Lecture 6 Nuclear Reactions (Cross Section)

SCPY322 Nuclear and Particle Physics
Second Semester 2020-21

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Basic Nuclear Reaction Description

- Nuclear reaction equation

$$a + A \to B + b + Q, \quad \text{or } A(a,b)B$$

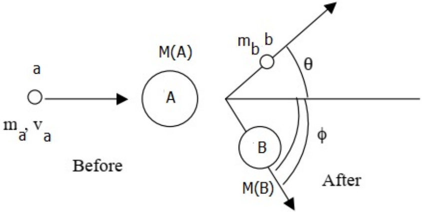
- Nuclear reaction energy Q (in the lab. frame (fixed target)):

$$Q = [m_a + M(A) - m_b - M(B)]c^2 = T_b + T_B - T_a$$

$Q > 0 \to$ exoergic, $Q < 0 \to$ endoergic.

Reaction	Measured Q (MeV)	Reaction	Measured Q (MeV)
$^{2}H(n,\gamma)^{3}H$	6.257 +/- 0.004	$^{9}Be(p,\alpha)^{6}Li$	2.132 +/- 0.006
$^{2}H(d,p)^{3}H$	4.032 +/- 0.004	$^{10}B(n,\alpha)^{7}Li$	2.793 +/- 0.003
$^{6}Li(p,\alpha)^{3}H$	4.016 +/- 0.005	$^{10}B(p,\alpha)^{7}Be$	1.148 +/- 0.003
$^{6}Li(d,p)^{7}Li$	5.020 +/- 0.006	$^{12}C(n,\gamma)^{13}C$	4.948 +/- 0.004
$^{7}Li(p,n)^{7}Be$	-1.645 +/- 0.001	$^{13}C(p,n)^{13}N$	-3.003 +/- 0.002

- In the lab. frame (fixed target)



- With the conservation of energy and momentum, the kinetic energy of emitted particle in any direction θ will be derived in term of Q in the form

$$T_b^{1/2} = \Big\{ [m_a m_b T_a]^{1/2} \cos\theta \pm [m_a m_b T_a \cos^2\theta$$
$$+ (m_b + M_B)\{M_B Q + (M_B - m_a) T_a\}]^{1/2} \Big\} \times \frac{1}{m_b + M_B} \tag{1}$$

- This is a quadratic function of T_b as function of the known T_a, Q, and all masses.

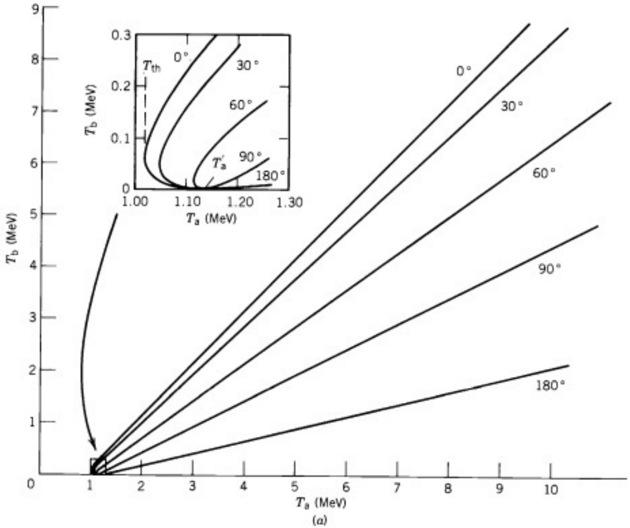


Figure 11.2 (*a*) T_a vs T_b for the reaction $^3H(p,n)^3He$. The inset shows the region of double-valued behavior near 1.0 MeV. (Krane)

- Let us determine an expression of T_b:
  - If $Q < 0$, there exists possibility of T_b to be double values at the angle $0 < \theta < \pi/2$. At the double-valued of T_b, T_a will fall into the range, determined at $\theta = 0$,

$$\frac{m_b + M_B}{M_B + m_b - m_a} < \frac{T_a}{|Q|} < \frac{M_B}{M_B - m_a}$$

The lower limit, shows the minimum of T_a for the reaction to occur. It is called *threshold energy* T_{th}, so that

$$T_{th} = \frac{-Q(m_b + M_B)}{M_B + m_b - m_a}$$

The upper limit of this range is called T_a', so that

$$T_a' = \frac{-QM_B}{M_B - m_a}$$

- Continue to determine T_b:
  - There is also exist the maximum scattering angle in the double-valued range of T_b, which is determined from the vanish of term in the square root, as

$$\cos^2\theta_{max} = -\frac{(m_b + M_B)\{M_B Q + (M_B - m_a)T_a\}}{m_a m_b T_a}$$

  - If $Q > 0$, then

$$\sqrt{m_a m_b T_a} > \sqrt{m_a m_b T_b \cos^2\theta + (m_b + M_B)\{M_B Q + (M_B - m_a)\}}$$

    Therefore , we always choose the positive sign in eq.(1) and there is no possibility for the double value of T_b, see the following figure.

- To determine Q from the known T_a, T_b in the experiment, we will have

$$Q = T_b\left(1 + \frac{m_b}{M_B}\right) - T_a\left(1 + \frac{m_a}{M_B}\right) - 2\cos\theta\left(\frac{m_a m_b}{M_B^2} T_a T_b\right)^{1/2}$$

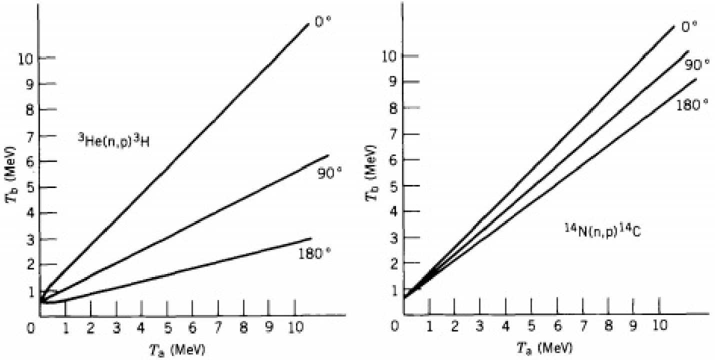
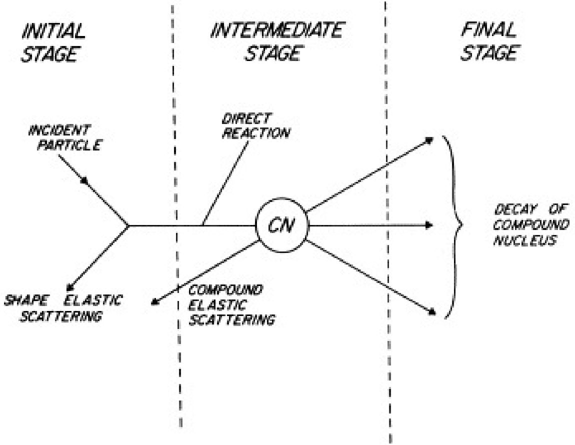


Figure 11.3 T_a vs T_b for the reactions ^{3}He(n, p)^{3}H and ^{14}N(n, p)^{14}C. No double-valued behavior occurs.

Nuclear Reactions Type

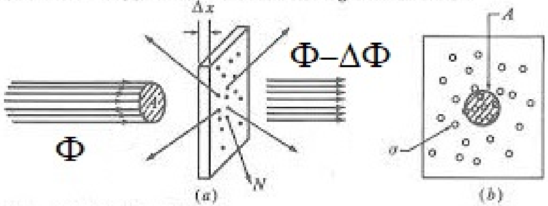
- According to W. Weisskopf, the nuclear reactions are classified into the three stages as



Reaction Cross Section

- According nuclear reaction experiment

Basic experimental arrangement to determine the cross section of a nuclear reaction. (a) Side view. (b) View along beam direction.



from "Elements of Nuclear Physics" by W.E. Meyerhof, Copyright 1967 McGraw-Hill

We can determine the beam flux reduction $\Delta\Phi$ as

$$\Delta\Phi = -\Phi n \sigma \Delta x, \quad n = \frac{N}{V} \tag{2}$$

where σ is called *cross section* of the nuclear reaction.

- From equation (2), by direct integration we get the transmission flux

$$\Phi_{trs}(x) = \Phi(0)e^{-n\sigma x} \tag{3}$$

So that the absorbed flux will be

$$\Phi_{abs}(x) = (1 - e^{-n\sigma x})\Phi(0) \tag{4}$$

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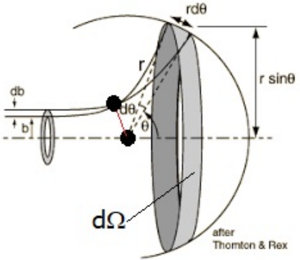
- The reaction cross section can be determined from single particle collision



- Its definition is

$$d\sigma = \frac{\text{transition rate into solid angle } d\Omega}{\text{incident flux } \Phi} \tag{5}$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{W_{fi}}{\Phi} \rightarrow \sigma_{tot} = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega \tag{6}$$

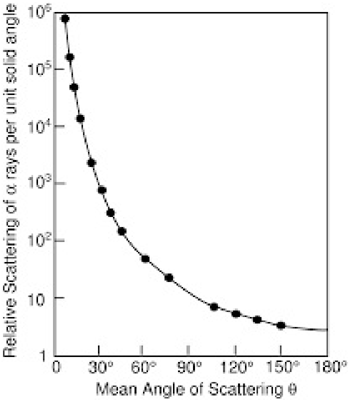
Note that $[\sigma] = m^2 \rightarrow 1\ barn = 10^{-28}m^2 = 10^{-24}cm^2 = 100fm^2$

- The number of particles will be detected, emitted from nuclear reactions, in particular direction per unit solid angle will be

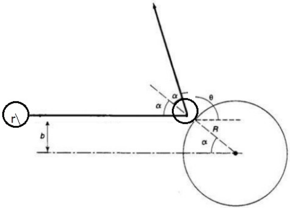
$$\frac{dN}{d\Omega} = \Phi n \left(\frac{d\sigma}{d\omega}\right) dx$$

- The elastic Coulomb collision differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{(Z_1 Z_2 k_e e^2)^2}{16E^2 \sin^4(\theta/2)}$$



- Direct nuclear reaction differential cross section can be determined from hard sphere collision as



- From classical theory, we will have

$$\sigma_{tot} = \pi(R + r)^2$$

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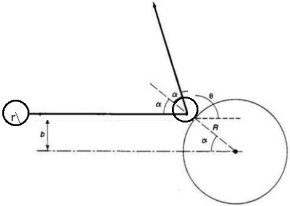

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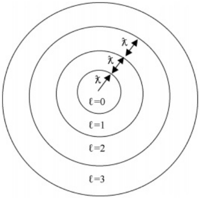


- From classical theory, we will have

$$\sigma_{tot} = \pi(R + r)^2$$

- From quantum theory, we will have

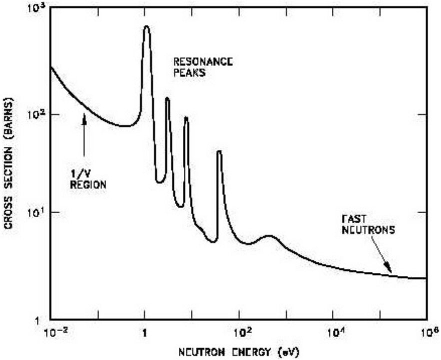
$$\sigma_{tot} = \pi\lambda\!\!\!^{-2}(l_{max} + 1)^2$$

where $\lambda\!\!\!^{-} = \frac{\lambda}{2\pi}$, $l_{max} = \frac{R}{\lambda\!\!\!^{-}}$

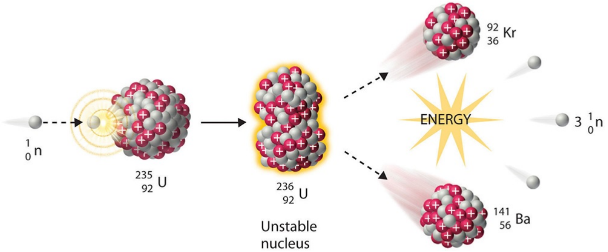
- Compound nuclear reaction cross section, derived from the optical theorem of complex potential $V = V_0 + iV_1$, in order to produce an absorption, we have

$$\sigma_{ab}(E,J) = \pi\lambda\!\!\!^{-2} \sum_{\lambda} \frac{\Gamma_{\lambda a}\Gamma_{\lambda b}}{(E-E_\lambda)^2 + \frac{1}{4}\Gamma_\lambda^2} \tag{7}$$

where $\Gamma_\lambda = \Gamma_{\lambda a} + \Gamma_{\lambda b}$. This is called *Breit-Wigner formula.*



Fission Reactions



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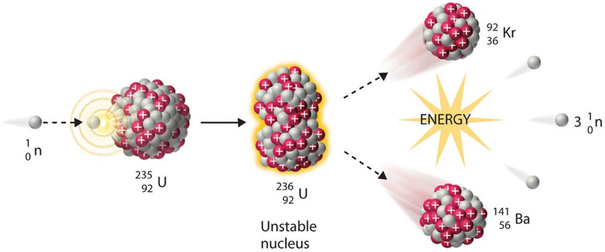


Exercises

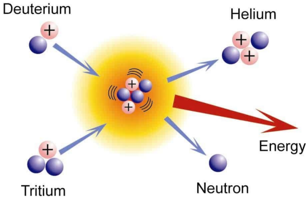
a. $^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + \underline{\ ?\ } + 3\ {}^{1}_{0}n$

b. $^{235}_{92}U + {}^{1}_{0}n \rightarrow \underline{\ ?\ } + {}^{137}_{52}Te + 2\ {}^{1}_{0}n$

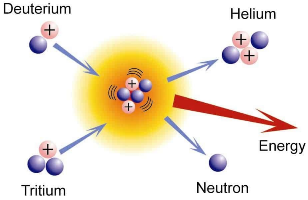
c. $^{235}_{92}U + {}^{1}_{0}n \rightarrow \underline{\ ?\ } + {}^{137}_{55}Cs + 3\ {}^{1}_{0}n$

d. $^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{94}_{38}Sr + {}^{140}_{54}Xe + \underline{\ ?\ }\ {}^{1}_{0}n$

Fission Reactions



Fission Reactions



Exercises

Complete the following fusion reactions:

(a) $^{2}_{1}H+^{2}_{1}H \rightarrow ^{3}_{2}He + \_\_\_$

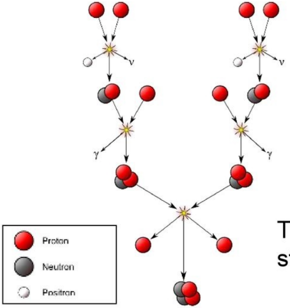
(b) $^{2}_{1}H + \_\_\_ \rightarrow ^{4}_{2}He + ^{1}_{0}n$

(c) $^{1}_{1}H+^{3}_{2}He \rightarrow \_\_\_ + ^{0}_{-1}e + ^{0}_{0}\nu$

(d) $^{2}_{1}H+^{2}_{1}H \rightarrow ^{3}_{1}H + \_\_\_$

Nucleosysnthesis on Star

Hydrogen Burning (Proton-proton chain)



- ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H$
- ${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He$
- ${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He$

This process is the longest stage in a star's life!

- Triple-alpha (He burning)

$$(1) \quad {}^{4}\mathrm{He} + {}^{4}\mathrm{He} \leftrightarrow {}^{8}\mathrm{Be}\,,$$
$$(2) \quad {}^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma\,.$$

- Carbon burning

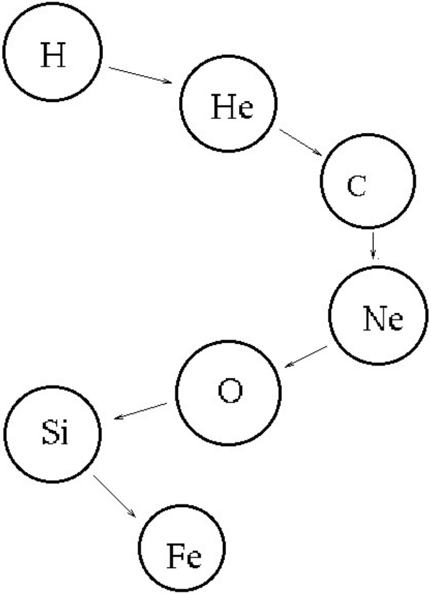
$$\begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{24}\mathrm{Mg} + \gamma \\ &\rightarrow {}^{23}\mathrm{Na} + {}^{1}\mathrm{H} \\ &\rightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \\ &\rightarrow {}^{23}\mathrm{Mg} + \mathrm{n} \\ &\rightarrow {}^{16}\mathrm{O} + 2\,{}^{4}\mathrm{He}\,. \end{aligned}$$

- Oxygen burning

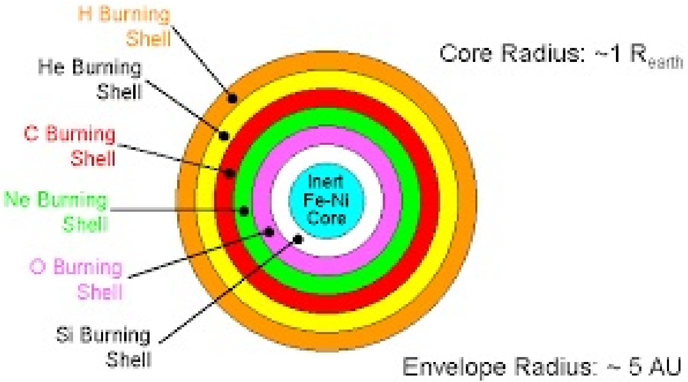
$$\begin{aligned} {}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{32}\mathrm{S} + \gamma \\ &\rightarrow {}^{31}\mathrm{P} + {}^{1}\mathrm{H} \\ &\rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} \\ &\rightarrow {}^{31}\mathrm{S} + \mathrm{n} \\ &\rightarrow {}^{24}\mathrm{Mg} + 2\,{}^{4}\mathrm{He}\,. \end{aligned}$$

- Silicon burning

$$\begin{aligned} {}^{28}\mathrm{Si} + {}^{28}\mathrm{Si} &\rightarrow {}^{56}\mathrm{Ni} + \gamma\,, \\ {}^{56}\mathrm{Ni} &\rightarrow {}^{56}\mathrm{Fe} + 2\,\mathrm{e}^{+} + 2\,\nu_{\mathrm{e}}\,. \end{aligned}$$


H
He
C
Ne
O
Si
Fe

At the end of Carbon burning



Nuclear Spallation