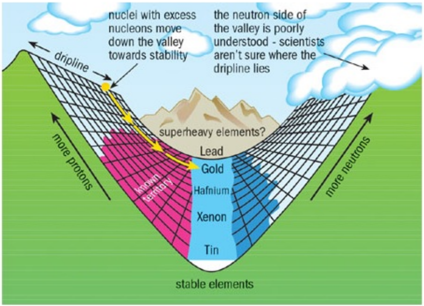
This is the nuclear mass density. (We have used $M_A = Au$, where u is atomic mass unit.)



Nuclear stability

In nature (even in the Laboratory), atomic nuclei appear in *the valley of stable nuclei*.





The unstable nuclei have spontaneous decays $(\alpha, \beta^{\pm})$ to be more stable one, may be in many steps to be the most stable.

We can determine the nuclear stability from the empirically calculated *nuclear binding energy*.

- Using *mass defect* formula, for a nucleus $(^A_Z X_N)$,

$$\delta M(X) = (ZM(^1H) + NM_n - M(X)) \tag{12}$$
$$E_b(X) = \delta M(X)c^2 \times 931.5 MeV/c^2 \tag{13}$$
$$\rightarrow e_b = \frac{E_b}{A}\ Mev/nuclei \tag{14}$$

where all masses are measured in $u = 1.66054 \times 10^{-27} kg$ and the energy is calculated in MeV, using conversion factor of $uc^2 = 931.5 MeV$. Some frequently used masses

$$m_e = 0.000549u = 0.512 MeV/c^2 \tag{15}$$
$$M_p = 1.00728u = 938.28 MeV/c^2 \tag{16}$$
$$M_n = 1.00867u = 939.57 MeV/c^2 \tag{17}$$
$$M(^1H) = 1.00783u = 938.79 MeV/c^2 \tag{18}$$

Exercise: Calculate e_b of ^4He and $^{40}_{20}Ca$, using mass defect formula.

- Using *Bethe-Weizsacker* formula (derived from liquid drop model)

$$E_b(MeV) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} + a_A \frac{(N-Z)^2}{A} + a_P \frac{1}{A^{1/2}} \quad (19)$$

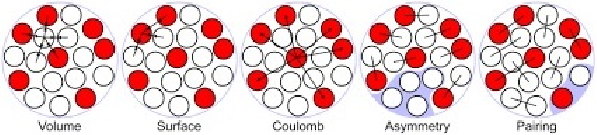
$$a_V(MeV) = 15.56 \quad (20)$$

$$a_S(MeV) = 17.23 \quad (21)$$

$$a_C(MeV) = 0.697 \quad (22)$$

$$a_A(MeV) = 23.285 \quad (23)$$

$$a_P(MeV) = \begin{cases} -12, & even - even \\ 0, & odd - even \\ +12, & odd - odd \end{cases} \quad (24)$$



Exercise: Calculate e_b of ^4He and $^{40}_{20}Ca$, using BW formula.

Nuclear decay

The unstable nuclei will decay into more stable one. The decay processes will be determined later. The reduce of number of unstable nuclei follows the *decay law*.

Let $N(t)$ is the number of nuclei at any time t its reduction rate under the decay will be

$$-\frac{dN(t)}{dt} \propto N(t) \rightarrow \frac{dN(t)}{dt} = -\lambda N(t) \quad in\ \#/s \tag{25}$$

$$\rightarrow N(t) = N(0)e^{-\lambda t}, \quad [\lambda] = 1/s \tag{26}$$

Note that λ is called *decay constant*, and it is characteristic of the decayed nucleus. Its value leads us to two time scales the corresponding nucleus:

- *half-life time:*

$$T_{1/2} = \frac{\ln 2}{\lambda} \rightarrow N(T_{1/2}) = \frac{1}{2}N(0) = 0.5N(0) \tag{27}$$

- *life-time:*

$$\tau = \frac{1}{\lambda} \rightarrow N(\tau) = \frac{1}{e}N(0) = 0.33N(0) \qquad (28)$$

Isotope	Half life	Decay constant (s^{-1})
Uranium 238	4.5×10^{9} years	5.0×10^{-18}
Plutonium 239	2.4×10^{4} years	9.2×10^{-13}
Carbon 14	5570 years	3.9×10^{-12}
Radium 226	1622 years	1.35×10^{-11}
Free neutron 239	15 minutes	1.1×10^{-3}
Radon 220	52 seconds	1.33×10^{-2}
Lithium 8	0.84 seconds	0.825
Bismuth 214	1.6×10^{-4} seconds	4.33×10^{3}
Lithium 8	6×10^{-20} seconds	1.2×10^{19}

- **Decay chain** Let us consider a decay series $N_1 \rightarrow N_2 \rightarrow N_3$(stable), the decay equations will be

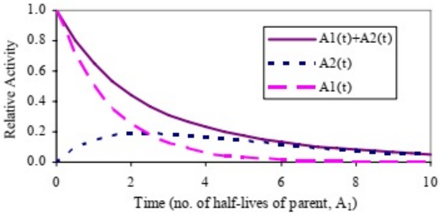
$$\frac{dN_1}{dt} = -\lambda_1 N_1 \tag{29}$$
$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \tag{30}$$
$$\frac{dN_3}{dt} = \lambda_2 N_2 \tag{31}$$

Solutions for all:

$$N_1(t) = N_1(0)e^{-\lambda_1 t} \tag{32}$$
$$\rightarrow \frac{dN_2(t)}{dt} + \lambda_2 N_2(t) - \lambda_1 N_1(0)e^{-\lambda_1 t} = 0$$
$$N_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} N_1(0)\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right), \text{ with } N_2(0) = 0 \tag{33}$$

Exercise: Evaluate $N_3(t)$, with the fact that $N_3(0) = 0$.

- **Decay chain (cont.)**
  - Non-equilibrium decay: for $\lambda_1 > \lambda_2$, $N_2(t)$ follows (33)



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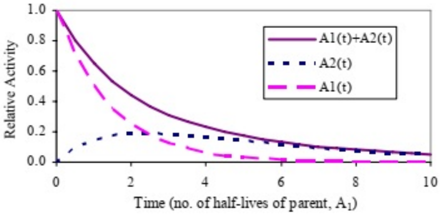
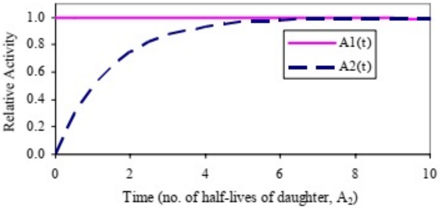


  - Secular equilibrium decay: for $\lambda_1 << \lambda_1$, $N_2(t) \simeq \frac{N_1(0)\lambda_1}{\lambda_2}\left[1 - e^{-\lambda_2 t}\right]$



- **Decay chain (cont.)** In general case of decay series

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} ... \xrightarrow{\lambda_{k-1}} N_k, \ \ N_i(0) = 0, i = 2, 3, .., k \tag{34}$$

We will face with a set of *Bateman equation*

$$\frac{dN_1}{dt} = -\lambda_2 N_1 \tag{35}$$

$$... \tag{36}$$

$$\frac{dN_i}{dt} = -\lambda_i N_i + \lambda_{i-1} N_{i-1} \tag{37}$$

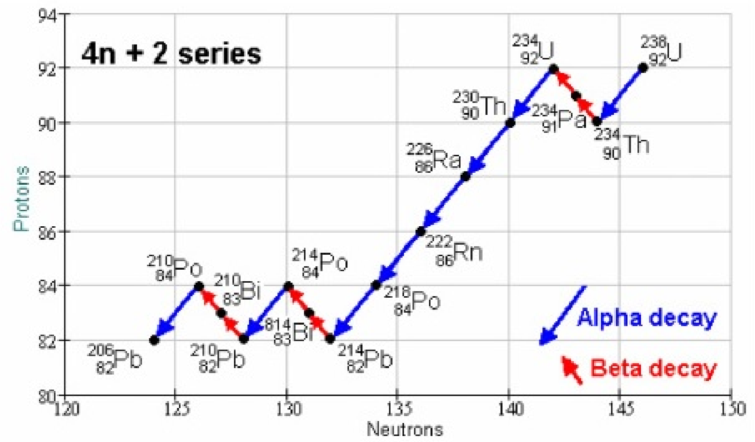
$$... \tag{38}$$

$$\frac{dN_k}{dt} = \lambda_{k-1} N_{k-1} \tag{39}$$

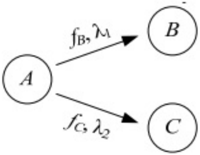
The solution for $N_i(t)$, with specified initial conditions, is

$$N_i(t) = \lambda_1 \lambda_2 ... \lambda_{i-1} N_1(0) \sum_{j=1}^{i} \left(\frac{e^{-\lambda_j t}}{\Pi_{k \neq j=1}^{i} (\lambda_k - \lambda_j)} \right) \tag{40}$$

- Decay chain of ^{238}U



- **Branching decay**, for a decay $A \to B(\lambda_1), C(\lambda_2)$, with branching ratios f_B, f_C, respectively.



Decay equation, and solutions are

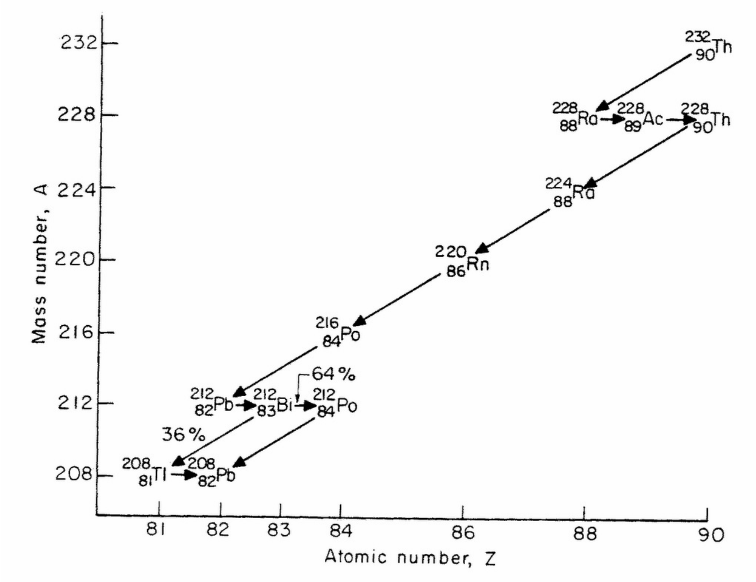
$$\frac{dN_A}{dt} = -\lambda N_A = -(\lambda_1 + \lambda_2)N_A \tag{41}$$

$$N_A(t) = N_A(0)e^{-(\lambda_1+\lambda_2)t} \tag{42}$$

$$N_B(t) = f_B N_A(0)(1 - e^{-\lambda_1 t}),\ N_B(0) = 0 \tag{43}$$

$$N_C(t) = f_C N_A(0)(1 - e^{-\lambda_2 t}),\ N_C(0) = 0 \tag{44}$$

- Decay chain + branching of ^{232}Th



Nuclear activity

- Nuclear activity is defined as the rate of nuclear decay, in unit of $\#/s$, as

$$R = \left|\frac{dN}{dt}\right| \rightarrow R(t) = \lambda N(t), \quad \#/s = Becquerel(Bq) \qquad (45)$$

$$1Ci(Curie) = 3.7 \times 10^{10} Bq \rightarrow 1Bq = 2.7 \times 10^{-11} Ci \qquad (46)$$

- Radiometric dating: ??

TABLE 22.2 Half-lives of Some Useful Radioisotopes

Radioisotope	Symbol	Radiation	Half-life	Use
Tritium	$^{3}_{1}H$	β^-	12.33 years	Biochemical tracer
Carbon-14	$^{14}_{6}C$	β^-	5730 years	Archaeological dating
Phosphorus-32	$^{32}_{15}P$	β^-	14.26 days	Leukemia therapy
Potassium-40	$^{40}_{19}K$	β^-	1.28×10^9 years	Geological dating
Cobalt-60	$^{60}_{27}Co$	β^-, γ	5.27 years	Cancer therapy
Technetium-99*m**	$^{99m}_{43}Tc$	γ	6.01 hours	Brain scans
Iodine-123	$^{123}_{53}I$	γ	13.27 hours	Thyroid therapy
Uranium-235	$^{235}_{92}U$	α, γ	7.04×10^8 years	Nuclear reactors

*The m in technetium-99m stands for metastable, meaning that it undergoes gamma emission but does not change its mass number or atomic number.