5 QCD Renormalization

5.1 QCD counter terms

The QCD bare Lagrangian is

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F^a_{0\mu\nu}F^{0a\mu\nu} + \bar{\psi}_{0i}(i\partial\!\!\!/ - m_0)_{ij}\psi_{0j} - \bar{c}^a_0\delta^{ab}\partial^2 c^b_0 \\ +g_0\bar{\psi}_{0i}\gamma^\mu t^a_{ij}\psi_{0j}A^a_{0\mu} - g_0 f^{abc}(\partial_\mu A^a_{0\nu})A^{b\mu}_0 A^{c\nu}_0 \\ -g_0^2 f^{abx}f^{cdx}A^a_{0\mu}A^b_{0\nu}A^{c\mu}_0 A^{d\nu}_0 - g_0 f^{abc}\bar{c}^a_0(\partial^\mu A^b_{0\mu}c^c_0)\end{aligned} \tag{5.1}$$

where ψ_{0i} is the bare quark field, $A^a_{0\mu}$ is the bare gluon field, c^a_0 is the bare ghost field, and q_0 is the bare color charge. The renormalization of QCD Lagrangian start by introducing the renormalization parameters $Z's$ into the Lagrangian of the form

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}Z_3F^a_{\mu\nu}F^{a\mu\nu} + Z_2\bar{\psi}_i(i\partial\!\!\!/ - m)_{ij}\psi_j - Z_c\bar{c}^a\partial^2\delta^{ab}c^b \\ +Z_1q\bar{\psi}_i\gamma^\mu t^a_{ij}\psi_j A^a_\mu - Z^{3g}_1 q f^{abc}(\partial_\mu A^a_\nu)A^{b\mu}A^{c\nu} \\ -Z^{4g}_1 q^2 f^{abx}f^{cdx}A^a_\mu A^b_\nu A^{c\mu}A^{d\nu} - Z^c_1 q f^{abc}\bar{c}^a(\partial^\mu A^b_\mu c^c)\end{aligned} \tag{5.2}$$

and then rewrite in the form

$$\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{CT} \tag{5.3}$$

$$\begin{aligned}\mathcal{L}_R = -\frac{1}{4}F^a_{\mu\nu}A^{a\mu\nu} + \bar{\psi}_i(i\partial\!\!\!/ - m)_{ij}\psi_j - \bar{c}^a\delta^{ab}\partial^2 c^b \\ +q\bar{\psi}_i\gamma^\mu t^a_{ij}\psi_j A^a_\mu - q f^{abc}(\partial_\mu A^a_\nu)A^{b\mu}A^{c\nu} \\ -q^2 f^{abx}f^{cdx}A^a_\mu A^b_\nu A^{c\mu}A^{d\nu} - q f^{abc}\bar{c}^a(\partial^\mu A^b_\mu c^c)\end{aligned} \tag{5.4}$$

while the counter term Lagrangian $\mathcal{L}_{CT}$ is derived from the following procedure:

$$\psi_0 = \sqrt{Z_2}\psi,\ A^\mu_0 = \sqrt{Z_3}A^\mu,\ c_0 = \sqrt{Z_c}c \tag{5.5}$$

$$m_0 = Z_m m,\ q_0 = Z_q q \tag{5.6}$$

$$\to q_0 = \frac{Z_1}{Z_2\sqrt{Z_3}}q = \frac{Z^{3g}_1}{\sqrt{Z^3_3}}q = \frac{\sqrt{Z^{4g}_1}}{Z_3}q = \frac{Z^c_1}{Z_c\sqrt{Z_3}}q \tag{5.7}$$

$$\tag{5.8}$$

Then assign

$$Z_i = 1 + \delta Z_i \tag{5.9}$$

The $\mathcal{L}_{CT}$ is derived by the expansion of (5.7) in (5.2) and containing δZ_i terms. These δZ_i terms are derived from loop-correction terms, and used to kill the divergence of $\mathcal{L}_R$ at loop level. On the other hand, δZ_i are used to determine the normalization of quark mass and color charge at the loop level, and their evolution, especially the color charge, to higher loop corrections in form of renormalization group equation.

5.2 Quark mass renormalization

5.3 Color charge renormalization

5.4 Renormalization group equation

5.5 Asymptotic freedom