5 Quantization of Dirac Spinor Field

5.1 Dirac spinor field

Dirac spinor field $\psi(x)$ is classical aspects of solution of Dirac equation derived from Dirac equation. For free field it is written in the form

$$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0 \tag{5.1}$$

where γ^μ is Dirac gamma matrix satisfies Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. Let us define *Dirac conjugation* as

$$\bar{\psi} = \psi^\dagger\gamma^0, \text{ since } (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu \mapsto \bar{\psi}(i\gamma^\mu\overleftarrow{\partial}_\mu + m) = 0 \tag{5.2}$$

Multiply (5.1) from the left with $\bar{\psi}$ and multiply (5.2) from the right with ψ and do the summation, we get

$$i\bar{\psi}\gamma^\mu\overleftarrow{\partial}_\mu\psi + i\bar{\psi}\gamma^\mu\partial_\mu\psi = \partial_\mu(i\bar{\psi}\gamma^\mu\psi) = 0 \tag{5.3}$$

$$\mapsto \partial_\mu j^\mu = 0, \; j^\mu = i\bar{\psi}\gamma^\mu\psi \tag{5.4}$$

It is the conserved Dirac current density.

This equation can be derived from the Dirac Lagrangian density of the form

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \tag{5.5}$$

Under insertion into Euler-Lagrange equation we will have

$$\frac{\partial\mathcal{L}}{\partial\partial_\mu\bar{\psi}} = 0, \frac{\partial\mathcal{L}}{\partial\bar{\psi}} = (i\gamma^\mu\partial_\mu - m)\psi = 0$$

The conjugate momentum field $\pi(x)$ of $\partial_0\psi$ is

$$\pi(x) = \frac{\partial\mathcal{L}}{\partial\partial_0\psi} = i\bar{\psi}\gamma^0 = i\psi^\dagger \tag{5.6}$$

Note that there is no $\partial_0\psi^\dagger$ term, so that there will be no its conjugate momentum $\pi^\dagger$. Then Dirac Hamiltonian will be derived in the form

$$\mathcal{H} = \pi\partial_0\psi - \mathcal{L} \mapsto H = \int d^3x\mathcal{H} = \int d^3x \left(-i\bar{\psi}\vec{\gamma}\cdot\nabla\psi + m\bar{\psi}\psi\right) \tag{5.7}$$

Free field solution is derived in the form

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3 2E_k}\sum_s \left(a(k,s)U(k,s)e^{-ik\cdot x} + b^* V(k,s)e^{ik\cdot x}\right)_{\omega=E_k} \tag{5.8}$$

where s is the spin degree of freedom, $U(k,s)$ is the positive energy spinor and $V(k,s)$ is the negative energy spinor. There are appear in the form

$$U(k,s) = \sqrt{E_k+m}\begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{k}}{E_k+m}\chi_s \end{pmatrix}, \; V(k,s) = \sqrt{E_k+m}\begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{k}}{E_k+m}\chi_s \\ \chi_s \end{pmatrix} \tag{5.9}$$

where χ_s is regular spinor basis. These U and V satisfy the othonarmality and completeness relations in the form

$$\sum_s U^\dagger(k,s)U(k,s) = 2E_k, \ \sum_s U(k,s)\bar{U}(k,s) = \not{k} + m \tag{5.10}$$

$$\sum_s V^\dagger(k,s)V(k,s) = 2E_k, \ \sum_s V(k,s)V(k,s) = \not{k} - m \tag{5.11}$$

where $\not{k} = \gamma^\mu k_\mu$ is known as *Dirac slashed.* For later using, one can write

$$\pi(x) = i\psi^\dagger(x) = i\int \frac{d^3k}{(2\pi)^3 2E_k}\sum_s \big(b(k,s)V^\dagger(k,s)e^{-ik\cdot x} + a^*(k,s)U^\dagger(k,s)e^{ik\cdot x}\big)_{\omega=E_k} \tag{5.12}$$

$$\nabla\psi(x) = i\int \frac{d^3k}{(2\pi)^3 2E_k}\vec{k}\sum_s \big(a(k,s)U(k,s)e^{-ik\cdot x} - b^*(k,s)V(k,s)e^{ik\cdot x}\big)_{\omega=E_k} \tag{5.13}$$

5.2 Canonical quantization

We just promote the field $\psi(x)$ and its conjugate momentum field $\pi(x)$ to be quantum operators satisfy the *equal time anti-commutation relation* of the form

$$\{\psi(x), \pi(y)\}_{x^0=y^0} = i\delta^{(3)}(\vec{x} - \vec{y}) \tag{5.14}$$

when the operator notion with *no hat* is understood for convenient. Let us check this by insertion their full expressions form above, we will have

$$\begin{aligned}
\{\psi(x), \pi(y)\}_{x^0=y^0=0} &= i\int \frac{d^3k}{(2\pi)^3 2E_k}\int \frac{d^3k'}{(2\pi)^3 2E_{k'}}\sum_{s,s'} \\
&\times \Big\{\Big(a(k,s)U(k,s)e^{i\vec{k}\cdot\vec{x}} + b^\dagger(k,s)V(k,s)e^{-i\vec{k}\cdot\vec{x}}\Big), \\
&\Big(a^\dagger(k',s')U^\dagger(k',s')e^{-i\vec{k}'\cdot\vec{y}} + b(k',s')V^\dagger(k',s')e^{i\vec{k}'\cdot\vec{y}}\Big)\Big\} \\
&= i\int \frac{d^3k}{(2\pi)^3 2E_k}\int \frac{d^3k'}{(2\pi)^3 2E_{k'}}\sum_{s,s'} \\
&\times \Big[\big\{a(k,s), a^\dagger(k',s')\big\}U(k,s)U^\dagger(k',s')e^{i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{y}} \\
&+ \{a(k,s), b(k',s')\}U(k,s)V^\dagger(k,s)e^{i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{y}} \\
&+ \big\{b^\dagger(k.s), a^\dagger(k',s')\big\}V(k,s)U^\dagger(k',s')e^{-i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{y}} \\
&\big\{b^\dagger(k,s), b(k',s')\big\}V(k,s)V^\dagger(k',s')e^{-i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{y}}\Big]
\end{aligned} \tag{5.15}$$

Let us assign the anti-commutation relations

$$\begin{aligned}
\{a(k,s),a(k',s')\} &= 0 = \{a^\dagger(k,s),a^\dagger(k',s')\} \\
\{b(k,s),b(k',s')\} &= 0 = \{b^\dagger(k,s),b^\dagger(k',s')\} \\
\{a(k,s),b(k',s')\} &= 0 = \{a^\dagger(k,s),b^\dagger(k',s')\} \\
\{a(k,s),a^\dagger(k',s')\} &= (2\pi)^3 2E_k \delta^{(3)}(k-k')\delta_{s,s'} \\
\{b(k,s),b^\dagger(k',s')\} &= (2\pi)^3 2E_k \delta^{(3)}(k-k')\delta_{s,s'} \\
\{a(k,s),b^\dagger(k',s')\} &= 0 = \{a^\dagger(k,s),b(k',s')\}
\end{aligned} \tag{5.16}$$

From (5.17), we will have

$$\begin{aligned}
\{\psi(x),\pi(y)\}_{x^0=y^0=0} &= i\int \frac{d^3k}{(2\pi)^3 2E_k}\sum_s [U(k,s)U^\dagger(k,s)e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \\
&\qquad + V(k,s)V^\dagger(k,s)e^{-i\vec{k}\cdot(\vec{(x)}-\vec{y})} \\
&= i\int \frac{d^3k}{(2\pi)^3 2E_k}[\underbrace{\sum_s U(k,s)\bar{U}(k,s)}_{=(\not{k}+m)}\gamma^0 e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \\
&\qquad + \underbrace{\sum_s V(k,s)\bar{V}(k,s)}_{=(\not{k}-m)}\gamma^0 e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}]
\end{aligned} \tag{5.17}$$

Using the fact that

$$\not{k} \pm m = \gamma^0 k_0 - \vec{\gamma}\cdot\vec{k} \pm m, \;\; k_0 = E_k$$

After changing sign $\vec{k} \to -\vec{k}$ of term from the V-spinor, we have

$$\{\psi(x),\pi(y)\}_{x^0=y^0=0} = i\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} = i\delta^{(3)}(\vec{x}-\vec{y}) \tag{5.18}$$

as require.

Next let us determine the Hamiltonian operator of Dirac spinor field. From

(5.7), let us evaluate H time $x^0 = 0$, we will have

$$
\begin{aligned}
H = & \int d^3x \int \frac{d^3k}{(2\pi)^3 2E_k} \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} \sum_{s,s'} \\
& \times \left[\left(a^\dagger(k,s)\bar{U}(k,s)e^{-i\vec{k}\cdot x} + b(k,s)\bar{V}(k,s)e^{i\vec{k}\cdot\vec{x}} \right) \right. \\
& \times(\vec{\gamma}\cdot\vec{k}') \left(a(k',s')U(k',s')e^{i\vec{k}'\cdot\vec{x}} - b^\dagger(k',s')V(k',s')e^{-i\vec{k}'\cdot\vec{x}} \right) \\
& +m \left(a^\dagger(k,s)\bar{U}(k,s)e^{-i\vec{k}\cdot\vec{x}} + b(k,s)\bar{V}(k,s)e^{+i\vec{k}\cdot\vec{x}} \right) \\
& \left. \times \left(a(k',s')U(k',s')e^{i\vec{k}'\cdot\vec{x}} + b^\dagger(k',s')V(k',s')e^{-i\vec{k}'\cdot\vec{x}} \right) \right] \\
= & \int d^3x \int \frac{d^3k}{(2\pi)^3 2E_k} \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} \sum_{s,s'} \\
& \times \left[a^\dagger(k,s)a(k',s')\bar{U}(k,s)(\vec{\gamma}\cdot\vec{k}')U(k',s')e^{-i(\vec{k}-\vec{k}')\cdot\vec{x}} \right. \\
& -b(k,s)b^\dagger(k',s')\bar{V}(k,s)(\vec{\gamma}\cdot\vec{k}')V(k',s')e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} \\
& +ma^\dagger(k,s)a(k',s')\bar{U}(k,s)U(k',s')e^{-i(\vec{k}-\vec{k}')\cdot\vec{x}} \\
& \left. + mb(k,s)b^\dagger(k',s')\bar{V}(k,s)V(k',s')e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} \right]
\end{aligned} \tag{5.19}
$$

After we have ignored the overlapping terms between positive and negative energy spinors. Integrate $\int d^3x$ we will get delta function $\delta^{(3)}(\vec{k}-\vec{k}')$ and do $\int d^3k'$ integration using this delta function. From Dirac equations of U and V spinors

$$(\gamma^\mu k_\mu - m)U(k,s) = 0 \mapsto (\vec{\gamma}\cdot\vec{k})U(k,s) = (\gamma^0 E_k - m)U(k,s)$$

$$(\gamma^\mu k_\mu + m)V(k,s) = 0 \mapsto (\vec{\gamma}\cdot\vec{k})V(k,s) = (\gamma^0 E_k + m)V(k,s)$$

Insertion these relations into (5.20), we can observe the cancellation of the mass terms. After using completeness relation of the U and V spinors, we will have

$$H = \int \frac{d^3k}{(2\pi)^3 2E_k} E_k \sum_s \left(a^\dagger(k,s)a(k,s) - b(k,s)b^\dagger(k,s) \right) \tag{5.20}$$

$$
\begin{aligned}
= & \int \frac{d^3k}{(2\pi)^3 2E_k} E_k \sum_s \left[a^\dagger(k,s)a(k,s) + b^\dagger(k,s)b(k,s) \right] \\
& - \int d^3k E_k \delta^{(3)}(0)
\end{aligned} \tag{5.21}
$$

5.3 Dirac hole theory

From (5.22), we observe the filled negative energy sea. Dirac was interpreted as the vacuum energy of fermionic oscillator. Two types of particles was created from this, one is particle created by $a^\dagger(k,s)$ and the other is anti-particle created as a hole in the sea by $b^\dagger(k,s)$. They are created in pair, particle has positive energy and anti-particle has negative energy. They fulfill momentum conservation by moving into opposite directions, i.e., particle has momentum $\vec{k}$ while anti-particle has momentum $-\vec{k}$. They also has opposite charges to fulfill charge conservation. From this point of view, Dirac particles will be created in pair from vacuum and also annihilate from pair into the vacuum.

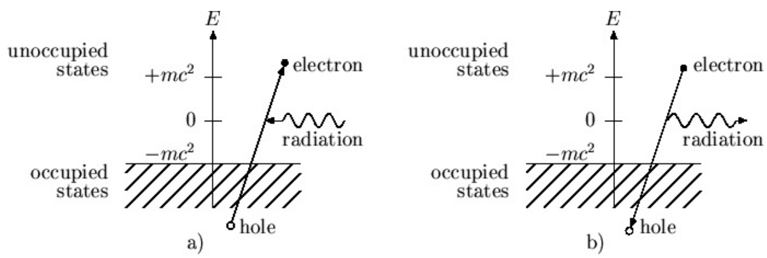

Figure 5.1: Dirac hole theory.

Let us denote $|0\rangle$ as the fermionic vacuum state, the particle and anti-particle will be created from this state as

$$a(k,s)|0\rangle = 0,\ a^\dagger(k,s)|0\rangle = |k,s\rangle \tag{5.22}$$

$$b(k',s')|0\rangle = 0,\ b^\dagger(k',s')|0\rangle = |k',s'\rangle \tag{5.23}$$

where $\vec{k}' = -\vec{k}$ when they are created in pair.

5.4 Spinor field propagators

Let us evaluate the spinor field propagator from time-ordered spinor field operators as

$$\begin{aligned}
\Delta(x,y) &= \langle 0|T[\psi(x)\bar{\psi}(y)]|0\rangle \\
&= \theta(x^0-y^0)\langle 0|\psi(x)\bar{\psi}(y)|0\rangle - \theta(y^0-x^0)\langle 0|\bar{\psi}(y)\psi(x)|0\rangle \\
&= \int \frac{d^3k}{(2\pi)^3 2E_k}\int \frac{d^3k'}{(2\pi)^3 2E_{k'}}\sum_{s,s'} \\
&\times \Big\{\theta(x^0-y^0)\langle 0|\left(a(k,s)U(k,s)e^{-ik\cdot x} + b^\dagger(k,s)V(k,s)e^{ik\cdot x}\right) \\
&\times \left(a^\dagger(k',s')\bar{U}(k',s')e^{ik'\cdot y} + b(k',s')\bar{V}(k',s')e^{-ik'\cdot y}\right)|0\rangle \\
&-\theta(y^0-x^0)\langle 0|\left(a^\dagger(k',s')\bar{U}(k',s')e^{ik'\cdot y} + b(k',s')\bar{V}(k',s')e^{-ik'\cdot y}\right) \\
&\times \left(a(k,s)U(k,s)e^{-ik\cdot x} + b^\dagger(k,s)V(k,s)e^{ik\cdot x}\right)|0\rangle\Big\}
\end{aligned} \tag{5.24}$$

The survival terms are

$$\Delta(x,y) = \int \frac{d^3k}{(2\pi)^3 2E_k} \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} \sum_{s,s'}$$

$$\Big\{ \theta(x^0 - y^0) \langle 0 | a(k,s) a^\dagger(k',s') | 0 \rangle U(k,s) \bar{U}(k',s') e^{-ik\cdot x + ik'\cdot y}$$

$$- \theta(y^0 - x^0) \langle 0 | b(k',s') b^\dagger(k,s) | 0 \rangle \bar{V}(k',s') V(k,s) e^{-ik'\cdot y + ik\cdot x} \Big\} \qquad (5.25)$$

After using their commutation relations and do integration $\int d^3k'$ using delta function and do the summation $\Sigma_{s'}$ using delta function $\delta_{s,s'}$, we get

$$\Delta(x,y) = \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_s \Big[\theta(x^0 - y^0) U(k,s) \bar{U}(k,s) e^{-ik\cdot(x-y)}$$

$$- \theta(y^0 - x^0) \bar{V}(k,s) V(k,s) e^{-ik\cdot(y-x)} \Big] \qquad (5.26)$$

Using the fact that

$$\sum_s U(k,s) \bar{U}(k,s) = \not{k} + m$$

$$\text{From DE} : (\not{k} + m) V(k,s) = 0 \mapsto m = -\not{k}$$

Then we have from above

$$\Delta(x,y) = \int \frac{d^3k}{(2\pi)^3 2E_k} \Big[\theta(x^0 - y^0)(\not{k} + m) e^{-ik\cdot(x-y)}$$

$$+ \theta(y^0 - x^0)(\not{k} - m) e^{-ik\cdot(y-x)} \Big] \qquad (5.27)$$

$$\begin{aligned} \mapsto \Delta(x,y) &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{i(\not{k} + m)}{\omega^2 - E_k^2 + i\epsilon} e^{-ik\cdot(x-y)} + \frac{i(\not{k} - m)}{\omega^2 - E_k^2 + i\epsilon} e^{-ik\cdot(y-x)} \right] \\ &= \int \frac{d^4k}{(2\pi)^4} \Delta(k) e^{-ik\cdot(x-y)} \qquad (5.28) \end{aligned}$$

where

$$\Delta(k) = \Delta^{(+)}(k) + \Delta^{(-)}(k) \qquad (5.29)$$

$$\Delta^{(+)}(k) = \frac{i(\not{k} + m)}{k^2 - m^2 + i\epsilon}, \text{ particle propagator} \qquad (5.30)$$

$$\Delta^{(-)}(k) = \frac{i(\not{k} - m)}{k^2 - m^2 + i\epsilon}, \text{ anti} - \text{particle propagator} \qquad (5.31)$$

After we have used the fact that $E_k = \sqrt{\vec{k}^2 + m^2}$ and $k^2 = \omega^2 - \vec{k}^2$. Note that particle propagator moves forward in time, while anti-particle propagator moves backward in time.

5.5 LSZ reduction formula of spinor field

From free field solution

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_s \left[a(k,s)U(k,s)e^{-ik\cdot x} + b^\dagger(k,s)V(k,s)e^{ik\cdot x}\right]_{\omega=E_k}$$

We observe that

$$\int d^3x e^{ik\cdot x}\psi(x) = \frac{1}{2E_k}\sum_s \left[a(k,s)U(k,s) + e^{2iE_k x^0} b^\dagger(-k,s)V(-k,s)\right] \quad (5.32)$$

$$\mapsto \int d^3x e^{ik\cdot x} U^\dagger(k,s')\psi(x) = \frac{1}{2E_k}\sum_s a(k,s)[\underbrace{U^\dagger(k,s')U(k,s)}_{=2E_k\delta_{ss'}}$$

$$+ e^{2iE_k x^0} b^\dagger(-k,s)\underbrace{U(k,s')V(-k,s)}_{=0}] \quad (5.33)$$

Then we have

$$a(k,s) = \int d^3x e^{ik\cdot x} U^\dagger(k,s)\psi(x) \equiv \int d^3x e^{ik\cdot x}\bar{U}(k,s)\gamma^0\psi(x) \quad (5.34)$$

$$a^\dagger(k,s) = \int d^3x e^{-ik\cdot x}\bar{\psi}(x)\gamma^0 U(k,s) \quad (5.35)$$

$$b(k,s) = \int d^3x e^{ik\cdot x}\bar{V}(k,s)\gamma^0\psi(x) \quad (5.36)$$

$$b^\dagger(k,s) = \int d^3k e^{-ik\cdot x}\bar{\psi}\gamma^0 V(k,s) \quad (5.37)$$

This we ll lead to the replacements

$$|k,s,+\rangle = a^\dagger(k,s)|0\rangle = \int d^3x e^{-ik\cdot x}\bar{\psi}(x)\gamma^0 U(k,s)|0\rangle$$

$$\simeq \int d^4x \partial_{x_0}\left\{e^{-ik\cdot x}\bar{\psi}(x)\right\}\gamma^0 U(k,s)|0\rangle$$

$$= \int d^4x \left\{\bar{\psi}(x)\left(\partial_{x_0}e^{-ik\cdot x}\right)\gamma^0 U(k,s) + e^{-ik\cdot x}\left(\bar{\psi}(x)\overleftarrow{\partial}_{x_0}\right)\gamma^0 U(k,s)\right\}|0\rangle \quad (5.38)$$

Using the fact that

$$(i\gamma^0\partial_0 + i\gamma^i\partial_i - m)e^{-ik\cdot x} = 0 \mapsto \partial_0 e^{-ik\cdot x} = -i(i\gamma^i\partial_i + m)\gamma^0 e^{-ik\cdot x}$$

We then have

$$|k,s,+\rangle = -i\int d^4x e^{-ik\cdot x}\left\{\bar{\psi}(x)(i\gamma^\mu\overleftarrow{\partial}_\mu + m)U(k,s)\right\}|0\rangle \quad (5.39)$$

Similarly we can have

$$\langle k,s,+| = \langle 0|a(k,s) = \int d^3x e^{ik\cdot x}\bar{U}(k,s)\gamma^0\langle 0|\psi(x) \tag{5.40}$$

$$\simeq \int d^4x \partial_0 \left\{ e^{ik\cdot x}\bar{U}(k,s)\gamma^0\langle 0|\partial_0\psi(x)\right\}$$

$$= \int d^4x \left\{ \left(\partial_0 e^{ik\cdot x}\right)\bar{U}(k,s)\gamma^0\langle 0|\psi(x) + e^{ik\cdot x}\bar{U}(k,s)\gamma^0\langle 0|\partial_0\psi(x)\right\} \tag{5.41}$$

From DE for $E < 0$:

$$(i\gamma^0\partial_0 + i\gamma^i\partial_i + m)e^{ik\cdot x} = 0 \mapsto \partial_0 e^{ik\cdot x} = -i\gamma^0(-i\gamma^i\partial_i - m)e^{ik\cdot x}$$

Then we have from (5.4)

$$\langle k,s,+| = -i\int d^4x e^{ik\cdot x}\bar{U}k,s)(i\gamma^\mu\partial_\mu - m)\langle 0|\psi(x) \tag{5.42}$$

In case of anti-particle state

$$|k,s,-\rangle = b^\dagger(k,s)|0\rangle = \int d^3x e^{ik\cdot x}\bar{V}(k,s)\gamma^0\psi(x)|0\rangle \tag{5.43}$$

$$\simeq \int d^4x \partial_0 \left\{ e^{ik\cdot x}\bar{V}(k,s)\gamma^0\psi(x)\right\}|0\rangle$$

$$= \int d^4x \left\{ \left(\partial_0 e^{ik\cdot x}\right)\bar{V}(k,s)\gamma^0\psi(x)|0\rangle + e^{ik\cdot x}\bar{V}(k,s)\gamma^0\partial_0\psi(x)|0\rangle\right\}$$

$$= -i\int d^4x e^{ik\cdot x}\bar{V}(k,s)(i\gamma^\mu\partial_\mu - m)\psi(x)|0\rangle \tag{5.44}$$

and

$$\langle k,s,-|b(k,s) = -i\int d^4x e^{-ik\cdot x}\langle 0|\bar{\psi}(x)(i\gamma^\mu\overleftarrow{\partial}_\mu + m)V(k,s) \tag{5.45}$$

For generic S-matrix of 2-particle to 2-particle scattering we can have its LSZ reduction formula in the form

$$\begin{aligned} S^{++,++}_{\alpha\beta} &\equiv \langle (k_1,s_1',+),(k_2,s_2',+),\beta|(p_1,s,+),(p_2,s,+),\alpha\rangle \\ = (-i)^4 &\int d^4x_1 \int d^4x_2 \int d^4y_1 \int d^4y_2 e^{-ip_1\cdot x_1 - ip_2\cdot x_2 + iy_1\cdot y_1 + iy_2\cdot y_2} \\ &\times \bar{U}(q_2,s_2')\bar{U}(q_1,s_1')(i\partial\!\!\!/_{y_1} - m_3)(i\partial\!\!\!/_{y_1} - m_3) \\ &\times \langle 0|T[\psi(y_1)\psi(y_2)\bar{\psi}(x_1)\bar{\psi}(x_2)]|0\rangle \\ &\times (i\overleftarrow{\partial\!\!\!/}_{x_1} + m_1)(i\overleftarrow{\partial\!\!\!/}_{x_2} + m_2)U(p_1,s_1)U(p_2,s_2) \end{aligned} \tag{5.46}$$