13 Spinor-Helicity Amplitude

13.1 Weyl spinors

From massless Dirac equation

$$i\partial\!\!\!/\psi(x) = 0 \tag{13.1}$$

where $\partial\!\!\!/ = \gamma^\mu\partial_\mu$ and γ^μ is Dirac gamma matrix satisfy Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. In Dirac representation of the gamma matrix, i.e.

$$\gamma^0 = \beta, \gamma^i = \beta\alpha^i,\ i = 1,2,3$$

$$\beta = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix},\ \alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

when 1_2 is 2x2 identity and $\{\sigma^i\}$ is a set of Pauli matrices, we cannot find a different solutions of Dirac positive energy $U(p)$ and negative energy $V(p)$ spinors, since they satisfy the same equation. On contrary we can have in Weyl representation of the Gamma matrix

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix},\ \sigma^\mu = (1_2, \vec{\sigma}) \text{ and } \bar{\sigma}^\mu = (1_2, -\vec{\sigma}) \tag{13.2}$$

Let us start with positive energy solution, with trial function

$$\psi(x) \propto U(p)e^{-ip\cdot x} \xrightarrow{(13.1)} p_\mu\gamma^\mu U(p) = 0 \tag{13.3}$$

$$U(p) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \xrightarrow{(13.2)} \begin{pmatrix} 0 & p_\mu\sigma^\mu \\ p_\mu\bar{\sigma}^\mu & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \tag{13.4}$$

$$\mapsto p_\mu\bar{\sigma}^\mu u_1 = (E + \vec{\sigma}\cdot\vec{p})u_1 = 0 \tag{13.5}$$

$$p_\mu\sigma^\mu u_2 = (E - \vec{\sigma}\cdot\vec{p})u_2 = 0 \tag{13.6}$$

Since p^μ is a null momentum, i.e. $p^2 = E^2 - \vec{p}^2 = 0 \mapsto E = |\vec{p}|$, and by definition of *helicity operator*

$$\hat{h} = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|} \tag{13.7}$$

Then we have from (13.5,6)

$$(1 + \hat{h})u_1 = 0 \mapsto \hat{h}u_1 = -u_1 \tag{13.8}$$

$$(1 - \hat{h})u_2 = 0 \mapsto \hat{h}u_2 = +u_2 \tag{13.9}$$

From this u_1 is said to be *left-handedness spinor*, and u_2 is said to be *right-handedness spinor*. The spinor symbols and indices are assigned in the form

$$u_1 = \psi_\alpha,\ \alpha = 1, 2 \text{ and } u_2 = \bar{\eta}^{\dot{\alpha}},\ \dot{\alpha} = \dot{1}, \dot{2} \tag{13.10}$$

Note that

$$(\psi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}},\ (\bar{\eta}^{\dot{\alpha}})^\dagger = \eta^\alpha \tag{13.11}$$

On the other hand we can lift or lower spinor index by using total anti-symmetric tensor as

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta,\ \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta \tag{13.12}$$

$$\bar{\eta}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\eta}^{\dot{\beta}},\ \bar{\eta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\eta}_{\dot{\beta}} \tag{13.13}$$

where

$$\epsilon^{12} = 1 = -\epsilon^{21} = -\epsilon_{12} = \epsilon_{21} \text{ and } \epsilon^{\dot{1}\dot{2}} = 1 = -\epsilon^{\dot{2}\dot{1}} = -\epsilon_{\dot{1}\dot{2}} = \epsilon_{\dot{2}\dot{1}}$$

We also assign spinor indices to the gamma matrix in the form

$$\sigma^\mu \mapsto \sigma^\mu_{\alpha\dot{\alpha}} \xrightarrow{(13.6)} p_\mu \sigma^\mu_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}} = 0 \tag{13.14}$$

$$\bar{\sigma}^\mu \mapsto \bar{\sigma}^{\mu\dot{\alpha}\alpha} \xrightarrow{(13.5)} p_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha}\psi_\alpha = 0 \tag{13.15}$$

From (13.4) one can write Dirac U spinor in the form

$$U = \begin{pmatrix} \psi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix} \tag{13.16}$$

Its Dirac conjugation is

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto \bar{U} = U^\dagger \gamma^0 = \begin{pmatrix} \eta^\alpha & \bar{\psi}_{\dot{\alpha}} \end{pmatrix} \tag{13.17}$$

Then we can observe the orthonomality and completeness relations as

$$\bar{U}U = \eta^\alpha \psi_\alpha + \bar{\psi}_{\dot{\alpha}}\bar{\eta}^{\dot{\alpha}} = 1 \tag{13.18}$$

$$U\bar{U} = \begin{pmatrix} \psi_\alpha\eta^\alpha & \psi_\alpha\bar{\psi}_{\dot{\alpha}} \\ \bar{\eta}^{\dot{\alpha}}\eta^\alpha & \bar{\eta}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}} \end{pmatrix} = \not{p} = \begin{pmatrix} 0 & p_\mu\sigma^\mu \\ p_\mu\bar{\sigma}^\mu & 0 \end{pmatrix} \tag{13.19}$$

$$\mapsto p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = p_{\alpha\dot{\alpha}} = \psi_\alpha \bar{\psi}_{\dot{\alpha}} \tag{13.20}$$

$$p_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha} = p^{\dot{\alpha}\alpha} = \bar{\eta}^{\dot{\alpha}}\eta^\alpha \tag{13.21}$$

and the other two products are zero. We can define the *helicity projection operator* in the form

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1_2 & 0 \\ 0 & 1_2 \end{pmatrix} \mapsto P_R = \frac{1}{2}(1_4 + \gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1_2 \end{pmatrix} \tag{13.22}$$

$$P_L = \frac{1}{2}(1_4 - \gamma_5) = \begin{pmatrix} 1_2 & 0 \\ 0 & 0 \end{pmatrix} \tag{13.23}$$

From (13.16) we will have

$$U_R = P_R U = \begin{pmatrix} 0 \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix},\ U_L = P_L U = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix} \tag{13.24}$$

$$\mapsto \bar{U}_R = \begin{pmatrix} \eta^\alpha & 0 \end{pmatrix},\ \bar{U}_L = \begin{pmatrix} 0 & \bar{\psi}_{\dot{\alpha}} \end{pmatrix} \tag{13.25}$$

The negative energy V spinor is defined in the form

$$V = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix},\ v_1 = u_2,\ v_2 = u_2 \mapsto V = \begin{pmatrix} \bar{\eta}^{\dot{\alpha}} \\ \psi_\alpha \end{pmatrix} \tag{13.26}$$

$$\bar{V} = \begin{pmatrix} \bar{\psi}_{\dot{\alpha}} & \eta^\alpha \end{pmatrix},\ V_R = \begin{pmatrix} \bar{\eta}^{\dot{\alpha}} \\ 0 \end{pmatrix},\ V_L = \begin{pmatrix} 0 \\ \psi_\alpha \end{pmatrix} \tag{13.27}$$

13.2 Spinor brackets

To be more practical we may define *spinor bravkets* in the form

$$\begin{aligned}
p\rangle = \quad & U_L(p) = \begin{pmatrix} \psi_\alpha(p) \\ 0 \end{pmatrix} & \equiv V_R(p) = \begin{pmatrix} 0 \\ \psi_\alpha(p) \end{pmatrix} && (13.28)\\
\langle p = \quad & \bar{U}_R(p) = \begin{pmatrix} \eta^\alpha(p) & 0 \end{pmatrix} & \equiv \bar{V}_L(p) = \begin{pmatrix} 0 & \eta^\alpha(p) \end{pmatrix} && (13.29)\\
p] = \quad & U_R(p) = \begin{pmatrix} 0 \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix} & \equiv V_L(p) = \begin{pmatrix} \bar{\eta}^{\dot{\alpha}}(p) \\ 0 \end{pmatrix} && (13.30)\\
[p = \quad & \bar{U}_L(p) = \begin{pmatrix} 0 & \bar{\psi}_{\dot{\alpha}}(p) \end{pmatrix} & \equiv \bar{V}_R(p) = \begin{pmatrix} \bar{\psi}_{\dot{\alpha}}(p) & 0 \end{pmatrix} && (13.31)
\end{aligned}$$

Let us determine their inner (scalar) products

$$\begin{aligned}
\langle pq\rangle &= \bar{U}_R(p)U_L(q) = \eta^\alpha(p)\psi_\alpha(q) \equiv \bar{V}_L(p)V_R(q) && (13.32)\\
[pq] &= \bar{U}_L(p)U_R(q) = \bar{\psi}_{\dot{\alpha}}(p)\bar{\eta}^{\dot{\alpha}}(q) \equiv \bar{V}_R(p)V_L(q) && (13.33)\\
\langle pq] &= \bar{U}_R(p)U_R(p) = 0 = \bar{V}_L(p)V_L(q) && (13.34)\\
[pq\rangle &= \bar{U}_L(p)U_L(q) = 0 = \bar{V}_R(p)V_R(q) && (13.35)
\end{aligned}$$

Note the anti-symmetric property

$$\begin{aligned}
\langle pq\rangle = \eta^\alpha(p)\psi_\alpha(q) = \epsilon^{\alpha\beta}\eta_\beta(p)\psi_\alpha(q) &= -\epsilon^{\alpha\beta}\psi_\alpha(q)\eta_\beta(p) \\
&= -\psi^\beta(q)\eta_\beta(p) = -\langle qp\rangle && (13.36)\\
&\mapsto \langle pp\rangle = 0 && (13.37)\\
\text{Also }\ [pq] = -[qp] &\mapsto [pp] = 0 && (13.38)
\end{aligned}$$

Their outer products

$$\begin{aligned}
p\rangle\langle q &= U_L(p)\bar{U}_R(q) = \psi_\alpha(p)\eta^\alpha(q) = 0 = V_R(p)\bar{V}_L(q) && (13.39)\\
p][q &= U_R(p)\bar{U}L(q) = \bar{\eta}^{\dot{\alpha}}(p)\bar{\psi}_{\dot{\alpha}}(q) = 0 = VL(p)\bar{V}_L(q) && (13.40)\\
p\rangle[q &= U_L(p)\bar{U}_L(q) = \psi_\alpha(p)\bar{\psi}_{\dot{\alpha}}(q) \equiv V_R(p)\bar{V}_R(q) && (13.41)\\
p]\langle q &= U_R(p)\bar{U}_R(q) = \bar{\eta}^{\dot{\alpha}}(p)\eta^\alpha(q) \equiv V_L(p)\bar{V}_L(q) && (13.42)
\end{aligned}$$

The inner products with gamma matrix

$$\langle p\gamma^\mu q] = \bar{U}_R(p)\gamma^\mu U_R(q) = \eta^\alpha(p)\sigma^\mu_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}}(q) \tag{13.43}$$
$$[p\gamma^\mu q\rangle = \bar{U}_L(p)\gamma^\mu U_L(q) = \bar{\psi}_{\dot{\alpha}}(p)\bar{\sigma}^{\mu\dot{\alpha}\alpha}\psi_\alpha(q) \tag{13.44}$$
$$\langle p\gamma^\mu q\rangle = \bar{U}_R(p)\gamma^\mu U_L(q) = 0 \tag{13.45}$$
$$[p\gamma^\nu q] = \bar{U}_L(p)\gamma^\mu U_R(q) = 0 \tag{13.46}$$

Let us determine

$$\begin{aligned}\langle p\gamma^\mu q]\langle r\gamma_\mu s] &= \eta^\alpha(p)\sigma^\mu_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}(q)\lambda^\beta(r)\sigma_{\mu\beta\dot{\beta}}\bar{\chi}^{\dot{\beta}}(s)\\ = \eta^\alpha(p)\bar{\psi}^{\dot{\beta}}(q)\lambda^\beta(r)\bar{\chi}^{\dot{\beta}}(s)2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} &= -2\eta^\alpha(p)\lambda_\alpha(r)\bar{\psi}_{\dot{\beta}}(q)\bar{\chi}^{\dot{\beta}}(s)\\ &= -2\langle pr\rangle[qs] = 2\langle pr\rangle[sq]\end{aligned} \tag{13.47}$$
$$\langle p\gamma^\mu q]\langle q\gamma_\mu p] = 2\langle pq\rangle[pq] \mapsto= 2|\langle pq\rangle|^2 \tag{13.48}$$

After we have used the fact that $\sigma^\mu_{\alpha\dot{\alpha}}\sigma_{\mu\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}$ and $[pq] = \langle pq\rangle^*$.

13.3 Spinor representation of vectors

13.3.1 Spinor representation of 4-momentum p^μ

From (13.19-21), one can write in more details as

$$p_\mu\sigma^\mu_{\alpha\dot{\alpha}} = p_{\alpha\dot{\alpha}} = \begin{pmatrix} p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & p^0 + p^3 \end{pmatrix} \tag{13.49}$$
$$= \begin{pmatrix} p^- & -p_\perp^- \\ -p_\perp^+ & p^+ \end{pmatrix} \equiv \lambda_\alpha(p)\bar{\lambda}_{\dot{\alpha}}(p) \tag{13.50}$$
$$\mapsto \det p_{\alpha\dot{\alpha}} = (p^0)^2 - (p^1)^2 - (p^2)^2 - (p^3)^2 = p^2 = 0 \tag{13.51}$$

where $p^- = p^0 - p^3,\ p^+ = p^0 + p^3,\ p_\perp^- = p^1 - ip^2,\ p_\perp^+ = p^1 + ip^2$. One can make a quest of $\lambda_\alpha(p)$ to satisfy (13.45) in the form

$$\lambda_\alpha(p) = \frac{1}{\sqrt{p^+}}\begin{pmatrix} -p_\perp^- \\ p^+ \end{pmatrix} = p\rangle \tag{13.52}$$
$$\mapsto \bar{\lambda}_{\dot{\alpha}}(p) = (\lambda_\alpha(p))^\dagger = \frac{1}{\sqrt{p^+}}\begin{pmatrix} -p_\perp^+ & p^+ \end{pmatrix} = [p \tag{13.53}$$
$$\lambda^\alpha(p) = \epsilon^{\alpha\beta}\lambda_\beta = \frac{1}{\sqrt{p^+}}\begin{pmatrix} -p^+ & -p_\perp^- \end{pmatrix} = \langle p \tag{13.54}$$
$$\bar{\lambda}^{\dot{\alpha}}(p) = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\lambda}_{\dot{\beta}} = \frac{1}{\sqrt{p^+}}\begin{pmatrix} p^+ \\ p_\perp^+ \end{pmatrix} = p] \tag{13.55}$$

Let us determine

$$\begin{aligned}\langle pq\rangle\,[qp] &= \lambda^\alpha(p)\mu_\alpha(q)\bar\mu_{\dot\beta}(q)\bar\mu^{\dot\beta}(p)\\ &= \frac{1}{p^+q^+}(p^+q_\perp^- - p_\perp^- q^+)(-q_\perp^+ p^+ + q^+ p_\perp^+)\\ &= -\frac{p^+}{q^+}[q_\perp^- q_\perp^+] + q_\perp^- p_\perp^+ + p_\perp^- q_\perp^+ - \frac{q^+}{p^+}[p_\perp^- p_\perp^+]\\ &= -\frac{p^+}{q^+}[(q^1)^2+(q^2)^2] + 2p^1q^1 + 2p^2q^2 - \frac{q^+}{p^+}[(p^1)^2+(p^2)^2]\\ &= 2q^1p^1 + 2q^2p^2 - q^+(p^0-p^3) - p^+(q^0-q^3)\\ &= -2q^0p^0 + 2q^1p^1 + 2q^2p^2 + 2q^3p^3 = -2p\cdot q \qquad (13.56)\end{aligned}$$

13.3.2 Spinor representation of 4-polarization ϵ^μ

Photon polarization vectors are $\epsilon^\mu_{+/-}(k)$, their spinor representations are defined in the form

$$\epsilon^\mu_+(k,r) = -\frac{\langle r\gamma^\mu k]}{\sqrt{2}\langle kr\rangle} \text{ and } \epsilon^\mu_-(k,r) = -\frac{\langle k\gamma^\mu r]}{\sqrt{2}[rk]} \qquad (13.57)$$

where r^μ is an arbitrary reference momentum in which $r\cdot k \neq 0$. We can observe their transversality with k^μ as

$$k_\mu\epsilon^\mu_+(k,r) = -\frac{\langle r\not{k}k]}{\sqrt{2}\langle kr\rangle} = -\frac{\langle rk]\langle kk] + \langle rk\rangle[kk]}{\sqrt{2}\langle kr\rangle} = 0 \qquad (13.58)$$

$$k_\mu\epsilon^\mu_-(k,r) = -\frac{\langle k\not{k}r]}{\sqrt{2}[rk]} = -\frac{\langle kk\rangle[kr] + \langle kk]\langle kr]}{\sqrt{2}[rk]} = 0 \qquad (13.59)$$

After we have used the completeness relation of Dirac U spinor

$$\sum_{h=R/L} U_h(p)\bar U_h(p) = p\rangle[p + p]\langle p = \not{p}$$

With the basic property $\epsilon^{\mu *}_\pm(k,r) = \epsilon^\mu_\mp(k,r)$, we can observe the completeness relation as

$$\begin{aligned}\sum_{h=\pm}\epsilon^\mu_h(k,r)\epsilon^{\nu *}_h(k,r) &= \epsilon^\mu_+(k,r)\epsilon^{\nu *}_+(k,r) + \epsilon^\mu_-(k,r)\epsilon^{\nu *}_-(k,r)\\ = \epsilon^\mu_+(k,r)\epsilon^\nu_-(k,r) + \epsilon^\mu_-(k,r)\epsilon^\nu_+(k,r) &= \frac{\langle r\gamma^\mu k]\langle k\gamma^\nu r]}{2\langle kr\rangle[rk]} + \frac{\langle k\gamma^\mu r]\langle r\gamma^\nu k]}{2[rk]\langle kr\rangle}\\ &= \frac{1}{2\langle kr\rangle[rk]}\Big\{\lambda^\alpha(r)\sigma^\mu_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}(k)\lambda^\beta(k)\sigma^\nu_{\beta\dot\beta}\bar\lambda^{\dot\beta}(r)\\ &\quad + \psi^\alpha(k)\sigma^\mu_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}(r)\lambda^\beta(r)\sigma^\nu_{\beta\dot\beta}\bar\psi^{\dot\beta}(k)\Big\}\\ &= \frac{1}{2\langle kr\rangle[rk]}\Big\{-\eta^{\mu\nu}\lambda^\alpha(r)\psi_\alpha(k)\bar\psi_{\dot\beta}(k)\bar\lambda^{\dot\beta}(r)\\ &\quad -\eta^{\mu\nu}\psi^\alpha(k)\lambda_\alpha(r)\bar\lambda_{\dot\beta}(r)\bar\psi^{\dot\beta}(k)\Big\} = -\eta^{\mu\nu} \qquad (13.60)\end{aligned}$$

13.4 Spinor-helicity amplitudes of QED

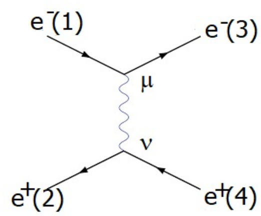

Figure 13.1: t-channel Bhabha scattering.

The t-channel Bhabha scattering appear in figure (13.1), an expression of its amplitude is

$$
\begin{aligned}
i\mathcal{M}_t(h_1,h_2,h_3,h_4) - &= (-ie)^2 \bar{U}(3)\gamma^\mu U(1) \frac{-i\eta_{\mu\nu}}{t} \bar{V}(2)\gamma^\nu V(4) \\
&= i\frac{e^2}{t}\bar{U}(3)\gamma^\mu U(1)\bar{V}(2)\gamma_\mu V(4) \qquad (13.61)
\end{aligned}
$$

where we have denoted $U/V(i) = U/V(p_i, h_i)$. The non-zero terms are list below

$$
\begin{aligned}
i\mathcal{M}_t(++--) &= i\frac{e^2}{t}\psi^\alpha(p_3)\sigma^\mu_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(p_1)\psi^\beta(p_2)\sigma_{\mu\beta\dot{\beta}}\bar{\lambda}^{\dot{\beta}}(p_4) \\
&= i\frac{e^2}{t}\langle p_3\gamma^\mu p_1]\langle p_2\gamma_\mu p_4] = -2i\frac{e^2}{t}\langle p_2 p_3\rangle[p_1 p_4] \qquad (13.62) \\
i\mathcal{M}_t(--++) &= i\frac{e^2}{t}\bar{\lambda}_{\dot{\alpha}}(p_3)\bar{\sigma}^{\mu\dot{\alpha}\alpha}\psi_\alpha(p_1)\bar{\lambda}_{\dot{\beta}}(p_2)\bar{\sigma}_\mu^{\dot{\beta}\beta}\psi_\beta(p_4) \\
&= i\frac{e^2}{t}[p_3\gamma^\mu p_1\rangle[p_2\gamma_\mu p_4\rangle = -2i\frac{e^2}{t}\langle p_2 p_3\rangle[p_1 p_4] \qquad (13.63) \\
i\mathcal{M}_t(+-+-) &= i\frac{e^2}{t}\psi^\alpha(p_3)\sigma^\mu_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(p_1)\bar{\lambda}_{\dot{\beta}}(p_4)\bar{\sigma}_\mu^{\dot{\beta}\beta}\psi_\beta(p_4) \\
&= i\frac{e^2}{t}\langle p_3\gamma^\mu p_1][p_2\gamma_\mu p_4\rangle = -2i\frac{e^2}{t}\langle p_3 p_4\rangle[p_2 p_1] \qquad (13.64) \\
i\mathcal{M}_t(-+-+) &= i\frac{e^2}{t}\bar{\lambda}_{\dot{\alpha}}(p_3)\bar{\sigma}^{\mu\dot{\alpha}\alpha}\psi_\alpha(p_1)\psi^\beta(p_2)\sigma_{\mu\beta\dot{\beta}}\bar{\lambda}^{\dot{\beta}}(p_4) \\
&= i\frac{e^2}{t}[p_3\gamma^\mu p_1\rangle\langle p_2\gamma_\mu p_4] = -2i\frac{e^2}{t}[p_3 p_4]\langle p_2 p_1\rangle \qquad (13.65)
\end{aligned}
$$

The mean amplitudes squared are

$$\overline{|\mathcal{M}_t(++--)|^2} = \frac{e^4}{t^2}\langle p_2p_3\rangle[p_3p_2][p_1p_4]\langle p_4p_1\rangle$$
$$= 4\frac{e^4}{t^2}(p_1\cdot p_3)(p_2\cdot p_4) = e^4 = \overline{|\mathcal{M}(--++)|^2} \tag{13.66}$$
$$\overline{|\mathcal{M}_t(+-+-)|^2} = \frac{e^4}{t^2}\langle p_3p_4\rangle[p_4p_3][p_2p_1]\langle p_1p_2\rangle$$
$$= 4\frac{e^4}{t^2}(p_3\cdot p_4)(p_1\cdot p_2) = e^4\frac{s^2}{t^2} = \overline{|\mathcal{M}_t(-+-+)|^2} \tag{13.67}$$

Finally we will have

$$\overline{|\mathcal{M}_t|^2} = e^4\left(\frac{s^2+t^2}{t^2}\right) \tag{13.68}$$