SCPY322 Nuclear and Particle Physics
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Lecture 4 - DIS, Partons and Quark Models of Hadrons

Reviews of the last lecture

We have classified all observed particles, according to their interactions and (decay) life-time, into *hardrons* and *leptons.* All leptons are fermions, while bosonic hardrons are called *meson* and fermionic hardrons are called *baryons.* We have assigned the quantum number to these particles, i.e., lepton number L and baryon number B, these must be conserved under particle processes according to their elementary properties.

Thing becomes more puzzle when more particles are observed, and some of them behave curios or *strange*, i.e., all are unstable, produced in pair, produced from strong force interaction and decay via weak force. Fro example

$$\pi^- + p^+ \to K^0 + \Lambda^0$$

$$K^0 \to \pi^+ + \pi^-$$

The appearance of strange particles show that proton, neutron and pion may not be elementary.

1 DIS project

In order to probe the inner structure of proton, SLAC-MIT initiate the DIS (deep inelastic scattering) project in 1968 by firing energetic electron, up to an energy of 20 Gev, on a proton target. The result was reported in the same year by Panovsky as

"*the apparent success of the parametrization of the cross-section in the variable $x-Q^2$ in at least indicative that point-like interactions are becoming involved.*"

1.1 e^-p^+-DIS

The SLAC-MIT DIS experiment, see Figure 1 below.
Diagram of $e^-p^+ \to e^- + X$ DIS, where X is anything, is illustrated as in the following Figure 2.

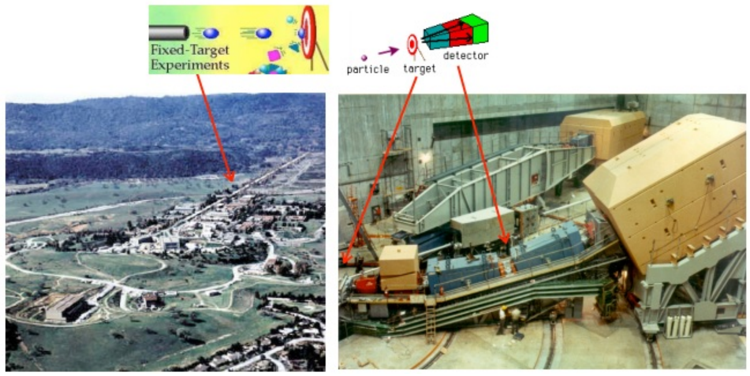

Figure 1: SLAC-MIT DIS Project.

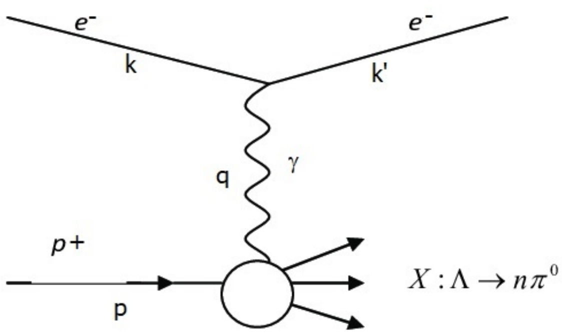

Figure 2: $ep \to eX$ DIS diagram.

1.2 DIS kinetamics

Let us denote

$$
\begin{aligned}
k^\mu = (E_e, \vec{k}),\ k'^\mu = (E'_e, \vec{k}'),\ q^\mu = (k-k')^\mu \qquad (1) \\
\to Q^2 = -q^2 = -(k^2 + k'^2 - 2k\cdot k') = -2m_e^2 + 2E_eE'_e - 2\vec{k}\cdot\vec{k}' \\
\simeq 2E_eE'_E(1-\cos\theta_e), \quad E_e \gg m_e^2,\ E_e \simeq |\vec{k}| \qquad (2) \\
y = \frac{p\cdot q}{p\cdot k} = 1 - \frac{E_e}{E'_e}\left(1 - \cos^2\frac{\theta_e}{2}\right) \qquad (3) \\
x = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{sy} \qquad (4)
\end{aligned}
$$

where x is called *Bjorken variable*, a fraction of proton momentum transfer to X and y is *inelasticity factor*.

Different beam energies provide access to different ranges in Q^2 and x, via the (approximate) relation $Q^2 = sxy$, see Figure 3 below.

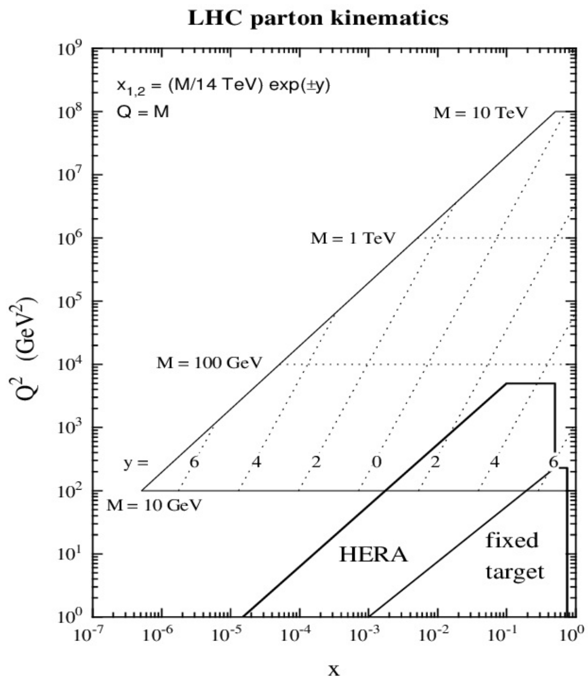

Figure 3: $Q^2 - x$ plot.

1.3 DIS cross section

Rutherford, low energy, elastic Coulomb scattering cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \tag{5}$$

Energetic Coulomb scattering, Mott, cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cos^2\frac{\theta}{2}, \quad m_e \ll M \tag{6}$$

Dirac (QFT) inelastic Coulomb scattering cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(\frac{E}{E'}\right) \left(1 - \frac{q^2}{2M^2}\tan^2\theta/2\right) \tag{7}$$

See Figure 4.

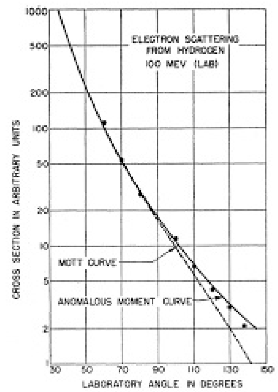

Figure 4: Dirac elastic Coulomb scattering cross section.

DIS cross section, see Figures 5 and 6 below.

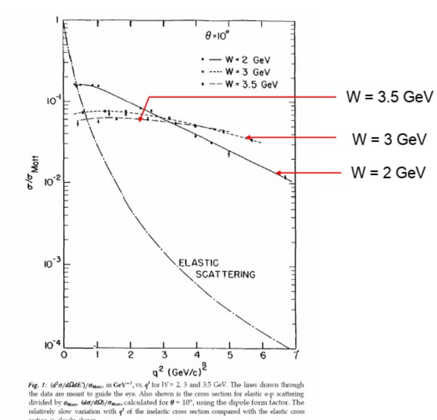

Figure 5: DIS cross section.

Scattering cross section from composite particle target

$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)_{point-like} |F(q^2)|^2 \tag{8}$$

where $|F(q^2)|^2$ is the *form factor*, for example of nuclei form factor see Figure 7 below.

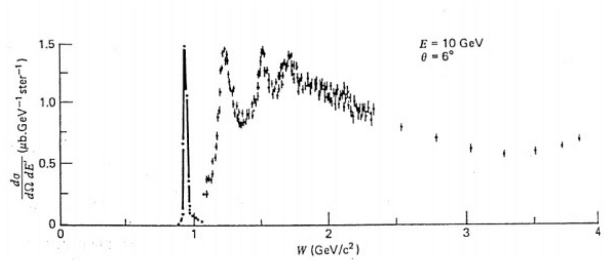

Figure 6: DIS resonance cross section.

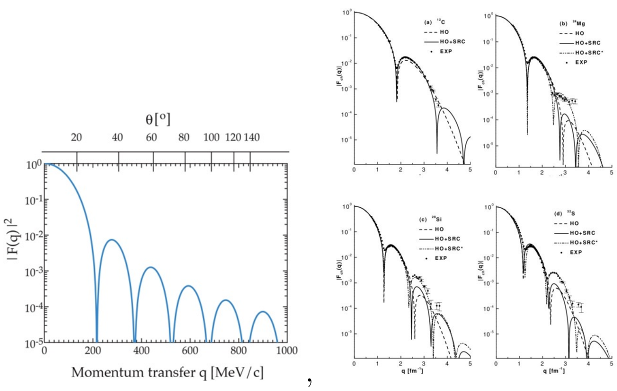

Figure 7: Nuclear form factor and elastic electron scattering cross section.

2 Partons

The parton model was proposed by Richard Feynman in 1969 as a way to analyze high-energy ep DIS experiment of hadron. Any hadron (for example, a proton) can be considered as a composition of a number of point-like constituents, termed ”partons”.

2.1 Parton distribution function of PDF

A parton distribution function (PDF) within so called collinear factorization is defined as the probability density for finding a particle with a certain longitudinal momentum fraction x at resolution scale Q^2.

The DIS cross section is derived (from QCD) in the form

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{DIS} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left\{W_2(Q^2,x) + 2W_1(Q^2,x)\tan^2\theta/2\right\} \tag{9}$$

$$F_1(Q^2,x) = m_P W_1(Q^2,x),\;\; F_2(Q^2,x) = \nu W_2(Q^2,x),\;\; \nu = \frac{p\cdot q}{m_p} \tag{10}$$

where ν is the exchanged boson energy in the proton rest frame. See Figure 6 below.

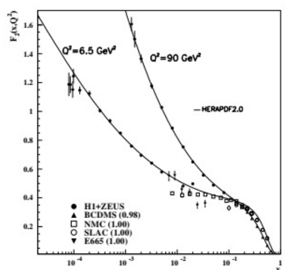

Figure 8: Parton distribution function.

Parton scattering cross section, see Figure 9 below.

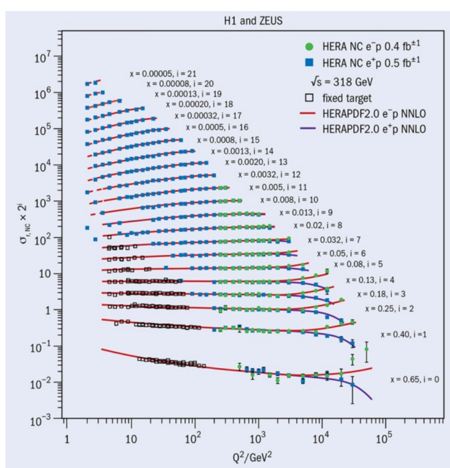

Figure 9: Parton scattering cross section.

3 Quark models of hardrons

George Zweig (1964), and Murry Gell-Mann (1964), independently proposed *quark models* of hadrons, using symmetry principle in quantum mechanics.

3.1 Symmetry principle in QM

Let O be symmetry operation, it means that

$$O|\psi >= |\psi' >, \ O^{\dagger}O = OO^{\dagger} = 1 \tag{11}$$

$$H|\psi >= E|\psi >\to OHO^{\dagger}O|\psi >= O(E|\psi >) = E(O|\psi >) \tag{12}$$

$$OHO^{\dagger} = H, \text{ or } OH = HO \to \ H|\psi' >= E|\psi' > \tag{13}$$

We can understand that O is a symmetry transformation iff $[O, H] = 0$, and O is unitary operator

$$O = e^{i\alpha t}, \ t^{\dagger} = t - \text{hermitian generator} \tag{14}$$

and α is a real transformation parameter.

3.1.1 Spcetime symmetry

Let us determine

a) Spatial translation

Let us consider

$$|x >\to |x + a >= |x > +ad_x|x > +... = |x > +iap_x|x > +... \tag{15}$$

$$\to |x + a >= (1 + iap_x + ..)|x >= e^{+iap_x}|x > \tag{16}$$

Note that $p_x = -id_x$, the momentum operator, is a generator of spatial translation. ($\hbar = 1$), i.e., $H = p^2/2m, [p, H] = 0$, the system is translation invariant.

b) Time evolution

Let us consider

$$|\psi(t + s) >= |\psi(t) > +sd_t|\psi(t) > +... \ |\psi(t) > -isH|\psi(t) > +.. \tag{17}$$

$$\to |\psi(t + s) >= (1 - isH + ..)|\psi(t) >= e^{-isH}|\psi(t) > \tag{18}$$

Note that $H = id_t$, the Hamiltonian operator, is a generator of time evolution, i.e., a system with $H \neq H(t)$, time-independent Hamiltonian, is stationary system, time-evolution invariant.

c) Spatial rotation

Let us consider

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \simeq \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (19)$$

So that

$$\begin{aligned} |x,y> \to |x+\theta y, y-\theta x> &= |x,y> + \theta x d_y |x,y> - \theta y d_x |x,y> + ... \\ &= (1 + i\theta(-ixd_y + iyd_x) + ...)|x,y> \\ &= (1 + i\theta L_z + ..)|x,y> = e^{i\theta L_z}|x,y> \quad (20) \end{aligned}$$

Note that L_z, the z-component angular momentum, is a generator of rotation on xy-plane. Let us denote

$$R_i(\theta_i) = e^{i\theta_i L_i}, \ i = 1,2,3 = (x,y,z) \quad (21)$$

$$\to [L_i, L_j] = i\epsilon_{ijk} L_k \quad (22)$$

It is the angular momentum (Lie type) algebra. A rotational symmetry is called *SO(3) symmetry*.

3.1.2 Internal symmetry

Let us determine the isopin doublet

$$|N> = \begin{pmatrix} p \\ n \end{pmatrix}, \text{ with } m_p \simeq m_n = 930MeV/c^2 \quad (23)$$

This state has degenerate mass, and it is invariant under unitary transformation

$$|N> \to |N'> = e^{i\alpha}|N> \quad (24)$$

a U(1) symmetry of constant phase change. And

$$|N> \to |N'> = e^{i\alpha^a t^a}|N>, \ a = 1,2,3, \ t^a = \frac{\sigma^a}{2} \quad (25)$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (26)$$

$$\{\sigma^a, \sigma^b\} = 2\delta^{ab}, \ [\sigma^a, \sigma^b] = \frac{i}{2}\epsilon^{abc}\sigma^c \quad (27)$$

Note that $g = e^{i\alpha^a t^a}$ is just a symmetry operator of rotation on *isopsin space*, $\{t^a\}$ is a set of hermitian generators of the transformation. This symmetry is called *SU(2) global unitary symmetry*.

3.2 Unitary groups and algebras

In general, the SU(N) symmetry is generate by a generator in the form

$$g = e^{i\alpha^a t^a},\ a = 1, 2, ..., N^2 - 1 \tag{28}$$

where t^a is NxN matrix, satisfy an algebra

$$[t^a, t^b] = if^{abc}t^c,\ Tr[t^a] = 0 \tag{29}$$

where f^{abc} is called *structure constant*, and t^a will transform N-plete inside its degenerate group of states. A set of $\{t^a\}$, together with identity 1_N, form a group called *SU(N)* group. The eigen-basis used for all representations is determined from the *Casimir operator*, constructed from the group generators as

$$C_i = \sum_a (t^a)^2,\ i = 1, 2, .., N - 1,\ C_{i3} = t^{a'} - diagonal \tag{30}$$

$$C_i|\lambda_i, m_i >= \lambda_i(\lambda_i + 1)|\lambda_i, m_i >,\ C_{i3}|\lambda_i, m_i >= m_i|\lambda_i, m_i > \tag{31}$$

Similar to the case of SO(3) group, i.e., $L^2 = L_x^2 + L_y^2 + L_z^2, L^2|l, m >= l(l + 1)|l, m >$.

Note that the Casimir has eigen-set similar to the angular momentum. Other importance quantity used to characterize the group algebra is its *weight*, determined from diagonal matrices generator. The weight diagram can be construct. The ladder operators, raising/lowering, can be constructed from the other with off-diagonal elements, used to determined changes of the weight points.

Let us determine some examples of SU(N) group.

a) SU(2) group, for isospin symmetry, its set of generators is

$$t^a = \frac{\sigma^a}{2},\ a = 1, 2, 3,\ I_3 \to t^3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \tag{32}$$

$$I^{\pm} = (t^1 \pm it^2) \to I^+|1/2, -1/2 >= |1/2, 1/2 > \tag{33}$$

$$I^-|1/2, 1/2 >= |1/2, -1/2 > \tag{34}$$

All of these are represented by weight diagram as appear in Figure (10).

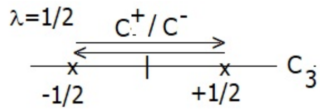

Figure 10:

b) SU(3) group, for color symmetry, its set of generators is

$$t^a = \frac{\lambda^a}{2},\ a = 1, 2, ..., 8 \tag{35}$$

where $\{\lambda^a\}$ is a set of Gell-Mann matrices, see Figure 11 below.

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \qquad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Figure 11:

The invariant *color multiplet* for this group is

$$|color\ quark\rangle = \begin{pmatrix} R \\ G \\ B \end{pmatrix} \tag{36}$$

From (35), we can assign the weights and ladders as

$$I_3 = t^3,\ Y = \frac{2}{\sqrt{3}} t^8 \to (I_3, Y) - \text{plane with three coordinates} \tag{37}$$

$$\left(\frac{1}{2}, \frac{1}{3}\right),\ \left(-\frac{1}{2}, \frac{1}{3}\right),\ \left(0, -\frac{2}{3}\right) \tag{38}$$

A sets of three ladder operators are defined as

$$I_\pm = (t^1 \pm it^2),\ V_\pm = t^4 + it^5,\ U_\pm = (t^6 \pm it^7) \tag{39}$$

See its weight diagram in Figure (12) below.

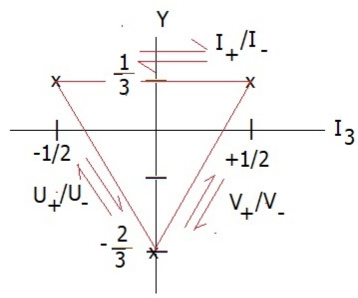

Figure 12:

3.3 Quark models of hadrons

Quark model of hadrons: mesons are quark-anti quark bound states, while baryons are three quarks bound states. Three quark state state is forbidden from Pauli's exclusion principle, except having additional quantum number. Color quantum numbers (R,G,B) is clever introduced with color rules:

"*only colorless (white) hadrons can exist in nature*"

Quark colors and anti-colors are appear in Figure (14) below.

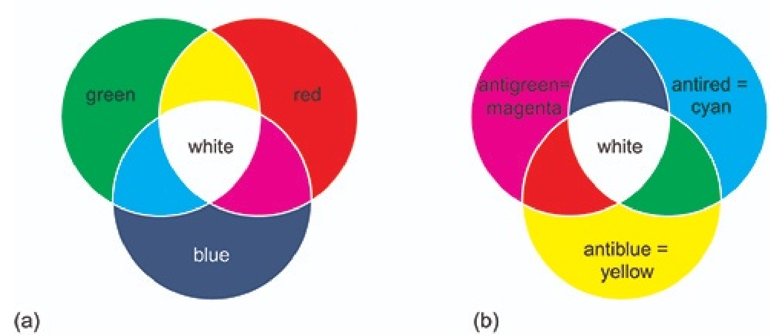

Figure 13:

Chronological list of observations:

- 1969, u, d and s quarks were observed at SLAC
- 1974, c quark was observed through J/Ψ meson at SLAC
- 1977, b quark was observed through Y-meson at Fermilab
- 1995, t quark was observed at Fermilab

List of the observed quarks appear in Figure 14 below.

Particle		Mass	J	B	Q (e)	I_3	C	S	T	B′	Antiparticle	
Name	Symbol	(MeV/c^2)*									Name	Symbol
First generation												
up	u	2.3 ±0.7 ± 0.5	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$	0	0	0	0	antiup	$\bar{u}$
down	d	4.8 ±0.5 ± 0.3	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0	antidown	$\bar{d}$
Second generation												
charm	c	1275 ± 25	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	+1	0	0	0	anticharm	$\bar{c}$
strange	s	95 ± 5	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	0	−1	0	0	antistrange	$\bar{s}$
Third generation												
top	t	173 210 ± 510 ± 710 *	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	0	0	+1	0	antitop	$\bar{t}$
bottom	b	4180 ± 30	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	0	−1	antibottom	$\bar{b}$

Figure 14:

3.3.1 SU(2) quark models

The model consist of u, d quarks (also with $\bar{u}, \bar{d}$ anti-quarks) We assign (u, d) with su(2) fundamental representation, and $(\bar{u}, \bar{d})$ with su(2) adjoint representation as appear in Figure (15) below.

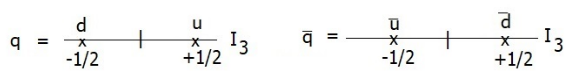

Figure 15:

a)Mesons, mesons are quark-antiquark state, i.e., $q\bar{q} = su(2) \otimes \overline{su}(2)$. The construction of this state appear in Figure (16) below.

This matches with a group of pions $(\pi^0, \pi^{\pm})$, with degenerate mass of 140 MeV/c^2, as

$$\pi^- = d\bar{u},\ \pi^0 = d\bar{d} + u\bar{u},\ \pi^+ = u\bar{d} \tag{40}$$

b) Hadrons, hadrons are three quark state, i.e., $qqq = su(2) \otimes su(2) \otimes su(2)$, as appear in Figure (17).

We observe that proton and neutron match with this model as

$$p(I_3 = +1/2) = \{uud\},\ n(I_3 = -1/2) = \{udd\}$$

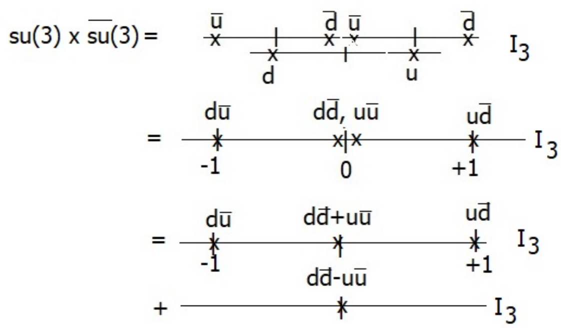

Figure 16:

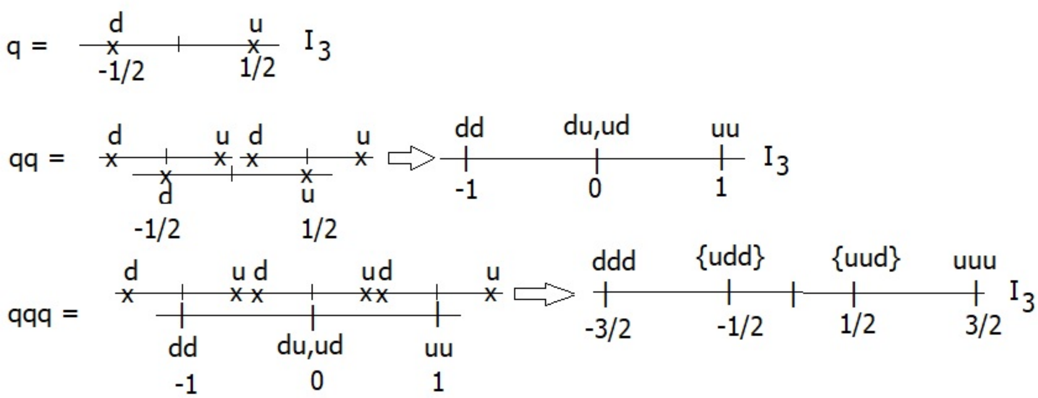

Figure 17:

with charge $Q = I_3 + \frac{1}{2}B$, i.e., $Q(p) = +1, Q(n) = 0$.

3.4 su(3) quark models

Quark content of in this model are (u, d, s), their represented by su(3) algebra as

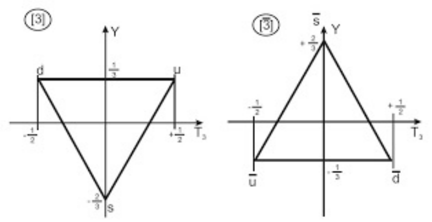

Figure 18:

a) Mesons, are constructed as

$$su(3) \otimes \overline{su(3)} = 9 = 8 \oplus 1$$

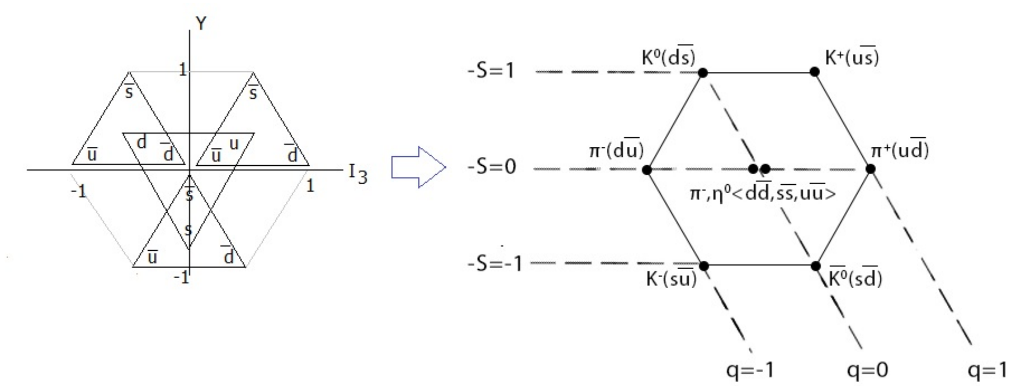

Figure 19:

b) Baryons, are constructed as

$$su(3) \otimes su(3) \otimes su(3) = 27 = 10 \oplus 8 \oplus 8 \oplus 1$$

List of mesons and baryons, see Figure (21).

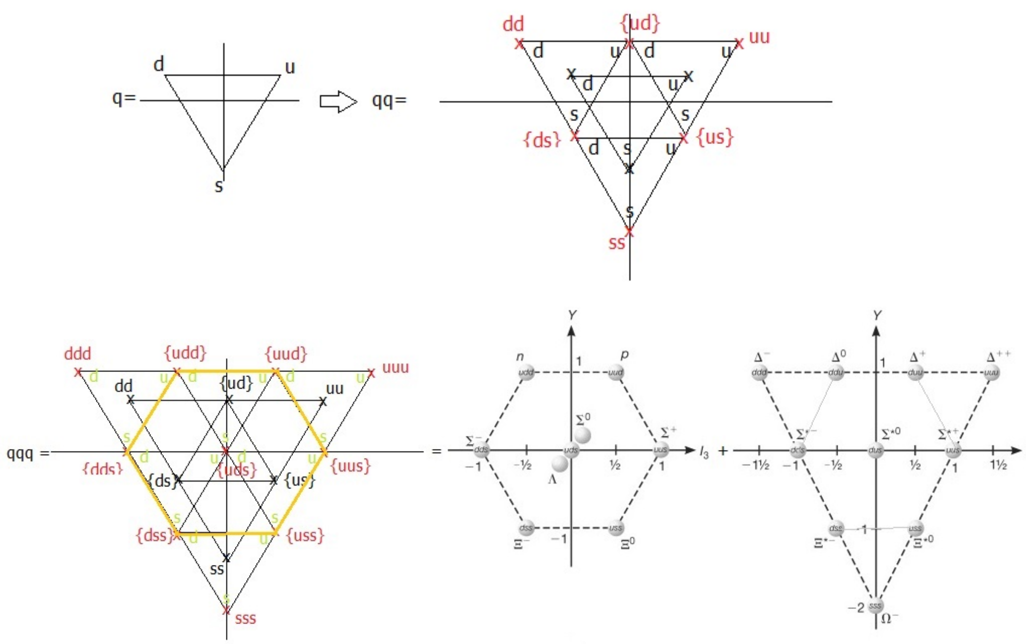

Figure 20:

$q_i\bar{q}_j$	$J=0$	$J=1$
$\lvert u\bar{d}\rangle$	$\pi^+(140)$	$\rho^+(770)$
$2^{-1/2}\lvert d\bar{d}-u\bar{u}\rangle$	$\pi^0(135)$	$\rho^0(770)$
$\lvert u\bar{d}\rangle$	$\pi^-(140)$	$\rho^-(770)$
$2^{-1/2}\lvert d\bar{d}+u\bar{u}\rangle$	$\eta(549)$	$\omega(783)$
$\lvert u\bar{s}\rangle$	$K^+(494)$	$K^{*+}(892)$
$\lvert d\bar{s}\rangle$	$K^0(498)$	$K^{*0}(892)$
$\lvert \bar{u}s\rangle$	$K^-(494)$	$K^{*-}(892)$
$\lvert \bar{d}s\rangle$	$\bar{K}^0(498)$	$\bar{K}^{*0}(892)$
$\lvert s\bar{s}\rangle$	$\eta'(958)$	$\phi(1020)$

,

$q_iq_jq_k$	$J=1/2$	$J=3/2$
$\lvert uuu\rangle$		$\Delta^{++}(1230)$
$\lvert uud\rangle$	$p(938)$	$\Delta^+(1231)$
$\lvert udd\rangle$	$n(940)$	$\Delta^0(1232)$
$\lvert ddd\rangle$		$\Delta^-(1234)$
$\lvert uus\rangle$	$\Sigma^+(1189)$	$\Sigma^+(1383)$
$2^{-1/2}\lvert(ud+du)s\rangle$	$\Sigma^0(1192)$	$\Sigma^0(1384)$
$\lvert dds\rangle$	$\Sigma^-(1197)$	$\Sigma^-(1387)$
$2^{-1/2}\lvert(ud-du)s\rangle$	$\Lambda(1116)$	
$\lvert uss\rangle$	$\Xi^0(1315)$	$\Xi^0(1532)$
$\lvert dss\rangle$	$\Xi^-(1321)$	$\Xi^-(1535)$
$\lvert sss\rangle$		$\Omega^-(1672)$

Figure 21: