scpy322 nuclear and particle physics
lecture 4 many-nucleon models (19/02/2021)

4 Many-Nucleon Models

4.1 Fermi gas model

For $N+Z$ nucleon system, from Fermi statistics, the system energy is

$$\epsilon_k = \frac{\hbar^2 k^2}{2M} \to E = \sum_{k\in[0,k_F]} \epsilon_k \tag{1}$$

From basic quantum mechanics of infinite 3D square well, we have

$$\vec{k} = (k_x = n_x\pi/L, n_y\pi/L, n_z\pi/L) \tag{2}$$

$$\epsilon_k \to \epsilon_{n_x n_y n_z} = \frac{\hbar^2}{2ML^2}(n_x^2 + n_y^2 + n_z^2) = \frac{\hbar^2 n^2}{2ML^2} \tag{3}$$

$$\vec{n} = (n_x, n_y, n_z) \to dn = \frac{1}{8}\frac{L^3}{\pi^3} d\vec{k} = \frac{V}{(2\pi\hbar)^3} d\vec{p}, \quad \vec{p} = \hbar\vec{k} \tag{4}$$

$$n = \frac{V}{(2\pi\hbar)^3}(4\pi)\int_0^{p_F} p^2 dp = \frac{V}{(2\pi\hbar)^3}\frac{4\pi p_F^3}{3} = \frac{Vp_F^3}{6\pi^2\hbar^3} \tag{5}$$

$$\text{With spin} : \to n = (2)\frac{Vp_F^3}{6\pi^2\hbar^3} = \frac{Vp_F^3}{3\pi^2\hbar^3} \tag{6}$$

Then we have

$$N = \frac{V(p_F^n)^3}{3\pi^2\hbar^3}, \quad Z = \frac{V(p_F^p)^3}{3\pi^2\hbar^3} \tag{7}$$

Let us determine

$$r = r_0 A^{1/3} \to V = \frac{4\pi r^3}{3} = \frac{4\pi r_0^3 A}{3} \tag{8}$$

$$\to n = \frac{4\pi r_0^3 A}{3} \cdot \frac{p_F^3}{3\pi^2\hbar^3} = \frac{4A}{9\pi}\frac{r_0^2 p_F^3}{\hbar^3} \to p_F = \left(\frac{9\pi n}{4A}\right)^{1/3}\frac{\hbar}{r_0} \tag{9}$$

In a simple case of $n = N = Z = A/2$, we will have

$$p_F = \left(\frac{9\pi}{8}\right)^{1/3}\frac{\hbar}{r_0} \simeq 250MeV/c \to E_f = \frac{p_F^2}{2M} \simeq 33MeV \tag{10}$$

The depth of nuclear potential well $-U_0$ can be estimated to be

$$U_0 = E_F + e_b \simeq 40MeV$$

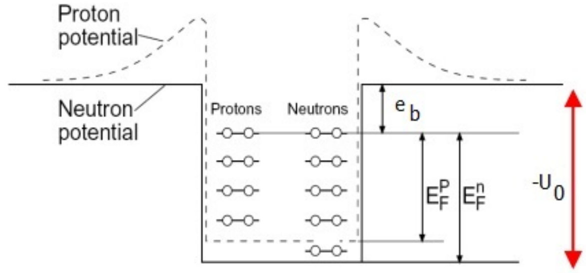

Figure 1:

4.2 Liquid drop model

In order to explain nuclear fission, the liquid drop nuclear model was proposed by George Gamow (1930)

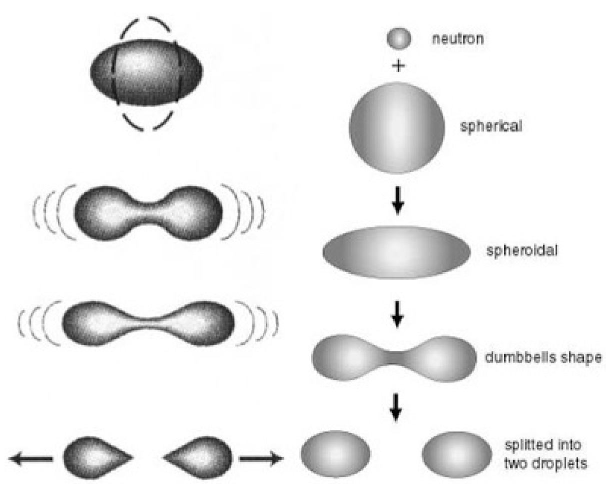

Figure 2:

The nuclear binding energy formula, determined from the liquid drop, was proposed by Carl Friedrich von Weizsäcker (1935), and known as *Bethe–Weizsäcker formula*, in the form

$$E_b = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z^2}{A^{1/3}} - a_A \cdot \frac{(Z-N)^2}{A} - \delta \cdot \frac{1}{A^{1/2}} \tag{11}$$

$$a_V \simeq 16MeV,\ a_S \simeq 20MeV,\ a_C \simeq 0.75MeV, a_A \simeq 21MeV \tag{12}$$

$$\delta = \begin{cases} -11.2MeV, & even-Z, even-N \\ 0, & odd-A \\ +11.2MeV, & odd-Z, odd-N \end{cases} \tag{13}$$

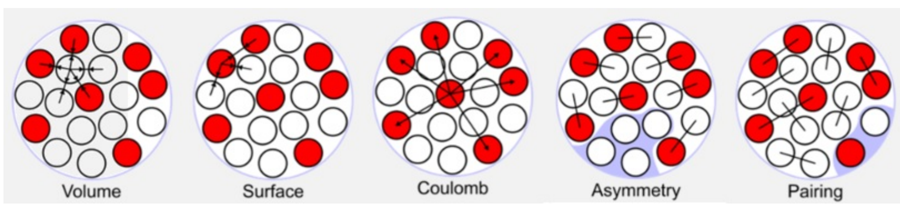

Figure 3:

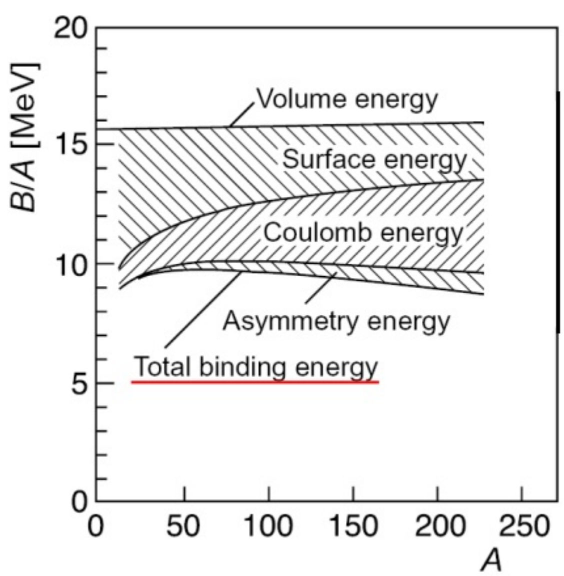

Figure 4:

4.3 Deformed sphere (Nilsson) model

Nuclear shape can be described by the surface function

$$R(\theta, \phi) = R_0 \left(1 + \sum_l \sum_{m=-l}^{+l} \alpha_{lm} Y^*_{lm}(\theta, \phi) \right) \tag{14}$$

$$l = 0 \text{ compression} \tag{15}$$

$$l = 1 \text{ translation} \tag{16}$$

$$l = 2 \text{ quadrupole deformation} \tag{17}$$

$$l = 3 \text{ octupole deformation} \tag{18}$$

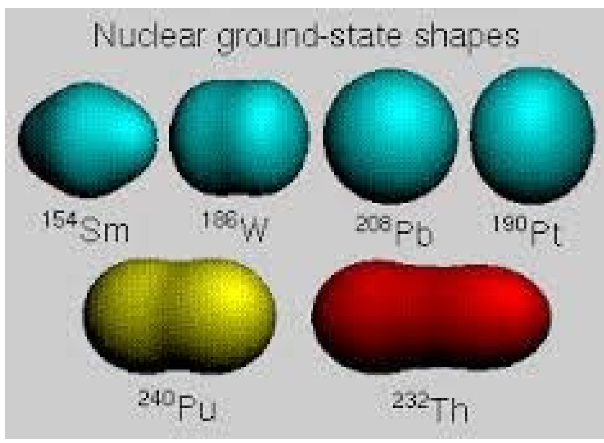

Figure 5:

The nuclear shapes can be determined from nuclear collective excitation, i.e., rotation and vibration.

4.4 Nuclear collective excitation

4.4.1 Rotation

4.4.2 Vibration