1 Motivations

1.1 Poincare symmetry of quantum field theory

1.2 Poincare algebra

Poincare symmetry is spacetime symmetry, consist of translation and Lorentz rotation symmetries. Its group compose of a set of generators $\{P_\mu, M_{\mu\nu}\}$ satisfy the Poincare algebra

$$\begin{aligned} [P_\mu, P_\nu] &= 0 && (1.1)\\ [M_{\mu\nu}, M_{\rho\sigma}] &= i(g_{\mu\rho}M_{\nu\sigma} + g_{\nu\sigma}M_{\mu\rho} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma}) && (1.2)\\ [M_{\mu\nu}, P_\rho] &= ig_{\mu\rho}P_\nu - ig_{\nu\rho}P_\mu && (1.3) \end{aligned}$$

The representation of Poincare algebra is determined from its twp Casimir operators

$$\begin{aligned} P_\mu \qquad &\to C_1 = P^2,\ |m^2(E)\rangle && (1.4)\\ W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma} \qquad &\to C_2 = W^2,\ |s(\lambda)\rangle && (1.5) \end{aligned}$$

with eigen-values of mass squared m^2 for massive (or energy E for massless) particle, and spin s for massive (or helicity λ for massless) particle. A state of quantum particle is represented by direct products of eigen-states of these Casimir, i.e., $|p\rangle = |m^2(E), s(\lambda)\rangle = |m^2(E)\rangle \otimes |s(\lambda)\rangle$, with $p^2 = m^2$.

Let $\phi(x)$ is the quantum field operator, its Fourier expansion is

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p}\left(a(p)e^{-ip\cdot x} + a^\dagger e^{ip\cdot x}\right) \qquad (1.6)$$

$$\pi(x) = \partial_0\phi(x) = -\frac{i}{2}\int \frac{d^3p}{(2\pi)^3}\left(a(p)e^{-ip\cdot x} - a^\dagger e^{ip\cdot x}\right) \qquad (1.7)$$

$$[\phi(x), \pi(y)]_{x^0=y^0} = i\delta^{(3)}(x-y) \qquad (1.8)$$

$$\to \left[a(p), a^\dagger(p')\right] = (2\pi)^3 2E_p\delta^{(3)}(p-p') \qquad (1.9)$$

$$a(p)|0\rangle = 0,\ a^\dagger(p)|0\rangle = |p\rangle \qquad (1.10)$$

where $|p\rangle$ is the one-particle Poincare invariant quantum state.

1.3 Gauge symmetry of gauge field theory

Let $A_\mu(x)$ is a gauge field, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is its field strength tensor. Its gauge symmetry is determined from a transformation

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \chi, \quad F'_{\mu\nu} = F_{\mu\nu} \tag{1.11}$$

where $\chi(x)$ is any real scalar function. A matter-coupled gauge field theory is determined from the Lagrangian

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1.12}$$

$$D_\mu \phi = (\partial_\mu + iqA_\mu)\phi \tag{1.13}$$

Let us the local unitary transformation on a scalar field, together the gauge transformation of a vector field

$$\phi(x) \to \phi'(x) = e^{i\alpha(x)} \phi(x) \tag{1.14}$$

$$A_\mu(x) \to A'_\mu(x) - \frac{i}{q} \partial_\mu \alpha(x) \tag{1.15}$$

we will get the invariant Lagrangian of (1.12).

Let G be a gauge group, and its generator is denoted as $g = e^{i\alpha}$. In case of α is a real constant, G is called global $U(1)$ symmetry, when $\alpha(x)$ is a real scalar function, G is called local $U(1)$ symmetry. In more general case

$$\alpha(x) = \alpha_a(x) t^a, \ \left[t^a, t^b\right] = i f^{abc} t^c, \text{ when } a, b, c = 1, 2, ..., N^2 - 1 \tag{1.16}$$

In this case G is called $SU(N)$ local symmetry, where $\{t^a\}$ is a set of its generators satisfy the $su(N)$ Lie algebra, with $\{f^{abc}\}$ is a set of structure constants. Note that for $N > 1$, the field $\phi(x)$ must be matrix-valued function, i.e., $N = 2$ the field ϕ is two-components column matrix and spinor.

1.4 The Coleman-Mandula "No-Go" theorem

In the gauge field theory of the standard model, we found that the gauge symmetry results to a set of degenerate mass multiplet, with a particle number of $N^2 - 1$. In the formulation of unified field theory, more set of particles is required into the theory, so the symmetry extension is looked for for the final theory.

Coleman and Mandula gave a constraint theorem on this criteria by simple stated that *the symmetry of the standard model theory (of particle with mass and spin) consist of a direct production of spacetime symmetry and gauge symmetry, where their generators are all bosonics. It cannot be extended by a symmetry with tensorial generators except for a symmetry with scalar generators.* The hidden reason is that we do not particle with spin higher than 2 in nature.

1.5 Hierarchy problem and naturalness

A hierarchy problem occurs when the fundamental value of some physical parameter, such as a coupling constant or a mass, in some Lagrangian is vastly different from its effective value, which is the value that gets measured in an experiment. This happens because the effective value is related to the fundamental value by a prescription known as *renormalization*, which applies corrections to it. Typically the renormalized value of parameters are close to their fundamental values, but in some cases, it appears that there has been a delicate cancellation between the fundamental quantity and the quantum corrections. The prove of this theorem based on a few assumptions and applied to the invariant of the S-matrix.

The naturalness is the property that the dimensionless ratios between free parameters or physical constants appearing in a physical theory should take values "of order 1" and that free parameters are not fine-tuned. But its has been observed that some parameters of the standard model theory vary by many orders of magnitude, and which require extensive "fine-tuning" of their current values of the models concerned.

More particles, out of the standard model particles, are required for the solution of loop corrections in the renormaliztion process.

1.6 The birth of supersymmetry

Some histories of *supersymmetry* was appear in the talk of P. Ramond in 2016 with a Tile of *Too Beautiful to Ignore*.
(https://cgc.physics.miami.edu/Miami2016/Ramond.pdf)

One can say that the significant idea of supersymmetry came from Yu A. Gol'fan and E.P. Likhtman (1971). They proposed the extension of spacetime symmetry beyond Poincare symmetry with a symmetric group of fermionic generators. This idea was later extended by R. Haag, J. Łopuszański, and

M. Sohnius (1975), which become the basic of supersymmetry formulation up to today.

Another major step of development given by A.Salam and J. Strathdee (1975), they induced *supercoordinates* of the *superspace* in order for doing the formulation of supersymmetric field theory.

1.7 Supersymmetric quantum mechanics

Here is an introduction to the idea of supersymmetry observed from the elementary quantum mechanics evaluation. First let us define the harmonic super-oscillator with Hamiltonian

$$H = a^\dagger a + c^\dagger c, \quad [a, a^\dagger] = 1, \ \{c, c^\dagger\} = 1 \tag{1.17}$$

Let $|n_B n_F\rangle$ is its eigen-basis, such that $n_B = \{0, 1, 2, ...\}$ and $n_F = \{0, 1\}$, and

$$a|n_b n_F\rangle = \sqrt{n_B}|n_b - 1, n_F\rangle, \ a|0 n_F\rangle = 0 \tag{1.18}$$
$$a^\dagger|n_B n_F\rangle = \sqrt{n_B + 1}|n_B + 1, n_F\rangle \tag{1.19}$$
$$c|n_B 0\rangle = 0, \ c^\dagger|n_B 0\rangle = |n_B 1\rangle \tag{1.20}$$
$$c|n_B 1\rangle = |n_B 0\rangle, \ c^\dagger|n_B 1\rangle = 0 \tag{1.21}$$
$$\text{and } H|n_B n_F\rangle = (n_B + n_F)|n_B n_F\rangle \tag{1.22}$$

Next we define

$$Q_+ = ac^\dagger, \qquad Q_- = ca^\dagger \tag{1.23}$$

We can observe that

$$Q_+|n_B n_F\rangle \sim |n_B - 1, n_F + 1\rangle \tag{1.24}$$
$$Q_-|n_B n_F\rangle \sim |n_B + 1, n_F - 1\rangle \tag{1.25}$$
$$\text{and } [H, Q_\pm] = 0 \tag{1.26}$$

Note that $Q_\pm$ are fermionic operators, according to summation theory, they change boson to fermion and vice versa. We can extend (1.17) to the form

$$\begin{aligned} H &= a^\dagger a + c^\dagger c + a^\dagger a c^\dagger c - c^\dagger c a^\dagger a \\ &= \{ac^\dagger, ca^\dagger\} = \{Q_+, Q_-\} \qquad (1.27) \\ &\to [H, Q_\pm] = 0 \qquad (1.28) \end{aligned}$$

Now we define

$$Q_1 = Q_+ + Q_-, \quad Q_2 = -i(Q_+ - Q_-) \tag{1.29}$$

$$H = Q_1^2 = Q_2^2 \tag{1.30}$$

$$[H, Q_i] = 0, \ \{Q_i, Q_j\} = 2H\delta_{ij} \tag{1.31}$$

This becomes the basic of supersymmetry algebra.