Lecture 5 (cont.) Nuclear Gamma Emission

SCPY322 Nuclear and Particle Physics
Second Semester 2020-21

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Friday 12, March 2021

Nuclear Gamma Emissions

- Energetic gamma emission equation

$$ {}^{A}_{Z}X^{*} \to {}^{A}_{Z}X + \gamma + Q $$

- Conservation of energy

$$ M^{*}_{X}c^{2} = M_{X}c^{2} + Q,\ Q = T_{r} + E_{\gamma} \simeq E_{\gamma} \tag{1} $$

- Momentum conservation, in the CM-frame,

$$ p_{X} = p_{\gamma} \tag{2} $$

$$ p_{\gamma} = \frac{E_{\gamma}}{c} \to T_{r} = \frac{p_{r}^{2}}{2M_{X}} = \frac{E_{\gamma}^{2}}{2M_{X}c^{2}} \tag{3} $$

- Example 1: γ-emission of ${}^{69}Zn^m \to {}^{69}Z_n$, with $E_\gamma = 0.439MeV$. Using $M(Zn) = 68.927u$

$$T_r = \frac{0.439MeV)^2}{2(68.927uc^2)(931.5MeV/c^2)} = 1.5eV \tag{4}$$

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- Example 2: γ-emission of $^{12}C^{*} \rightarrow {}^{12}C$, with $E_{\gamma} = 15.1MeV$. Using $M(C) = 12.000u$

$$T_r = \frac{(15.1MeV)^2}{2(12.000uc^2)(931.5MeV/c^2)} = 10.2keV \tag{5}$$

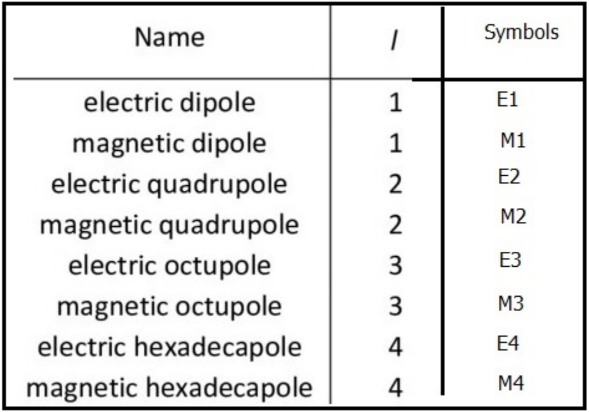
Classification of Gamma Emissions

- Photon has carry spin 1 (unit of $\hbar$)
- Let the angular momenta of initial and final states of a nucleus are I_i and I_f (unit of $\hbar$). The change of intrinsic angular momentum is $\Delta I = |I_i - I_f| = l$.
- The angular momentum take away by photon should be in the range

  $$|I_i - I_f| \leq l \leq |I_i + I_f|$$

  where $l = 0$ is forbidden, $l = 1$ is called *dipole photon*, $l = 2$ is called *quadrupole photon*, and so on.
- Transitions with the maximum change in the angular momentum of the nuclear states are called *stretched transitions.*

- Nomenclature of γ-emission selection rules



Name	l	Symbols
electric dipole	1	E1
magnetic dipole	1	M1
electric quadrupole	2	E2
magnetic quadrupole	2	M2
electric octupole	3	E3
magnetic octupole	3	M3
electric hexadecapole	4	E4
magnetic hexadecapole	4	M4

Emission Rate

- From Fermi's golden rule, the γ-emission rate is

$$\lambda = \frac{2\pi}{\hbar}|V_{fi}|^2\rho(E_f) \tag{6}$$

- After some extensive calculus and input from the theory of electromagnetism we come to an expression for the γ-emission rate in the form

$$\lambda(I_i, I_f, l) = \frac{8\pi(l+1)}{l[(2l+1)!!]^2}\frac{k^{2l+1}}{\hbar}B(I_i, I_f, l) \tag{7}$$

where $B(I_i, I_f, l)$ is derived by W. Weisskopf (single-particle approximation) in the form

$$B_{sp}(I_i, I_f, l) = \frac{1}{4\pi}\left[\frac{3}{(l+3)}\right]^2 r_0^{2l}A^{2l/3}e^2(fm)^{2l} \tag{8}$$

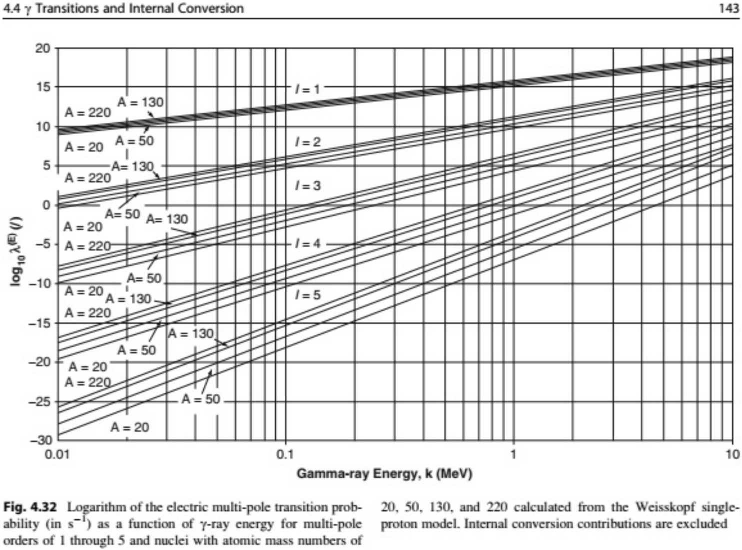

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Fig. 4.32 Logarithm of the electric multi-pole transition probability (in s^{-1}) as a function of γ-ray energy for multi-pole orders of 1 through 5 and nuclei with atomic mass numbers of 20, 50, 130, and 220 calculated from the Weisskopf single-proton model. Internal conversion contributions are excluded

Ref: B.J. McParland, *Nuclear Medicine Radiation Dosimetry* (Springer, 2020, Chapter 4) Download-able inside MU.