SCPY322 Nuclear and Particle Physics
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Lecture 3. Friday 9, April 2021

Reviews from the last lecture

We have determined the relativistic kinematics of three-body decays. The decay kinematics is written in therms of Lorentz covariant variables $s's$, and we have defined *Jackson frame*, the CM-frame of two particles used for the third party. The decay products are predicted by Dalitz plot.

The two particle collision kinematics is also written in terms of the Lorentz covariant Mandelstam variables (s, t, u). These variables are first calculated in the CM frame, and then LT to the LAB frame for making the prediction of the collision outcome.

I have left to talk about the decay rate and collision cross section.

Decay rate

From the NRQM, the transition rate is evaluated from the Fermi's golden rule ($\hbar = 1$)

$$\Gamma_{fi} = 2\pi |M_{fi}|^2 \rho(E_f) \tag{0.1}$$

In the unit of s^{-1}. Note that

$$\rho(E_f) = \left|\frac{dn}{dE}\right|_{E_f} = \int \frac{dn}{dE}\delta(E_f - E)dE = \int \delta(E_f - E)dn$$
$$= \int \delta(E_i - E)dn, \quad E_i = E_f \tag{0.2}$$

For N-particle decay, with energy-momentum conservation $p_i^\mu = \sum_{n=1}^N p_n^\mu$, we have

$$dn = \Pi_{n=1}^{N-1} dn_n = \Pi_{n=1}^{N-1} \frac{d^3p_n}{(2\pi)^3} \to \Pi_{n=1}^{N-1} \frac{d^3p_n}{(2\pi)^3} \delta\left(\vec{p} - \sum_{n=1}^{N} \vec{p}_n\right) d^3p_N$$
$$= (2\pi)^3 \Pi_{n=1}^{N} \frac{d^3p_n}{(2\pi)^3} \delta\left(\vec{p}_i - \sum_{n=1}^{N} \vec{p}_n\right) \tag{0.3}$$

And

$$M_{fi} = \langle f|V|i\rangle = \langle \psi_1\psi_2...\psi_N|V|\psi_i\rangle$$
$$\xrightarrow{relativistic} (2E_1)^{1/2}(2E_2)^{1/2}...(2E_N)^{1/2}(2E_i)^{1/2}\langle \psi_1'\psi_2'...\psi_N'|V|\psi_i'\rangle \tag{0.4}$$

where we have used relativistic normalization $\langle\psi_a|\psi_a\rangle = 2E_a$. From above we will have relativistic expression of decay rate

$$d\Gamma_{fi} = \frac{(2\pi)^4}{2E_i}|M_{fi}|^2\delta^{(4)}\left(p_i - \sum_{n=1}^{N} p_n\right)\underbrace{\Pi_{n=1}^{N}\frac{d^3p_n}{(2\pi)^3 2E_n}}_{dLIPS} \tag{0.5}$$

where *dLIPS* = *Lorentz Invariant Phase Space* measure. For example of two-particle decay, $a \to 1 + 2$, we will have

$$d\Gamma_{fi} = \frac{(2\pi)^4}{2E_a}|M_{fi}|^2\delta^{(4)}(p_a - p_1 - p_2)\frac{d^3p_1 d^3p_2}{(2\pi)^6 2E_1 2E_2} \tag{0.6}$$

In the CM-frame, $E_a = m_a, \vec{p}_a = 0$, we will have from above

$$d\Gamma_{fi} = \frac{1}{8\pi^2 m_a}|M_{fi}|^2\delta(m_a - E_1 - E_2)\delta^{(3)}(\vec{p}_1 + \vec{p}_2)\frac{d^3p_1 d^3p_2}{4E_1E_2} \tag{0.7}$$

$$\to \Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int\frac{|M_{fi}|^2}{4E_1E_2}\delta(m_a - E_1 - E_2)d^3\vec{p}_1 \tag{0.8}$$

Since

$$d^3\vec{p}_1 = p_1^2 dp_1 d\Omega \tag{0.9}$$

$$g(p_1) = \frac{p_1^2}{4E_1E_2} = \frac{p_1^2}{\sqrt{(p_1^2 + m_1^2)(p_1^2 + m_2^2)}} \tag{0.10}$$

$$f(p_1) = m_a - \sqrt{p_1^2 + m_1^2} - \sqrt{p_1^2 + m_2^2} \tag{0.11}$$

$$\to \Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int|M_{fi}|^2 g(p_1)\delta(f(p_1))dp_1 d\Omega \tag{0.12}$$

Evaluate

$$\int|M_{fi}|^2 g(p_1)\delta(f(p_1))dp_1 = |M_{fi}|^2 g(p^*)|f'(p^*)|^{-1} \tag{0.13}$$

$$\text{where } \delta(f(p_1)) = |f'(p^*)|^{-1}\delta(p_1 - p^*) \tag{0.14}$$

$$|f'(p^*)|^{-1} = p^*\frac{E_1 + E_2}{E_1E_2} \to g(p^*)|f'(p^*)|^{-1} = \frac{p^*}{4m_a} \tag{0.15}$$

Finally we have

$$d\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2}|M_{fi}^2|d\Omega \tag{0.16}$$

$$p^* = \frac{1}{2m_a}\sqrt{[m_a^2 - (m_1 + m_2)^2][m_a^2 - (m_1 - m_2)^2]} \tag{0.17}$$

Note that the amplitude $|M_{fi}|^2$ depends on fundamental interactions (EM,strong, weak) and is always written in terms of Lorentz invariant variable $s's$ and appear in particle data web page (www.pdg.org).

From multi-channels decays, we have

$$\Gamma = \sum_j \Gamma_j, \quad \tau = \frac{1}{\Gamma} \tag{0.18}$$

is the mean life time of the decay.

Cross section

The cross section is defined as

$$\sigma = \frac{\text{scattering rate}}{\text{Incident flux } F} = \frac{W_{fi}}{F} \tag{0.19}$$

In unit of m^{-2}, and the defined unit is 1 $barn = 10^{-24}\ cm^{-2}$. In case of 2-to-2 particle collision, $a + b \to 1 + 2$, we will have

$$F = |\psi_a|^2|\psi_b|^2(v_a + v_b) = 4E_1E_b\left(\frac{p_a}{E_a} + \frac{p_b}{E_b}\right) = 4(E_ap_b + E_bp_a) \tag{0.20}$$

$$F^2 = 16(E_a^2p_b^2 + E_b^2p_a^2 + 2p_ap_bE_aE_b)$$

$$= 16[(p_a \cdot p_b)^2 - (E_a^2 - p_a^2)(E_b^2 - p_b^2)] = 16[(p_a \cdot p_b)^2 - m_a^2m_b^2] \tag{0.21}$$

$$F = 4[(p_a \cdot p_b)^2 - m_a^2m_b^2]^{1/2} \tag{0.22}$$

In the CM-frame $p_a = -p_b = p_i^*,\ \sqrt{s} = E_a^* + E_b^*$, so that

$$F = 4p_i^*\sqrt{s} \tag{0.23}$$

So that the Lorentz invariant cross section, using Lorentz invariant Fermi's golden rule we have derived above, will be

$$\sigma = \frac{1}{(2\pi)^2 4\sqrt{s}p_i^*}\int |M_{fi}|^2\delta(\sqrt{s} - E_1 - E_2)\delta^{(3)}(\vec{p}_1 + \vec{p}_2)\frac{d^3p_1d^3p_2}{2E_12E_2} \tag{0.24}$$

Using previous analysis, we have

$$\sigma = \frac{1}{(2\pi)^2 4\sqrt{s}p_i^*}\frac{p_f^*}{4\sqrt{s}}\int |M_{fi}|^2 d\Omega^* \tag{0.25}$$

$$\to \frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|M_{fi}|^2 \tag{0.26}$$

The differential cross section determined in the CM-frame. Note that p^* can be written in terms of Lorentz invariant variables, and for elastic collision $p_f^* = p_i^*$.

3 Particle Accelerators and Detectors

3.1 Particle accelerators

Particle accelerators work from the action of Lorentz force on the charged particle

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \tag{3.1}$$

Particle get accelerated by the electric field and changed direction, without acceleration, by magnetic field. At high energy, the acceleration of a particle is reduced by relativistic factor as [1]

$$\vec{F} = m\vec{a} + m\gamma^2(\vec{\beta} \cdot \vec{a})\vec{\beta}, \text{ where } \vec{\beta} = \frac{\vec{v}}{c}, \ \gamma = (1 - \beta^2)^{-1/2} \tag{3.2}$$

The energy consumption rate of a particle from the electric field is

$$dP = \vec{F} \cdot d\vec{v} \tag{3.3}$$

while the particle velocity is related to its energy as $E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\beta^2}}$.

On the other hand, accelerated charged particle emit radiations with the loosing rate of [2]

$$P = \frac{2q^2\gamma^4}{3c^2}(\gamma^2 a_{||}^2 + a_{\perp}^2) \tag{3.4}$$

This shows that linear acceleration lose more energy than circular acceleration.

List of accelerators from the past:

- Van-de Graaff (1930) - static
- Walton-Cockroft (1932) - static
- Linacs - linear RF synchronous
- Cyclotron - circular RF synchronous
- Synchrotron - circular synchronous
- Collider - circular synchronous

See the list of all accelerators at *https://en.wikipedia.org/wiki/List of accelerators in particle physics.*

[1] See J.G. Pereira, arXiv: 1806.08680[physics.class-ph]

[2] See: http://astro.osu.edu/ ryden/ast822/week7.pdf

3.2 Particle detectors

Particle detectors work on the basis of particle interaction with matter, i.e., cross section and energy lose rate. There will be photon, charged particle and neutral particle.

3.3 Photon interaction with matter

Penetration depth is defined as

$$I(x) = I_0 e^{-\mu x}, \quad \mu - \text{linear attenuation coef.} \tag{3.5}$$

$$I(x) = I_0 e^{-(\mu/\rho)t}, \quad \frac{\mu}{\rho} - \text{mass attenuation coef} \tag{3.6}$$

$$t = \rho x \quad [gm/cm^2] - thickness \tag{3.7}$$

$$t_{1/2} \to I(t_{1/2}) = \frac{1}{2} I_0 \to \text{half} - \text{value thickness} \tag{3.8}$$

The energy lose processes of a photon are photoelectric effect, Compton scattering, pair production.

3.4 Charged particle interaction with matter

The interaction is characterize by *energy lose rate* $-dE/dx$. For Coulomb interaction between a particle of charge ze with target particle of charge Ze, we will have

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4 \rho}{m_e c^2 \beta^2} N_A \frac{Z}{A} \ln \frac{b_{max}}{b_{min}}, \quad b_{min} = \frac{ze^2}{\gamma m_e c^2 \beta^2} \tag{3.9}$$

$$b_{max} = \frac{ze^2}{\beta c} \sqrt{\frac{2}{m_e I}} \tag{3.10}$$

Full RQM derivation appear in form of Bethe-Bloch Formula.

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2 NZ}{(4\pi\varepsilon_0)^2 M_e v^2} \left[\ln\left(\frac{2M_e v^2}{I}\right) - \ln(1-\beta^2) - \beta^2 \right]$$

See figure (3.1) for muon stopping power.

List of particle detectors see *https://en.wikipedia.org/wiki/Particle detector*

Modern particle detectors (CMS for example)

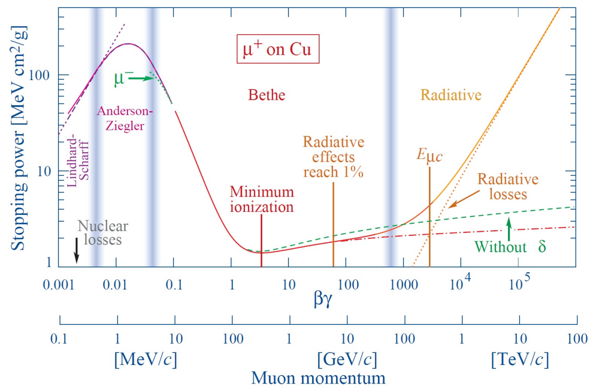

Figure 3.1:

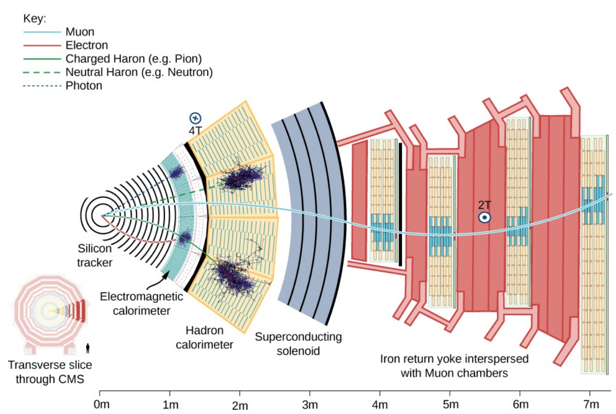

Figure 3.2:

4 Particle Classification

Observed elementary particles are classified by their interactions, as appear in figure (4.1). Normally, nuclear particles are identified with $|M, s, I, I_3>$, where I is isospin, and I_3 is its third component.

Some additional quantum numbers are assigned as

- Lepton quantum numbers L, see figure (4.2)
- Baryon quantum number B, see figure (4.3)

These numbers are conserved under particle processes., see figure (4.4)

Exercise 4.1 From figure (4.5), show that the following particle processes are possible or not, using quantum number conservation.

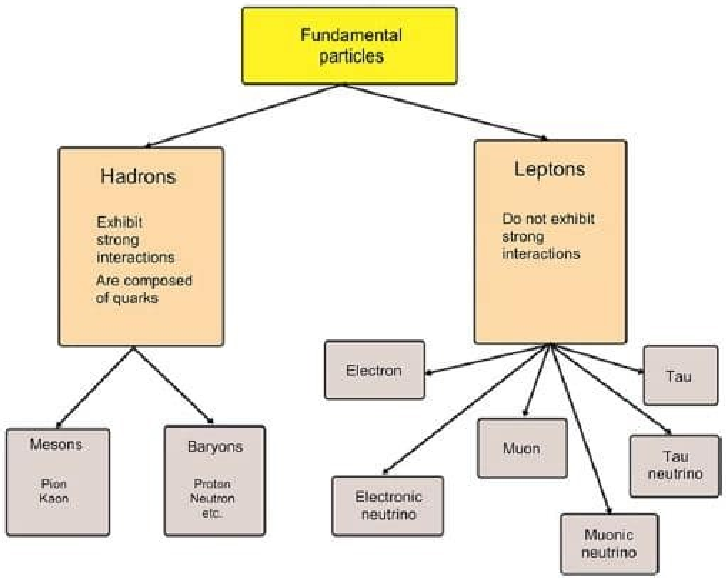

Figure 4.1:

For baryons, its charge Q is derived from *Gell-Mann-Nishijima formula* as

$$Q = I_3 + \frac{1}{2}B \tag{4.1}$$

For example of $p \to I_3 = 1/2, n \to I_3 = -1/2$, so that $Q_p = +1, Q_n = 0$.

Particle	L_e	L_μ	L_τ
e^-	+1	0	0
ν_e	+1	0	0
μ^-	0	+1	0
ν_μ	0	+1	0
τ^-	0	0	+1
ν_τ	0	0	+1

Figure 4.2:

baryons: (e.g. protons and neutrons)	+ 1
antibaryons: (e.g. antiproton)	- 1
non-baryons (e.g. mesons and leptons)	0

Figure 4.3:

	muon		muon neutrino		electron		e^- antineutrino
equation:	μ	$\rightarrow$	ν_μ	+	e^-	+	$\bar{\nu}_e$
electron number:	0	=	0	+	1	+	-1
muon number:	1	=	1	+	0	+	0
tau number:	0	=	0	+	0	+	0

Figure 4.4:

A. $\nu_e + p \rightarrow e^- + \pi^+ + p$

B. $\nu_\mu + p \rightarrow \mu^+ + n$

C. $\Lambda^0 \rightarrow e^- + \pi^+ + \nu_e$

D. $\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu$

Figure 4.5: